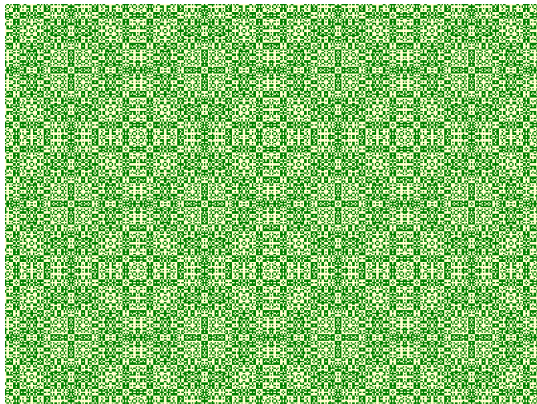


Hidden Patterns in Functions

Brian Heinold

Mount St. Mary's University



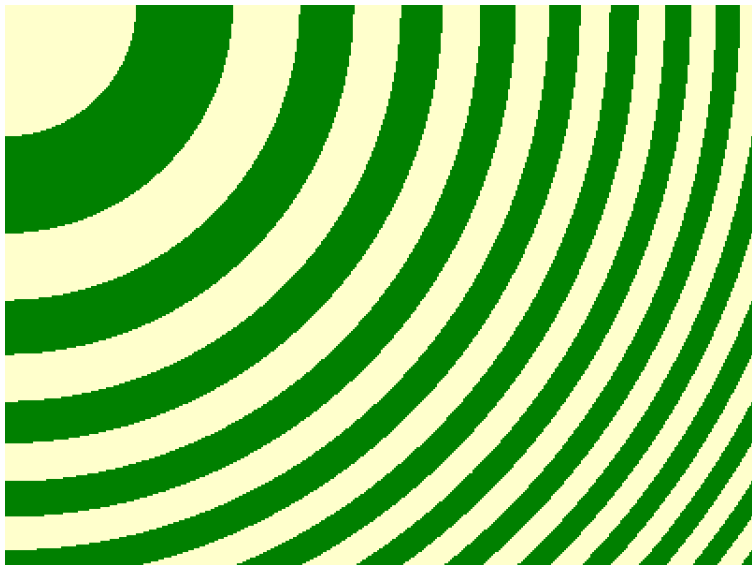
Given a function $f(x, y)$

Fix $a, b \in \mathbb{R}$. Lattice of points $\{(ja, kb) : j, k \in \mathbb{Z}^+\}$

- 1 Compute $f(x, y)$ at each lattice point
- 2 Round the result to the nearest integer
- 3 Mod this result by 2

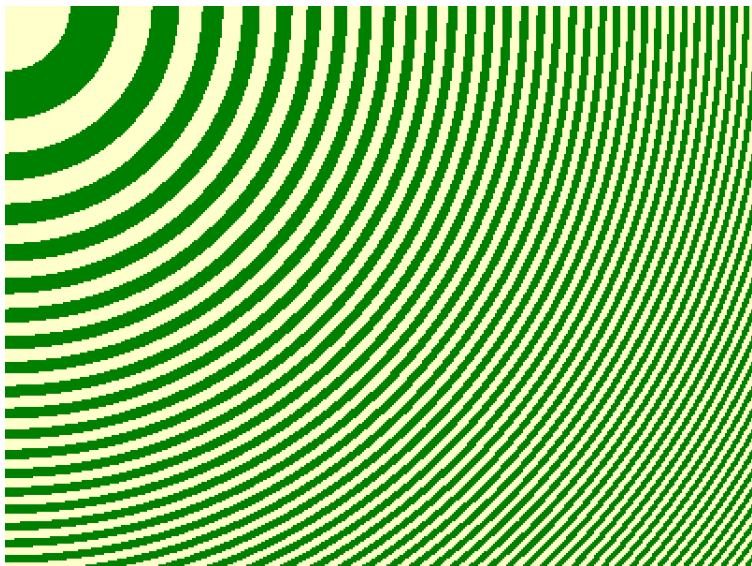
$$x^2 + y^2$$

0.01 0.01 2



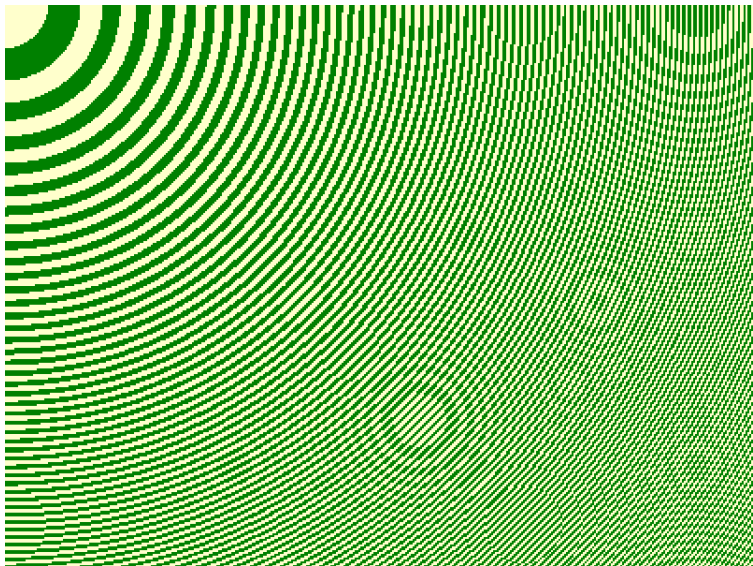
$$x^2 + y^2$$

0.02 0.02 2



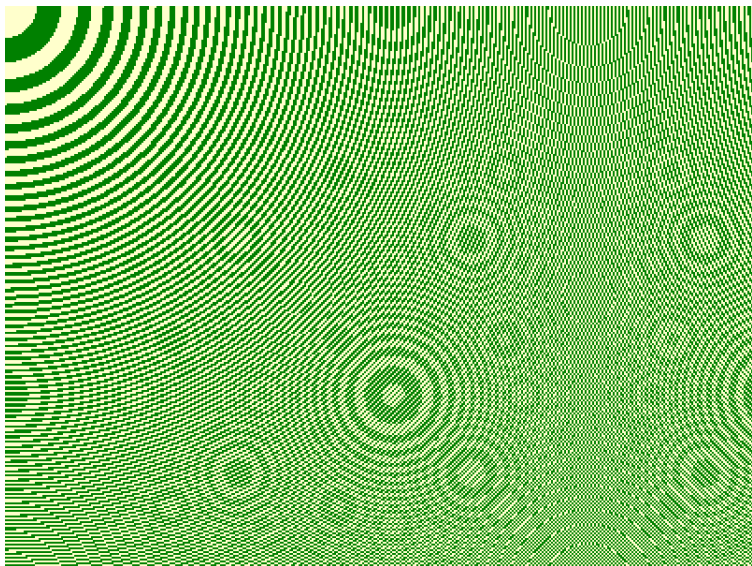
$$x^2 + y^2$$

0.03 0.03 2



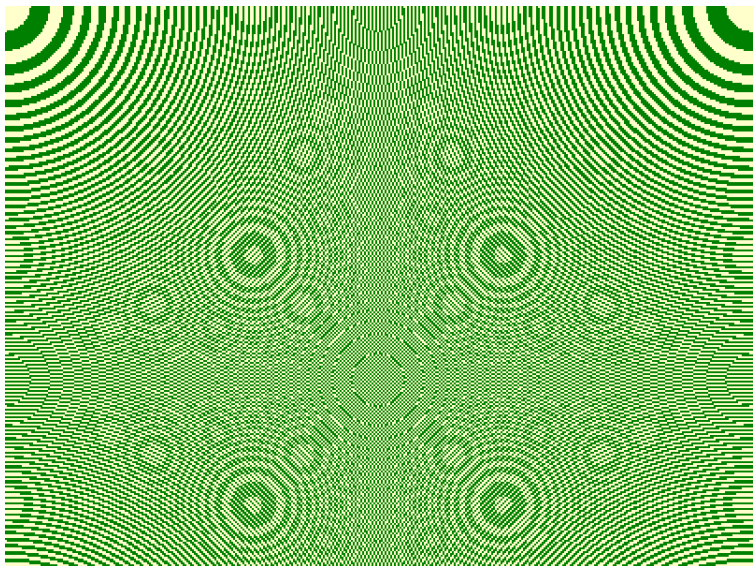
$$x^2 + y^2$$

0.04 0.04 2



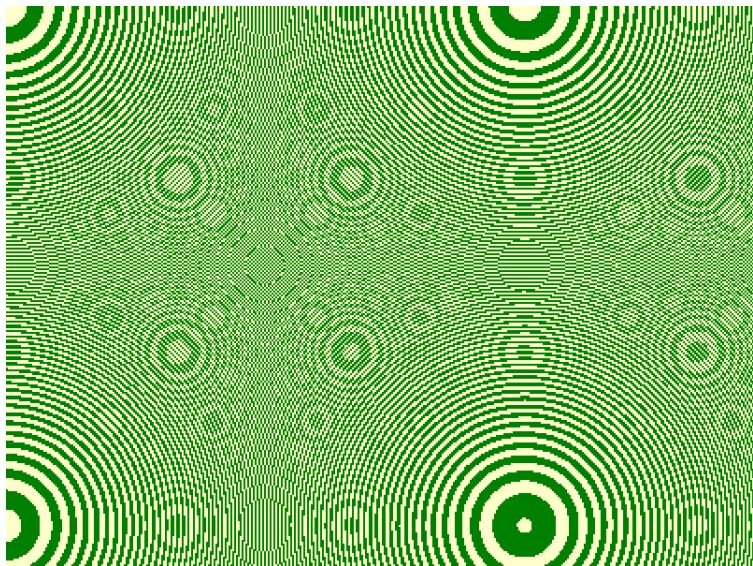
$$x^2 + y^2$$

0.05 0.05 2



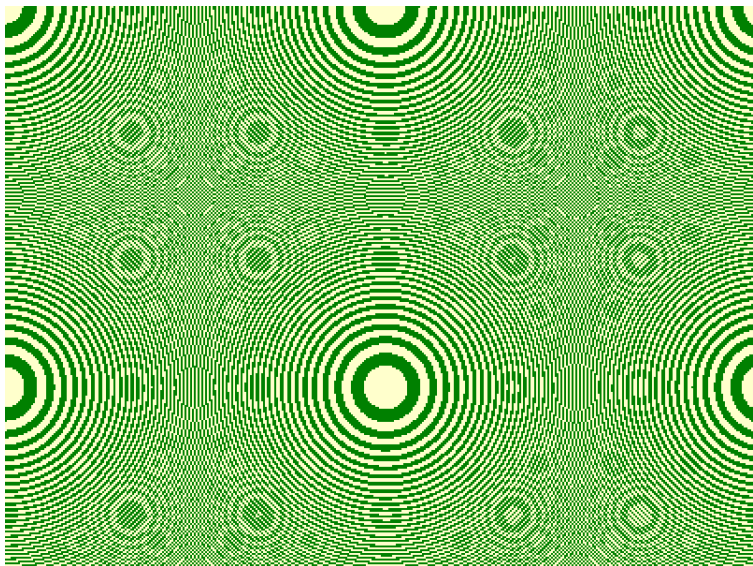
$$x^2 + y^2$$

0.06 0.06 2



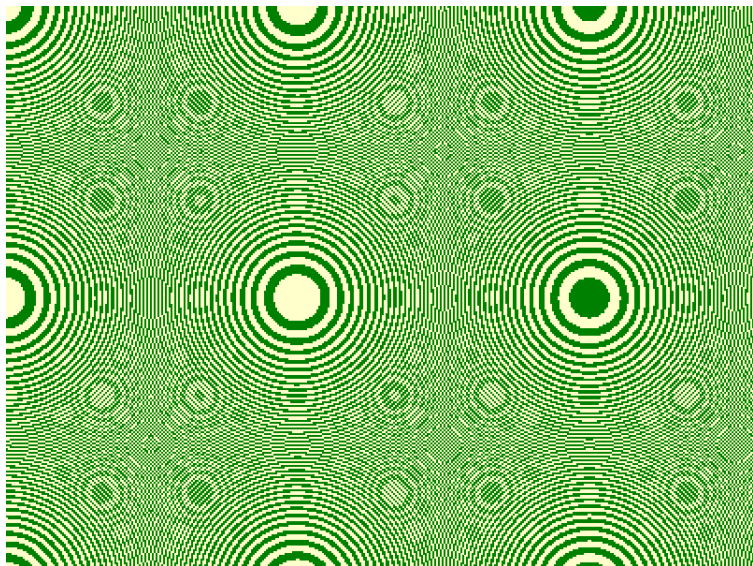
$$x^2 + y^2$$

0.07 0.07 2



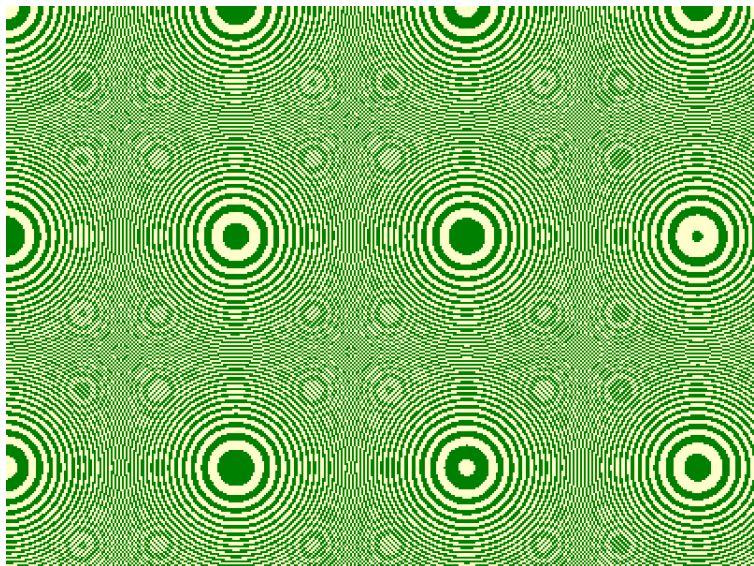
$$x^2 + y^2$$

0.08 0.08 2



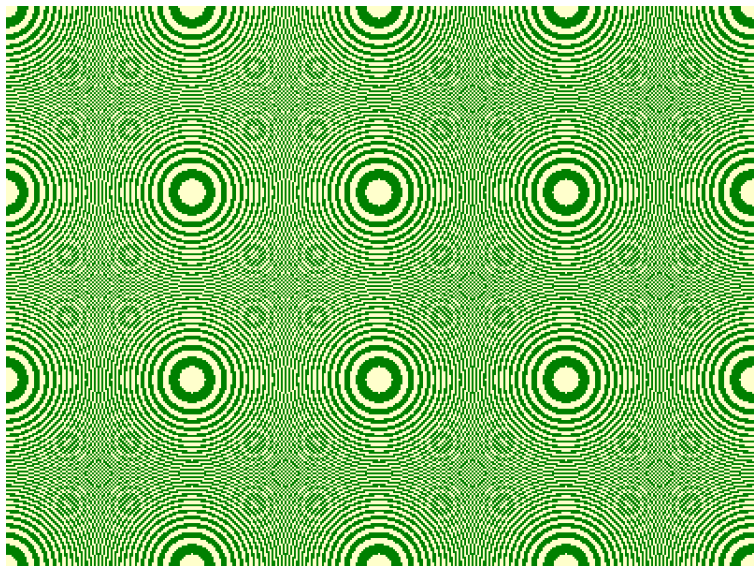
$$x^2 + y^2$$

0.09 0.09 2



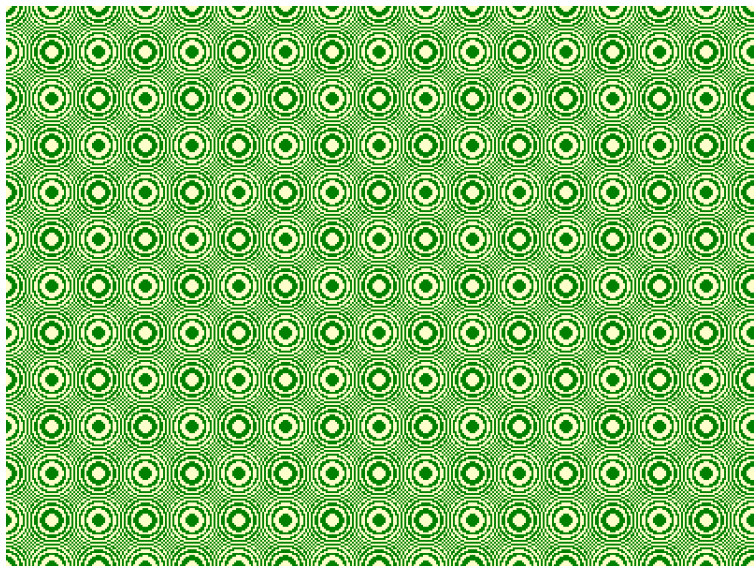
$$x^2 + y^2$$

0.1 0.1 2



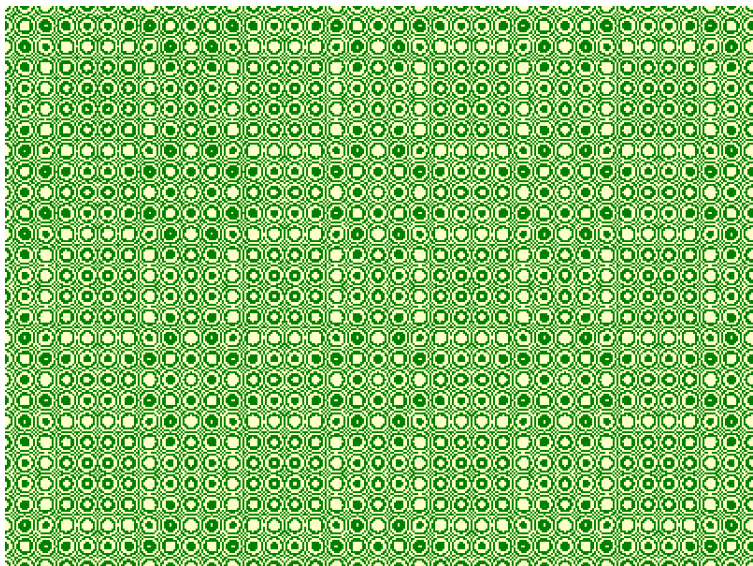
$$x^2 + y^2$$

0.2 0.2 2



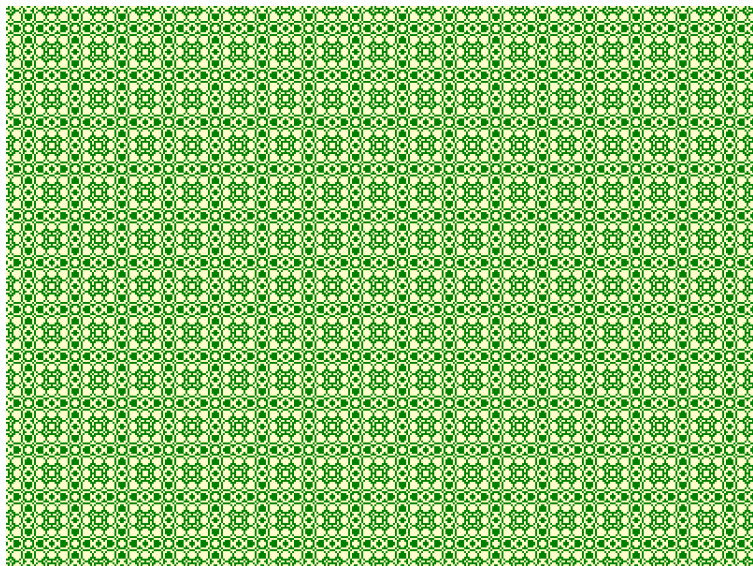
$$x^2 + y^2$$

0.3 0.3 2



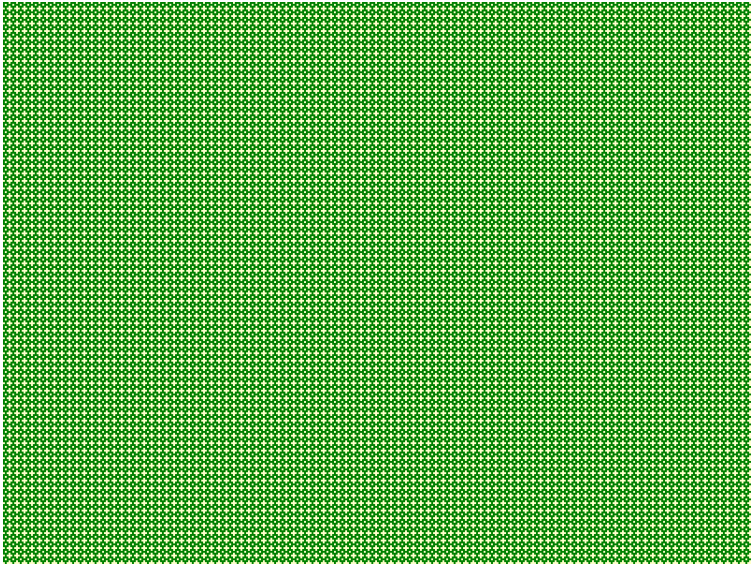
$$x^2 + y^2$$

0.4 0.4 2



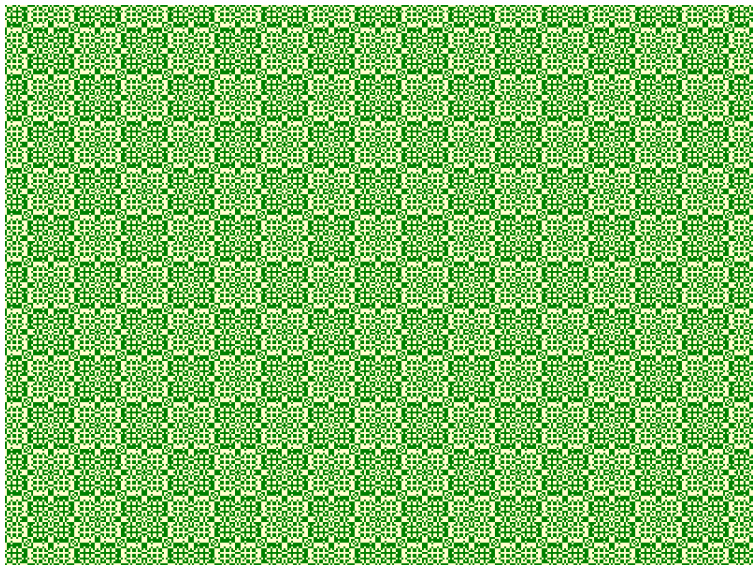
$$x^2 + y^2$$

0.5 0.5 2



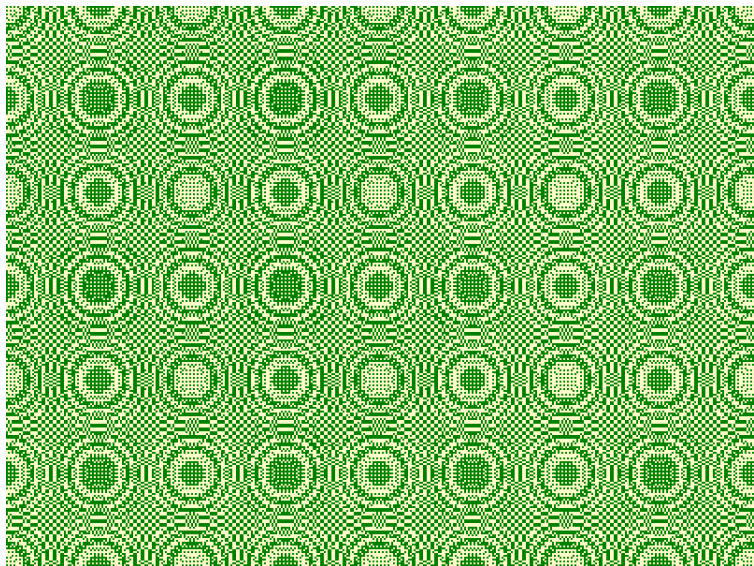
$$x^2 + y^2$$

0.6 0.6 2



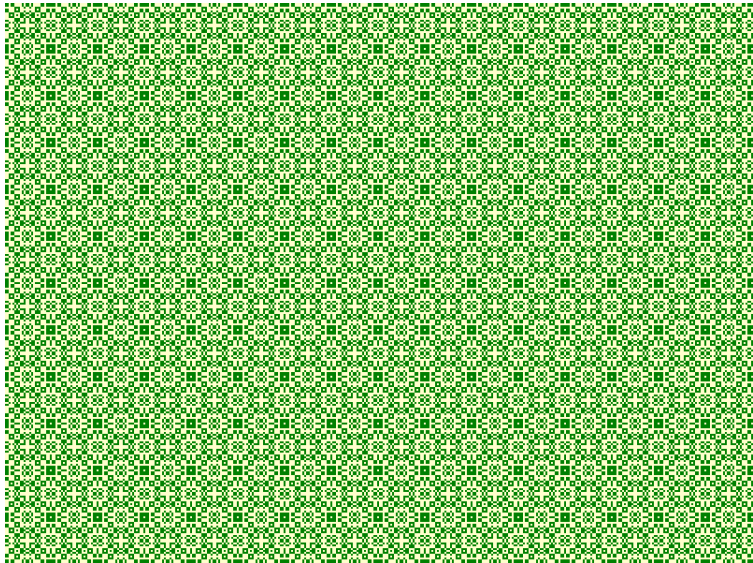
$$x^2 + y^2$$

0.7 0.7 2



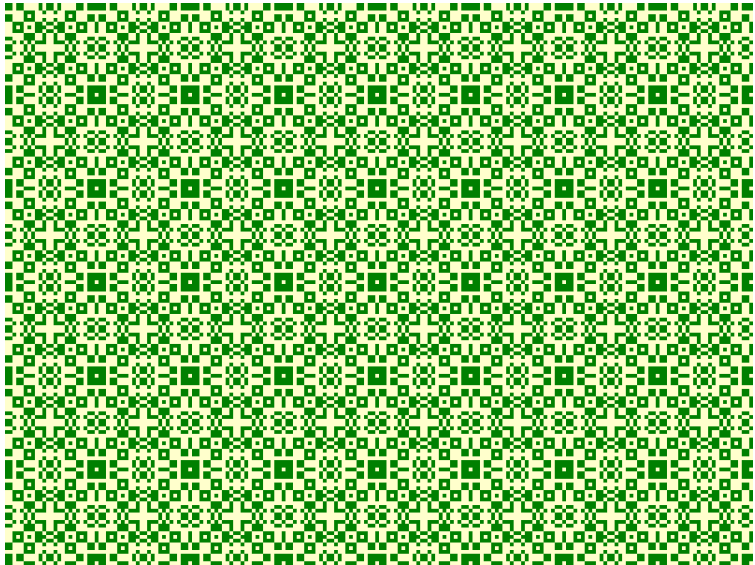
$$x^2 + y^2$$

0.8 0.8 2



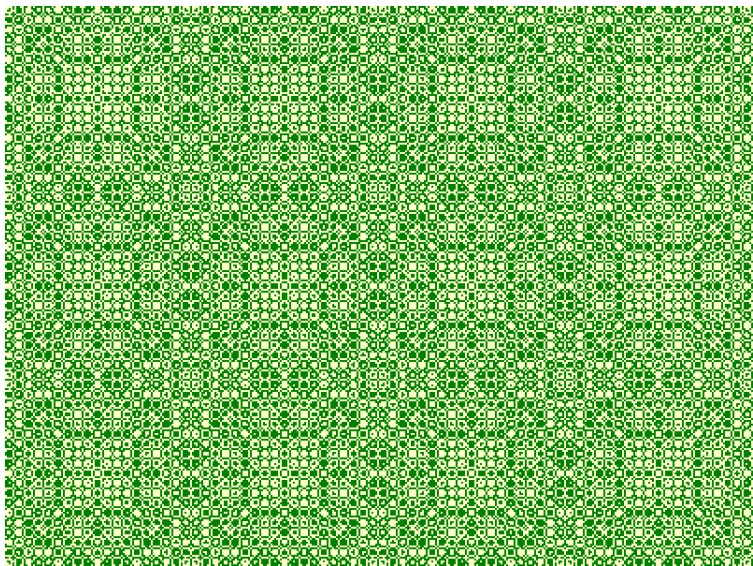
$$x^2 + y^2$$

0.8 0.8 4



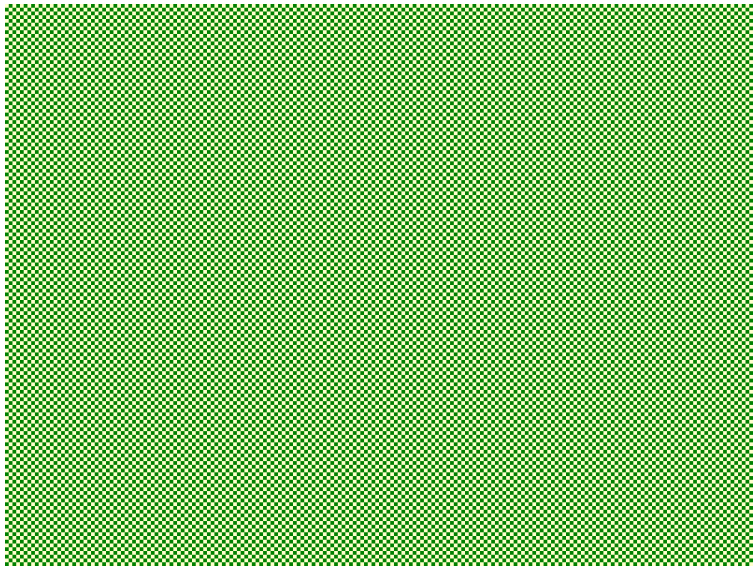
$$x^2 + y^2$$

0.9 0.9 2



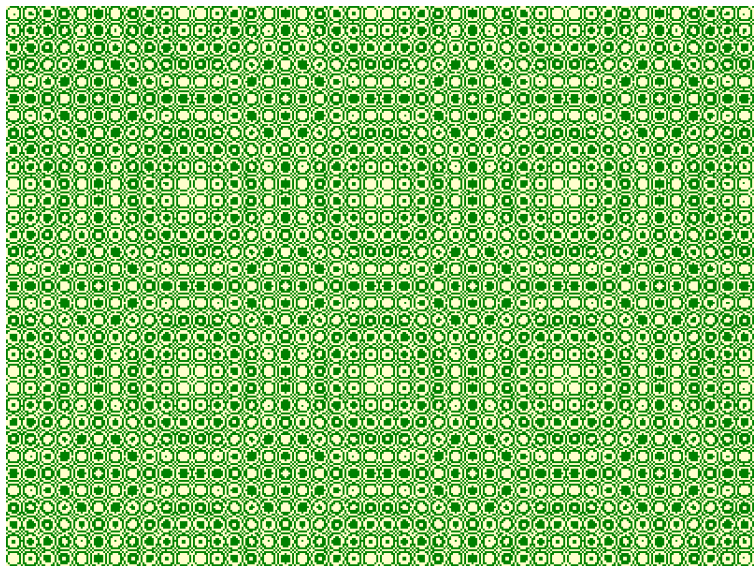
$$x^2 + y^2$$

1.0 1.0 4



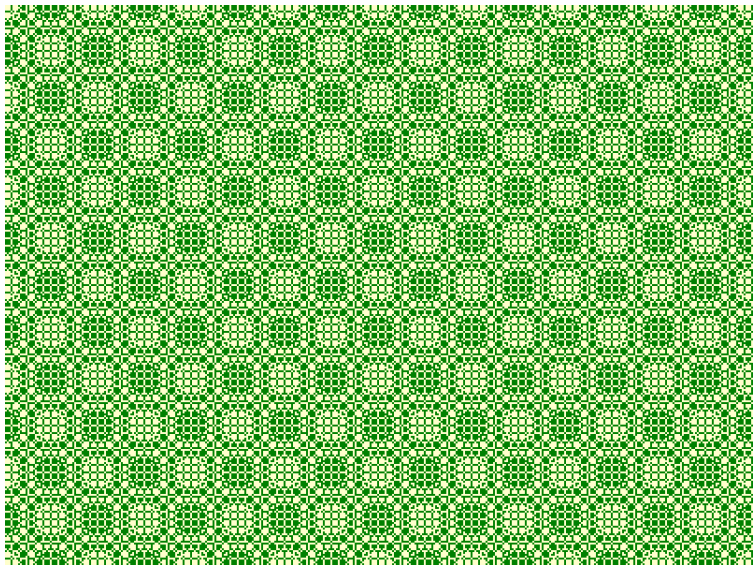
$$x^2 + y^2$$

1.7 1.7 2



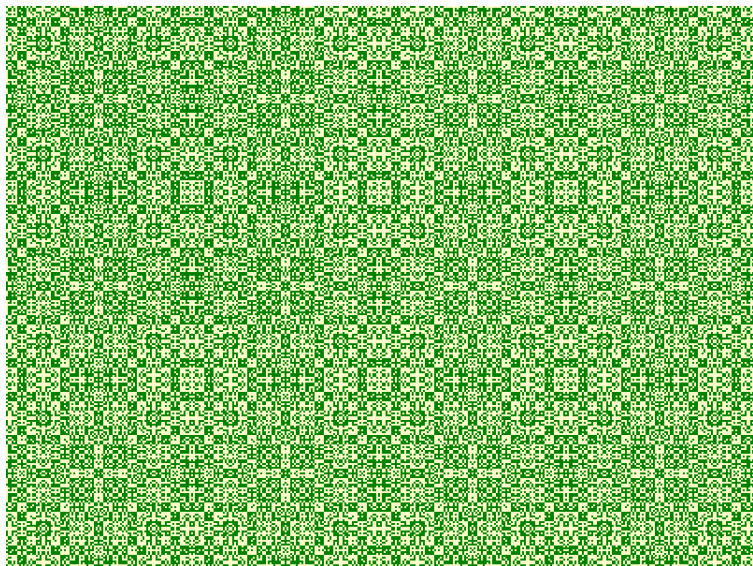
$$x^2 + y^2$$

1.8 1.8 2

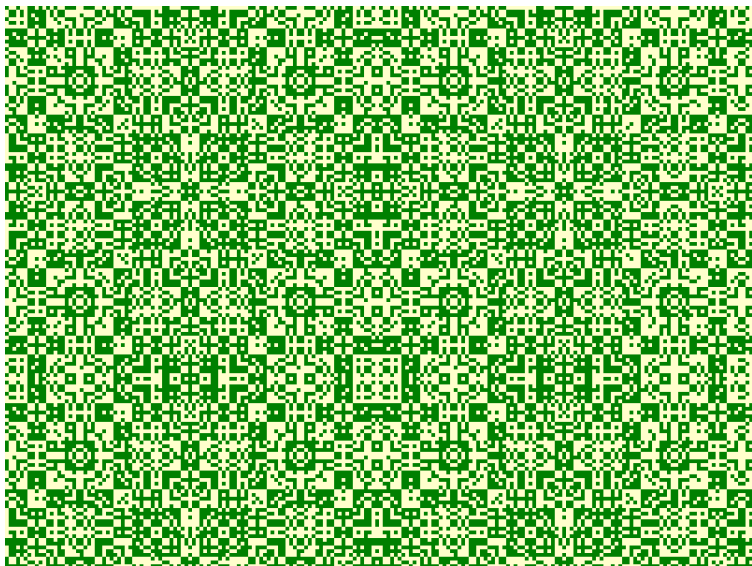


$$x^2 + y^2$$

1.9 1.9 2

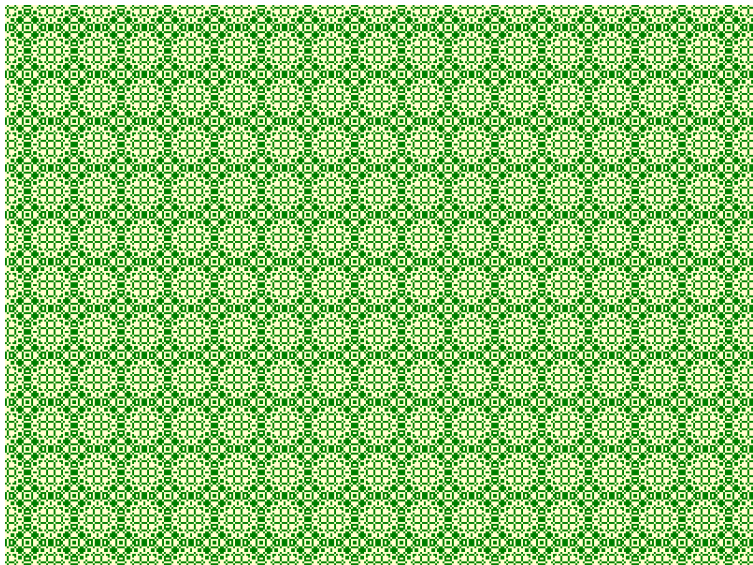


$$x^2 + y^2 \quad 1.9 \quad 1.9 \quad 4$$



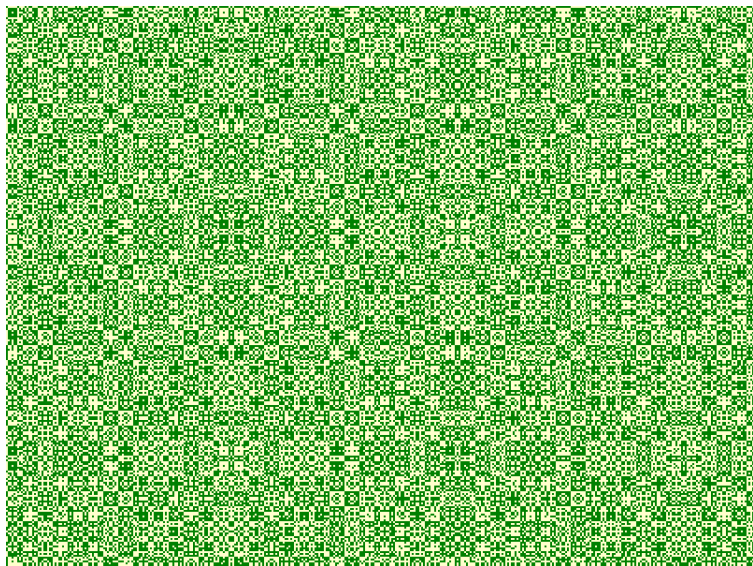
$$x^2 + y^2$$

2.4 2.4 2



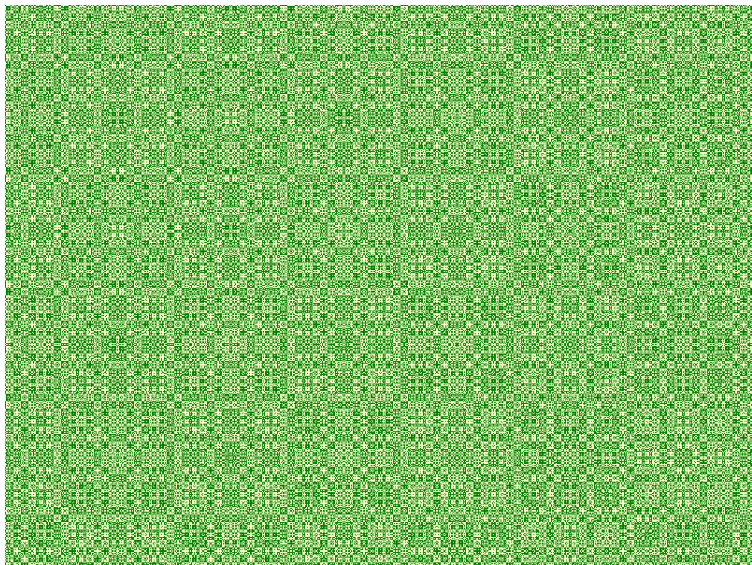
$$x^2 + y^2$$

3.41 3.41 2



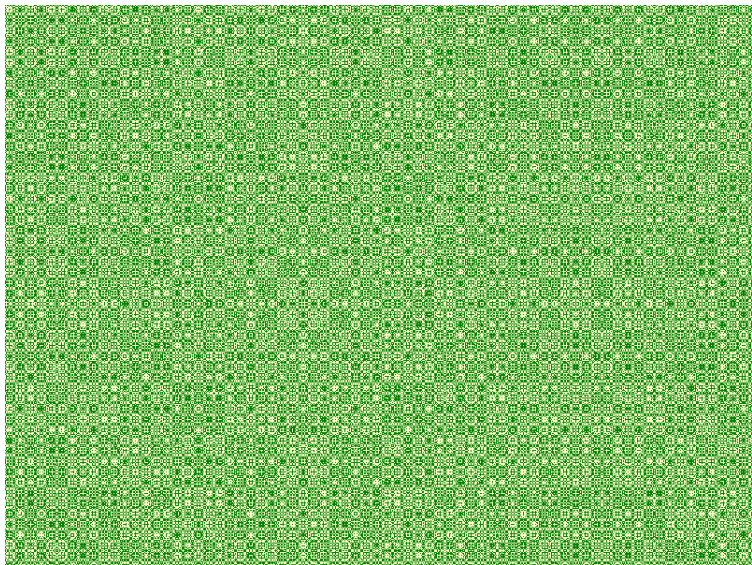
$$x^2 + y^2$$

3.41 3.41 1



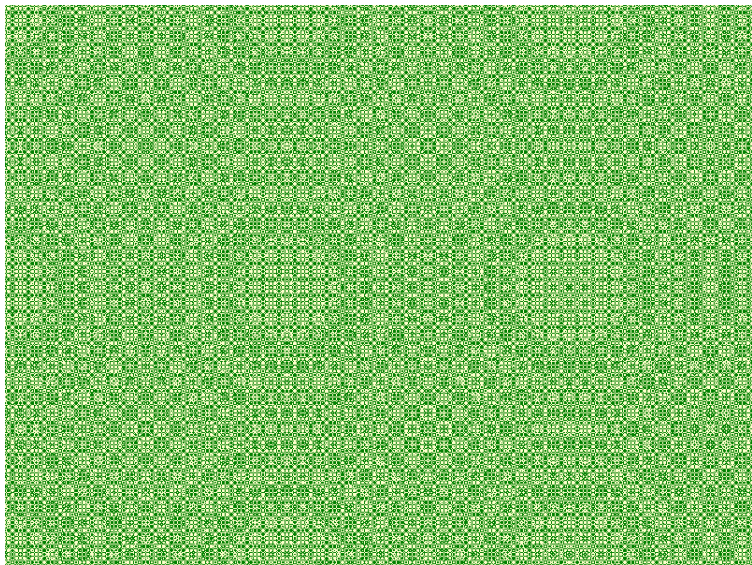
$$x^2 + y^2$$

3.42 3.42 1



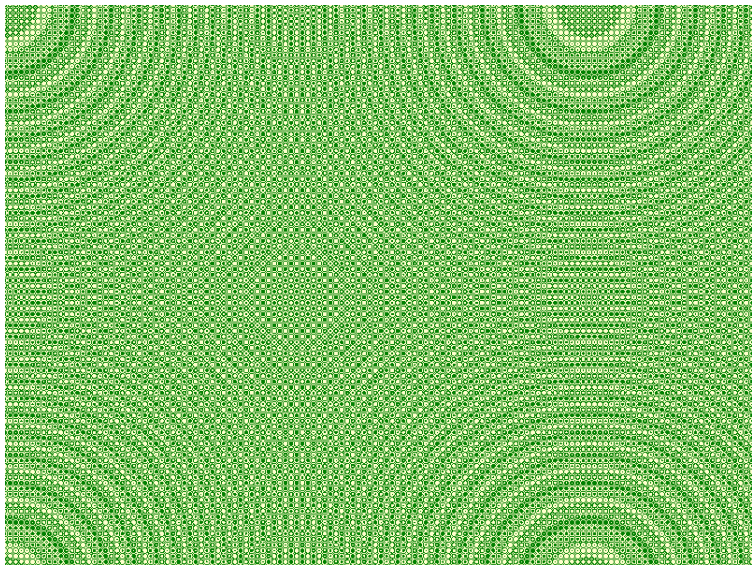
$$x^2 + y^2$$

3.43 3.43 1

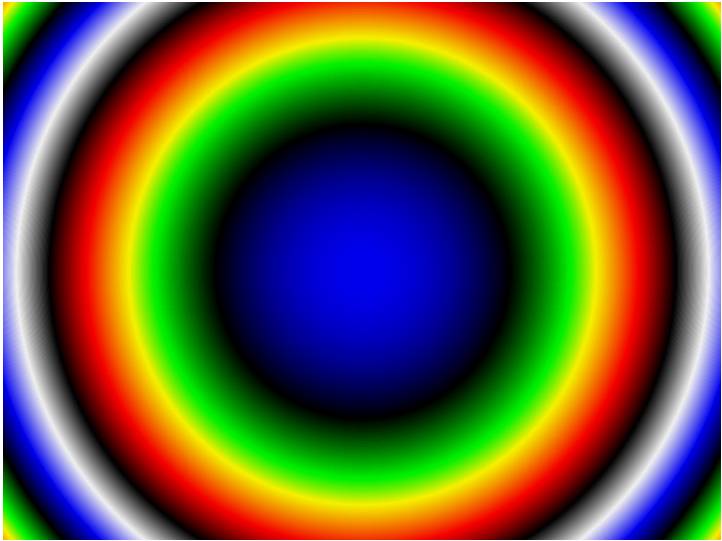


$$x^2 + y^2$$

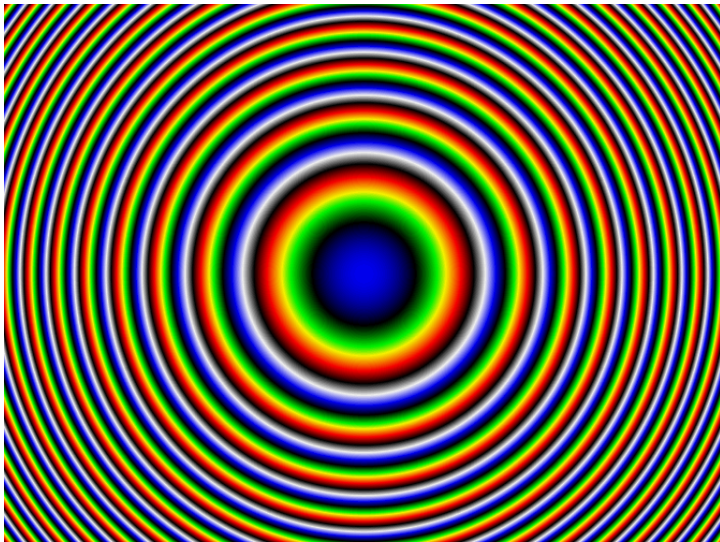
3.44 3.44 1



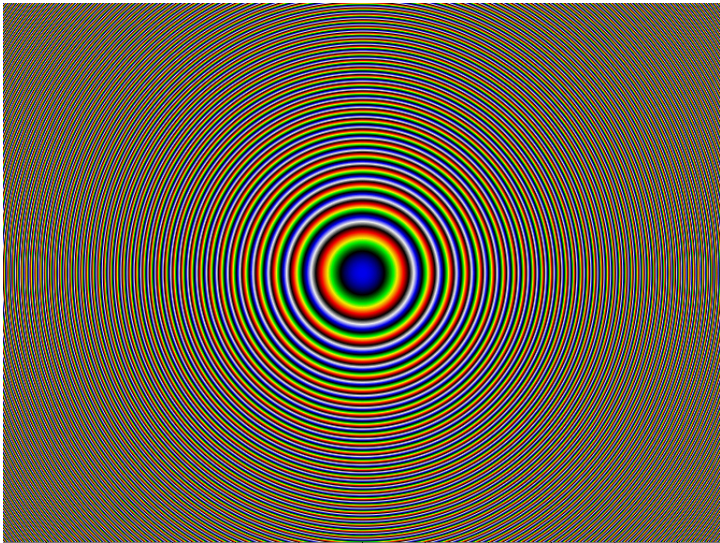
Contour map



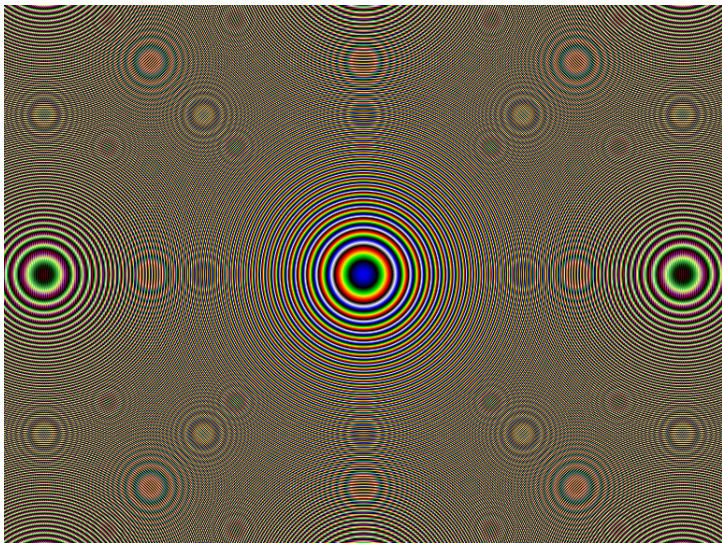
Contour map



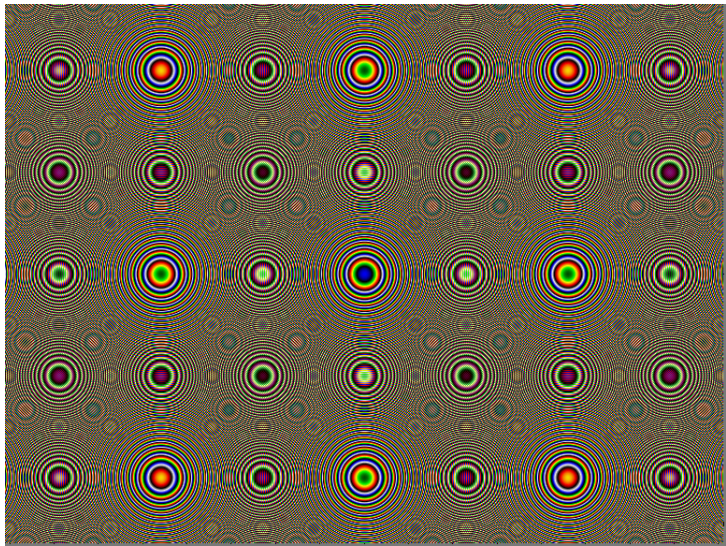
Contour map



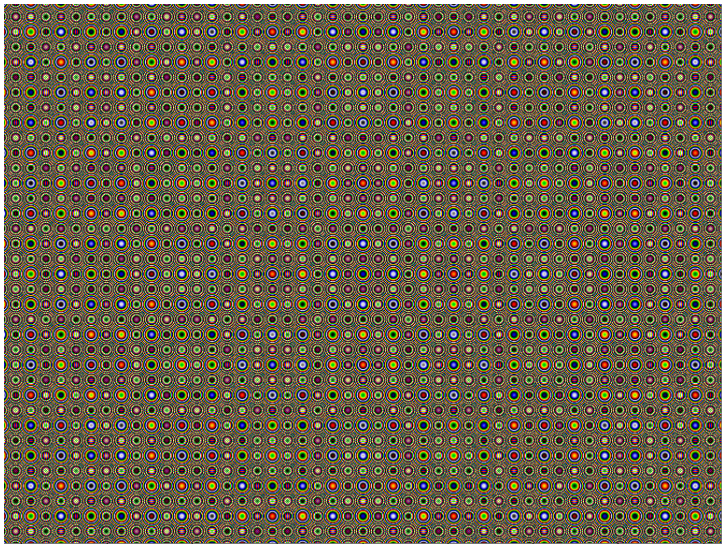
Contour map



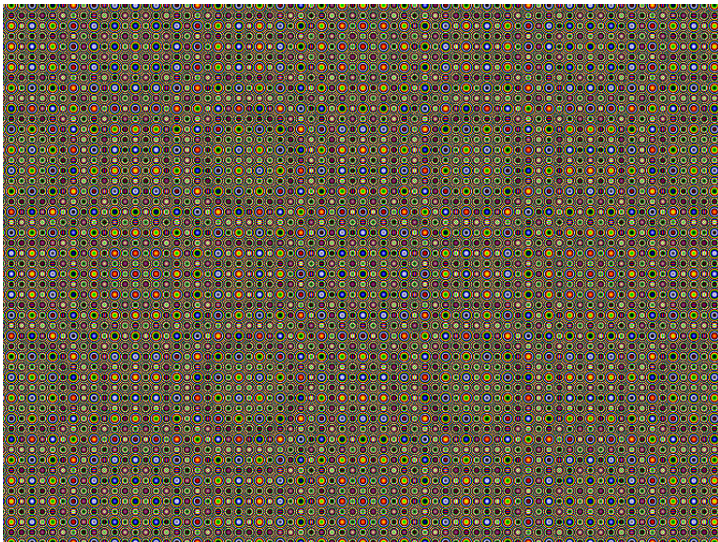
Contour map



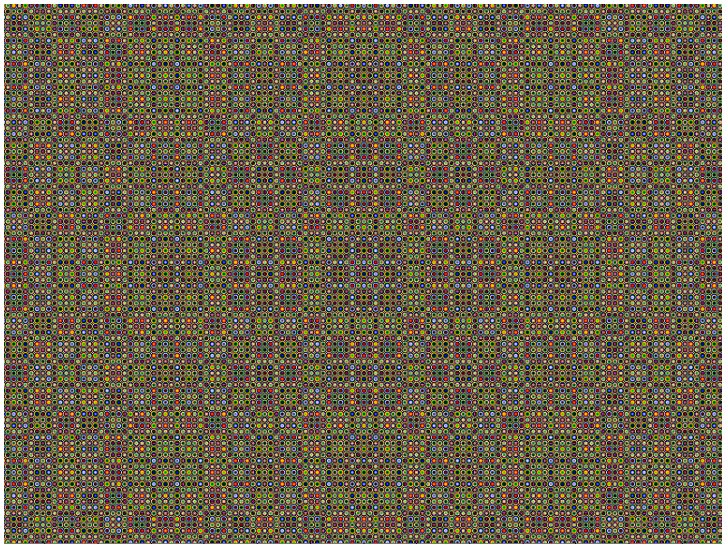
Contour map



Contour map

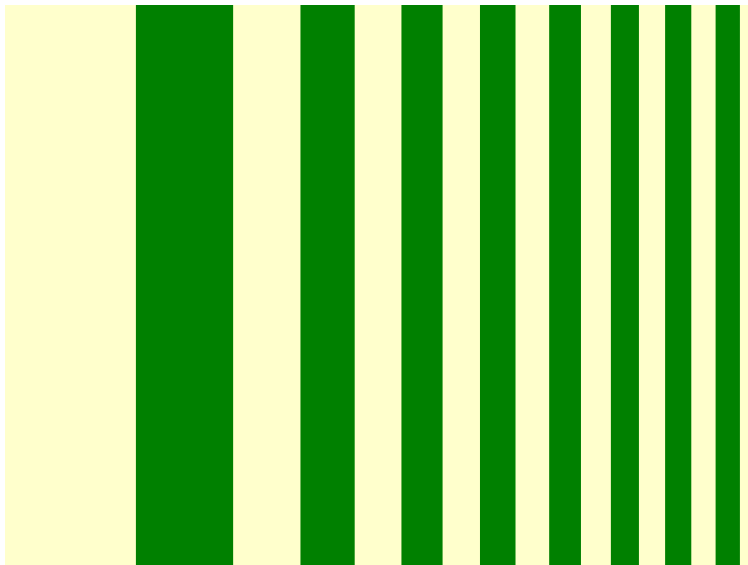


Contour map



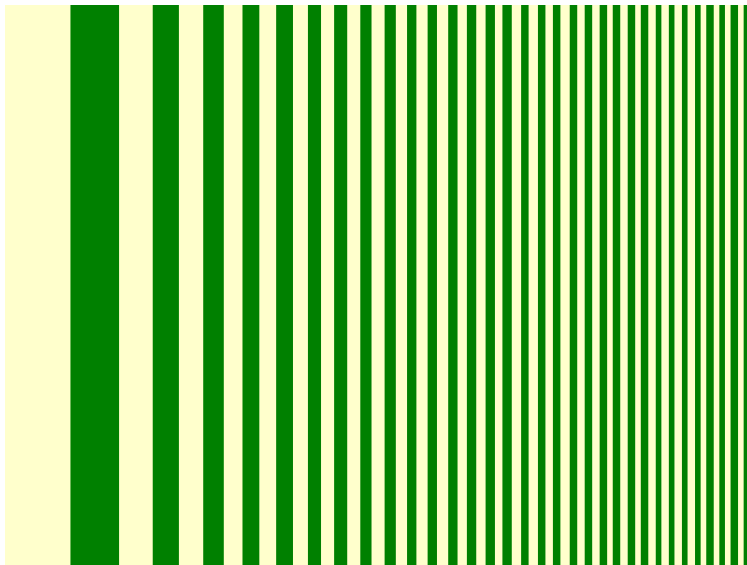
x^2

0.01 0.0 2



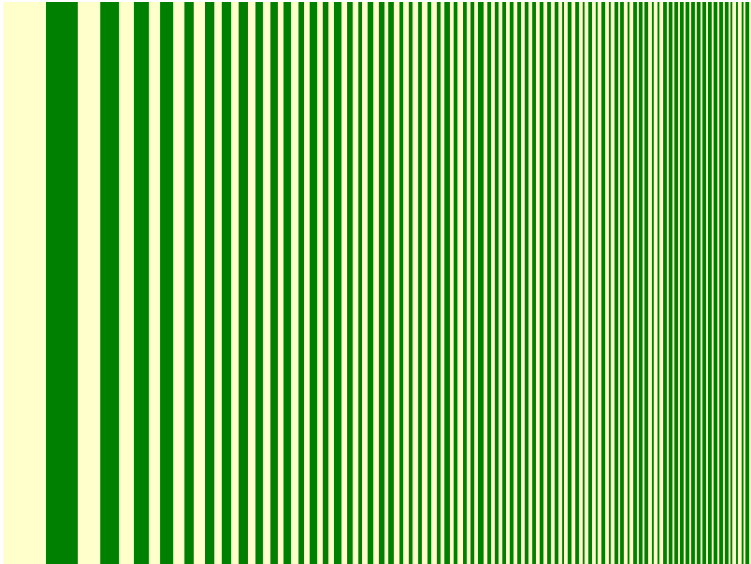
x^2

0.02 0.0 2



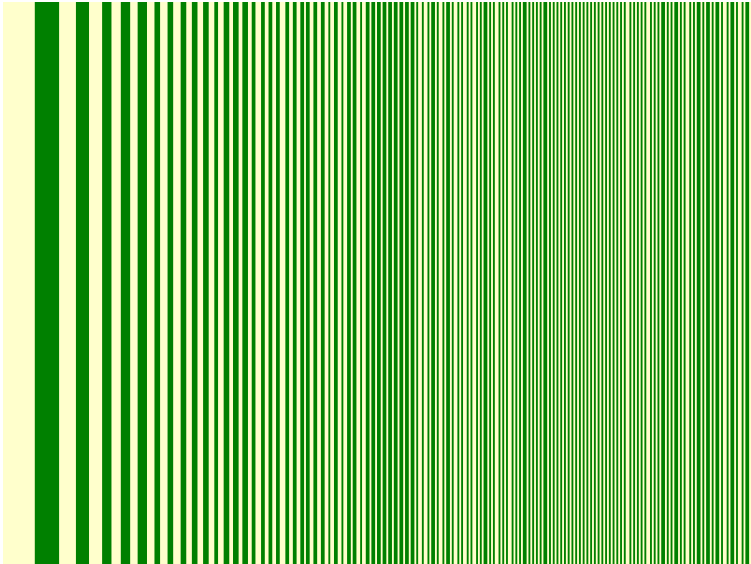
x^2

0.03 0.0 2



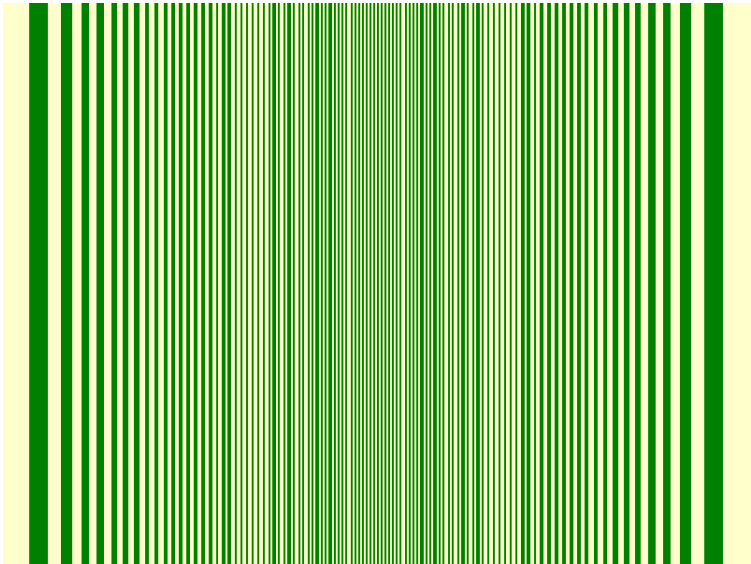
x^2

0.04 0.0 2



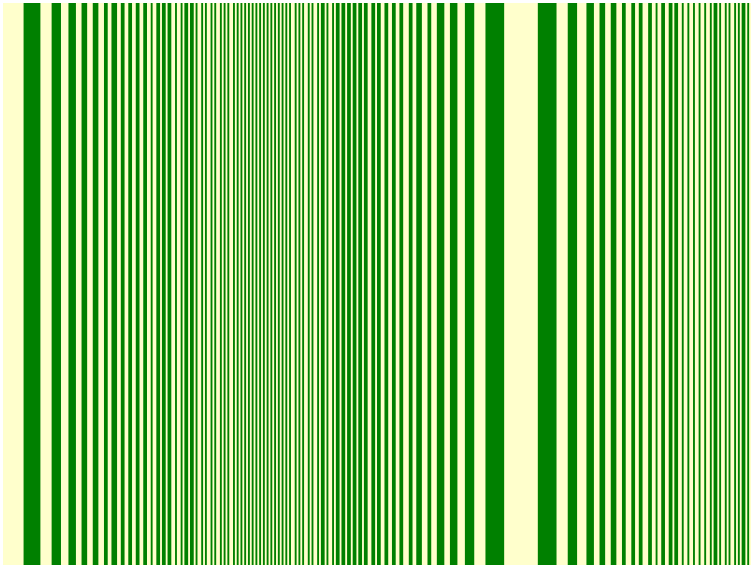
x^2

0.05 0.0 2



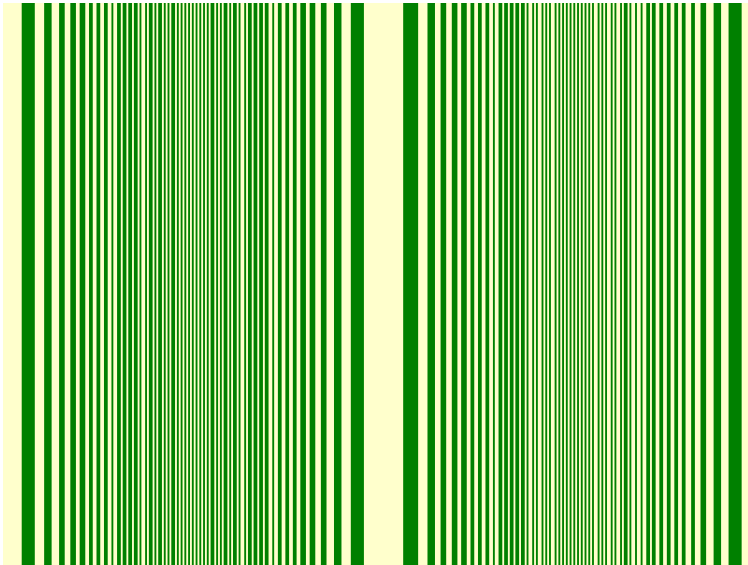
x^2

0.06 0.0 2



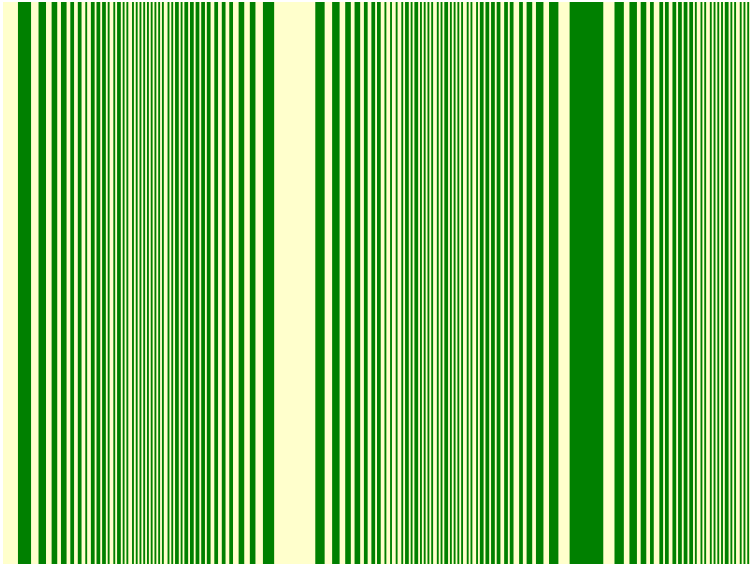
x^2

0.07 0.0 2



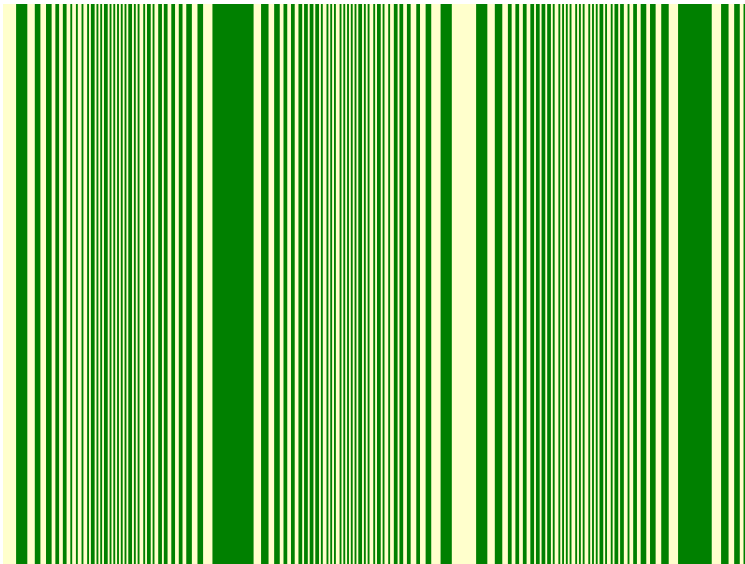
x^2

0.08 0.0 2



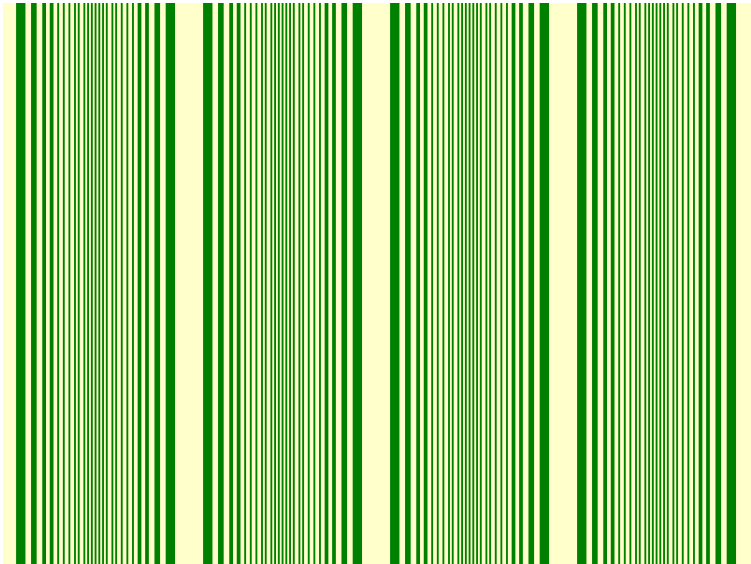
x^2

0.09 0.0 2



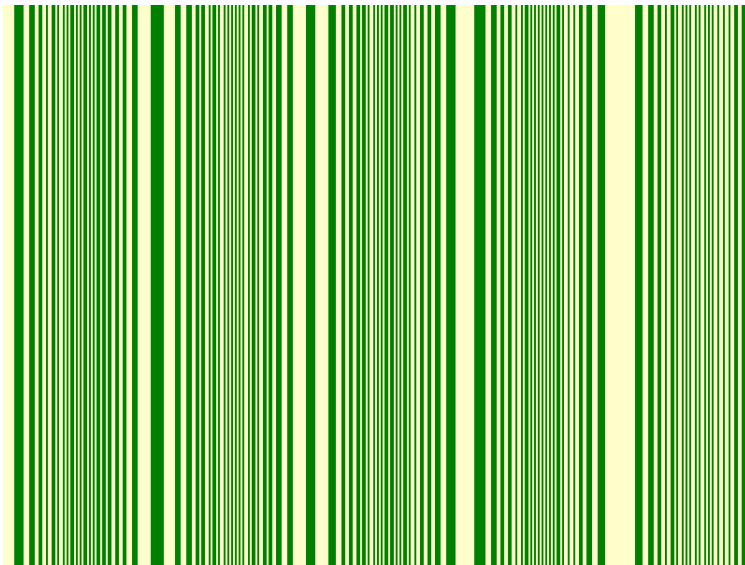
x^2

0.1 0.0 2



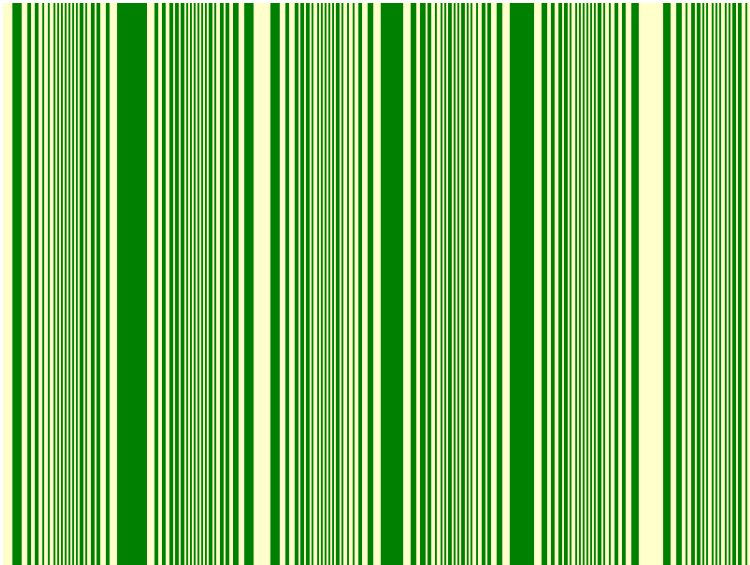
x^2

0.11 0.0 2



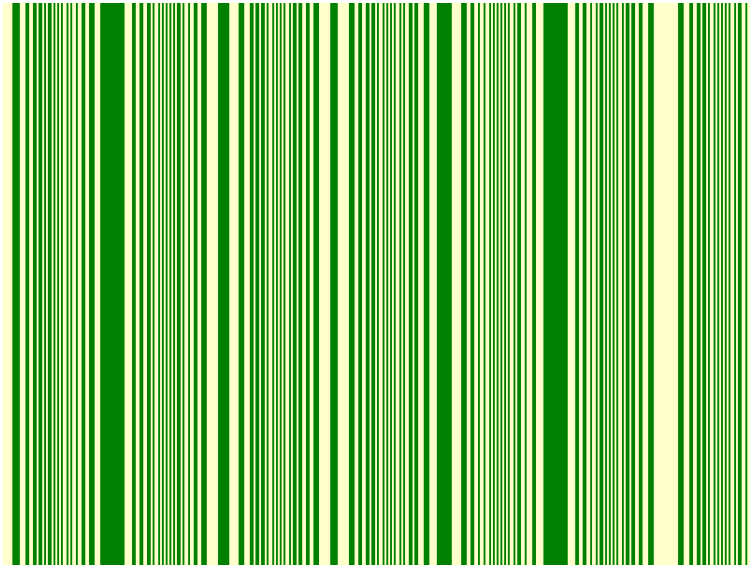
x^2

0.12 0.0 2



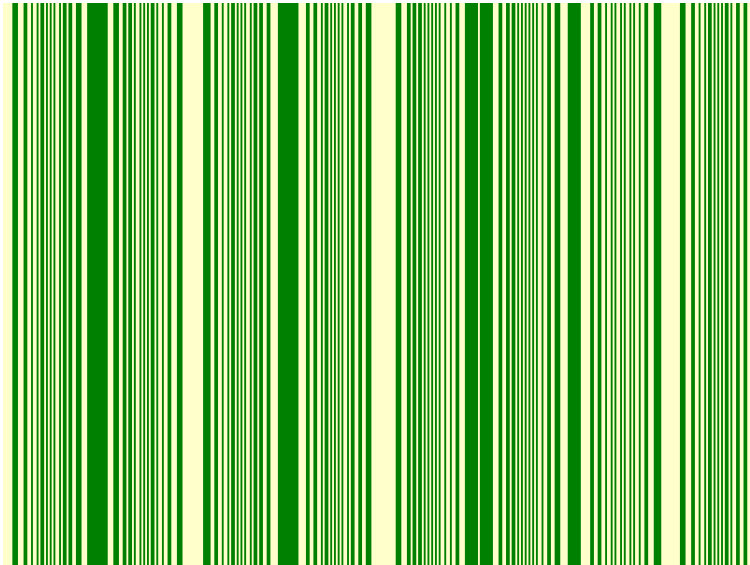
x^2

0.13 0.0 2



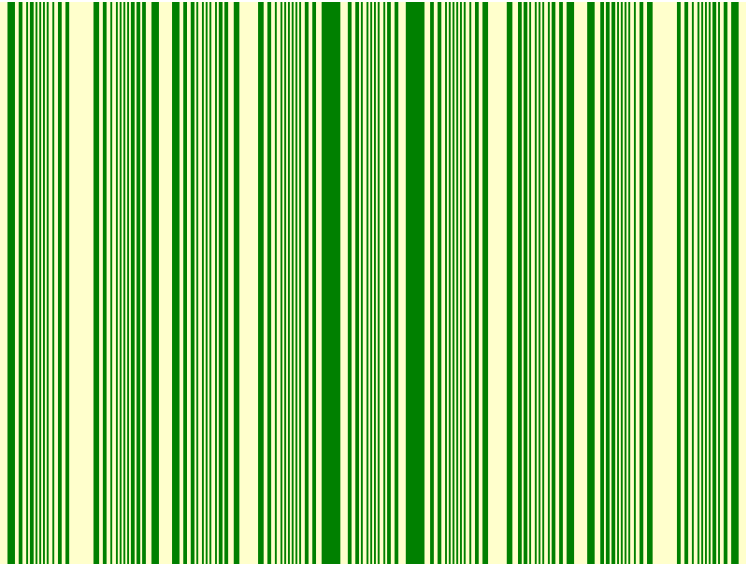
x^2

0.14 0.0 2



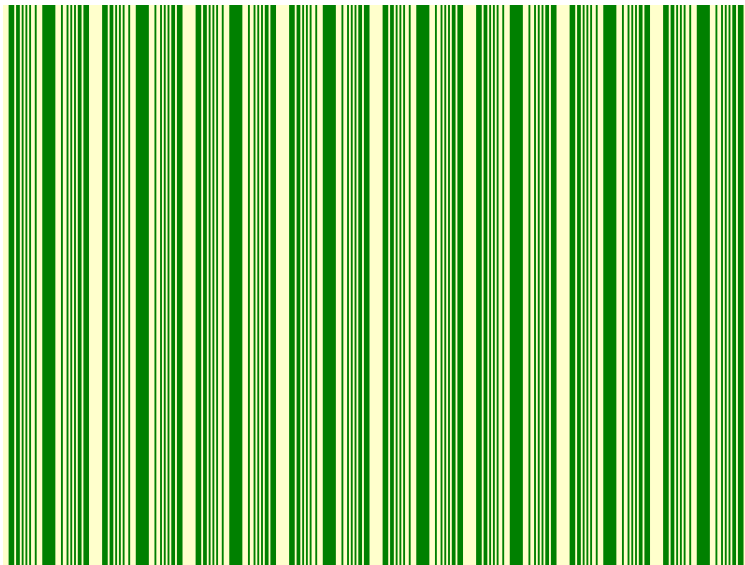
x^2

0.15 0.0 2



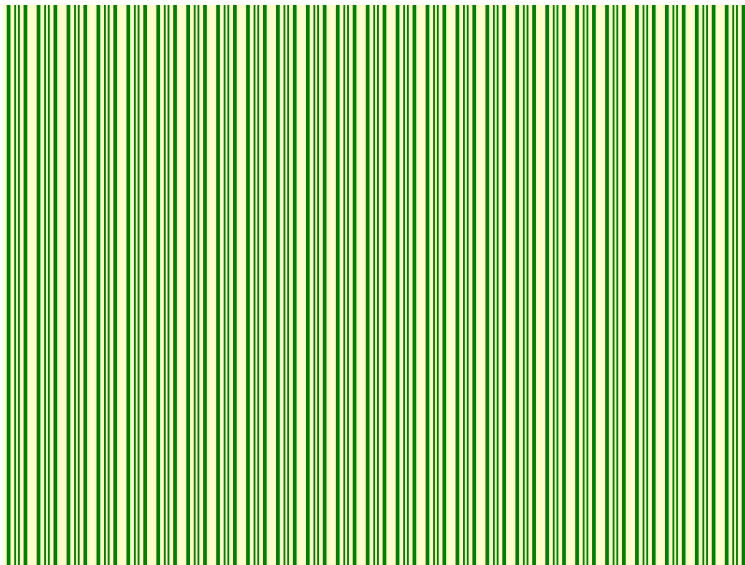
x^2

0.2 0.0 2



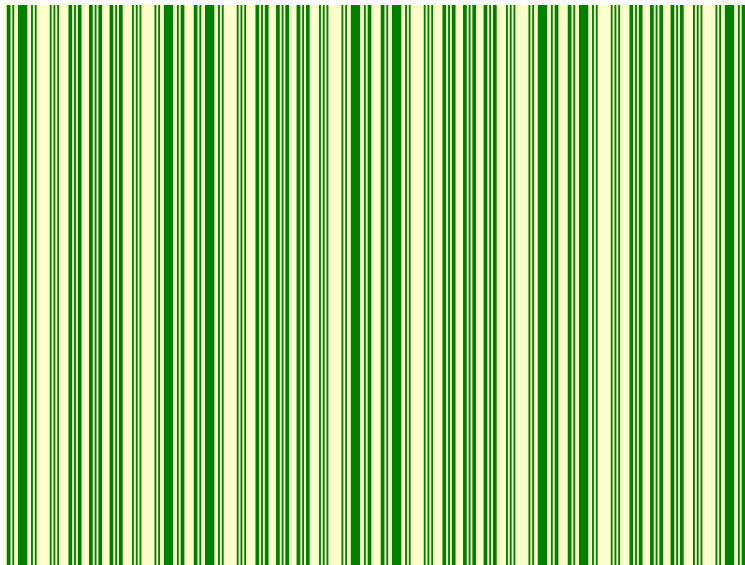
x^2

0.25 0.0 2



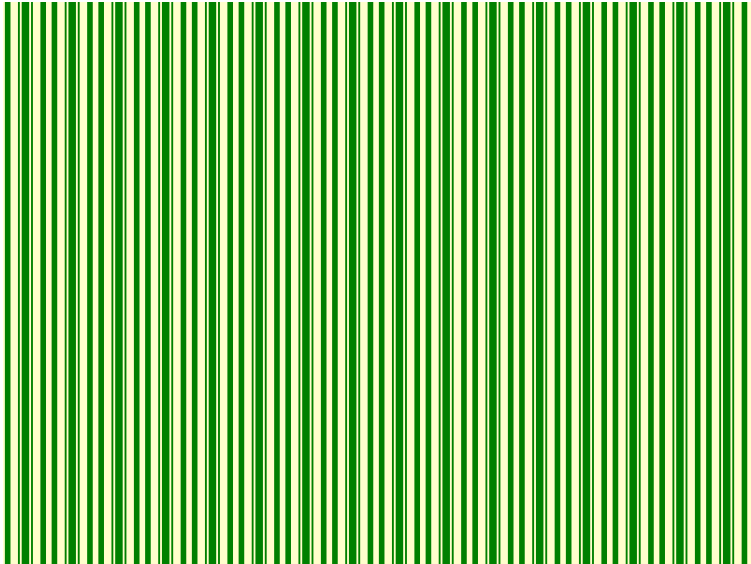
x^2

0.3 0.0 2



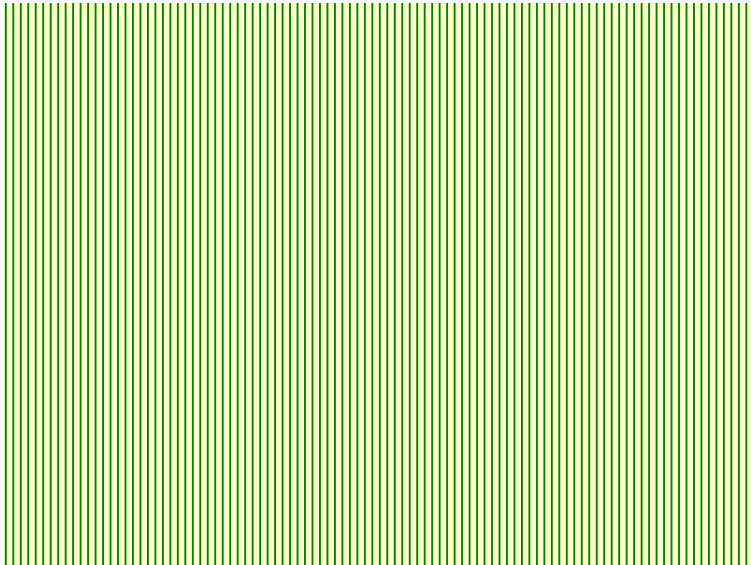
x^2

0.4 0.0 2



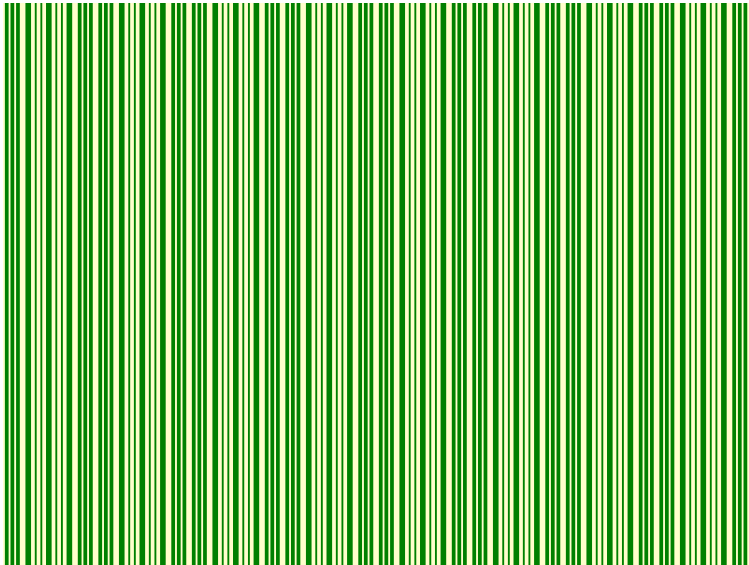
x^2

0.5 0.0 2



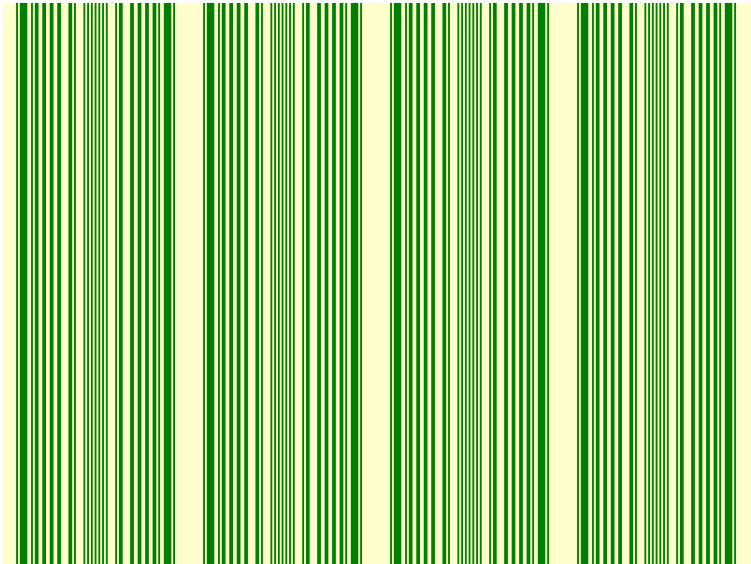
x^2

0.6 0.0 2



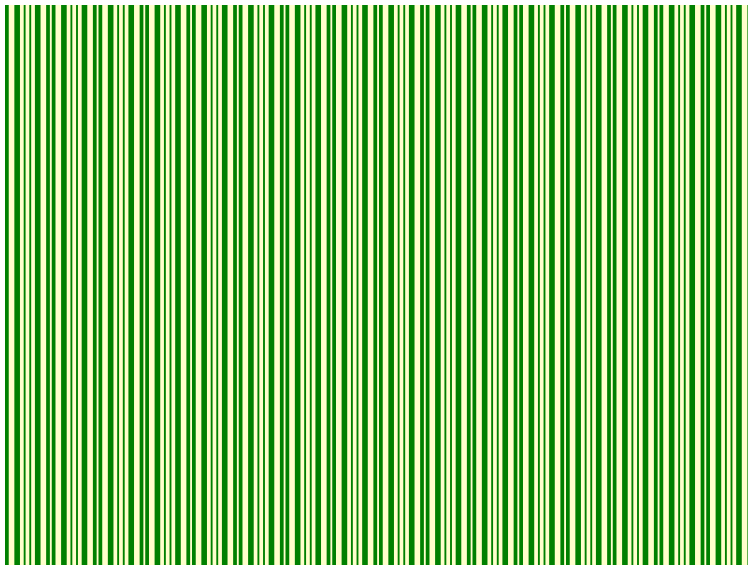
x^2

0.7 0.0 2



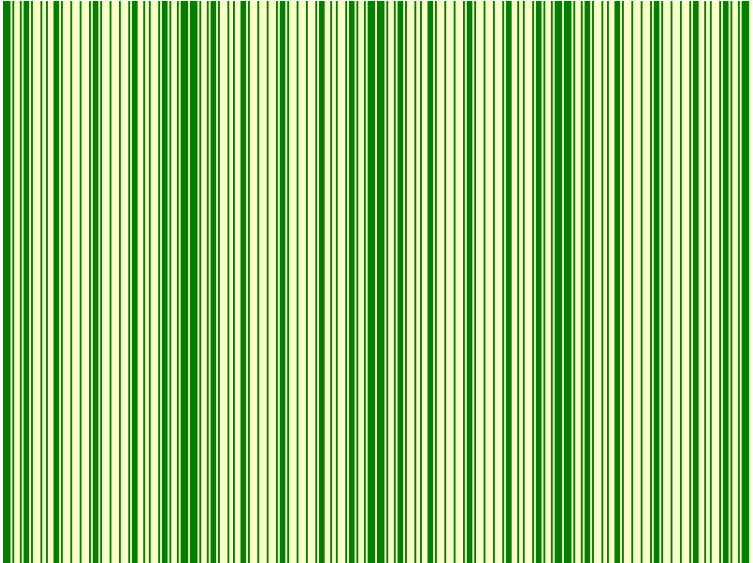
x^2

0.8 0.0 2



x^2

0.9 0.0 2



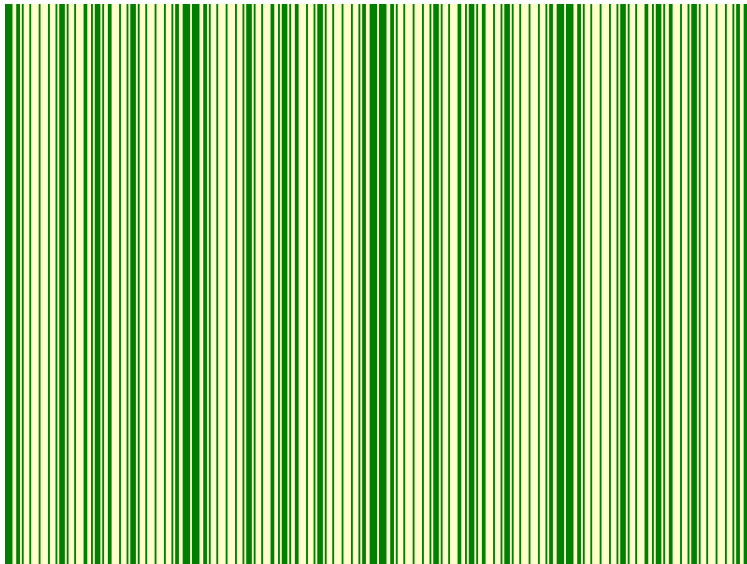
x^2

1.0 0.0 2



x^2

1.1 0.0 2



Let increment be p/q in lowest terms

Period: $\alpha = q^2$ if one of p, q even, and $\alpha = 2q^2$ otherwise

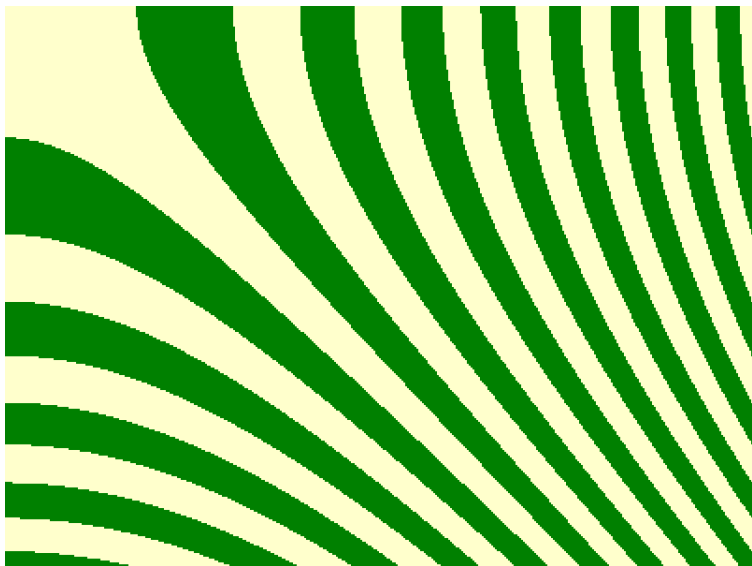
$$\left(k\frac{p}{q} + \alpha\frac{p}{q}\right)^2 - \left(k\frac{p}{q}\right)^2 = 2k\frac{p^2}{q^2}\alpha + \alpha^2\frac{p^2}{q^2}$$

Want this to be an integer divisible by 2

This means $q^2|\alpha$, and if p, q are both odd, then $2|\alpha$.

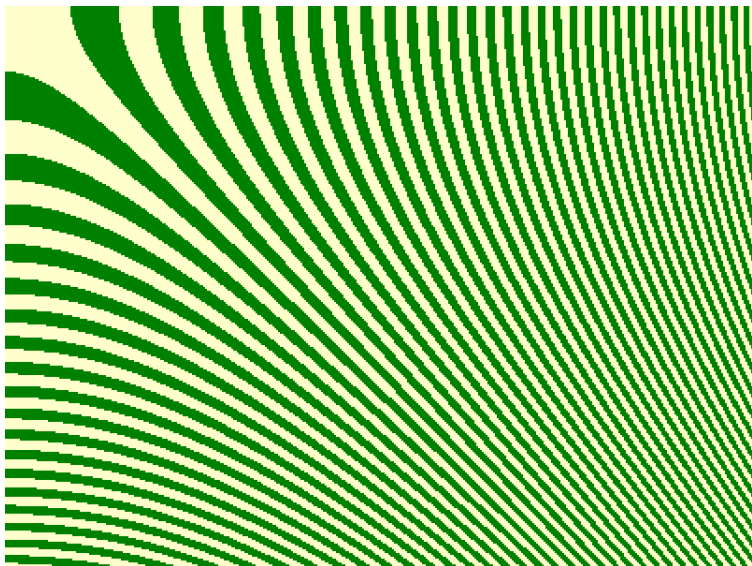
$$x^2 - y^2$$

0.01 0.01 2

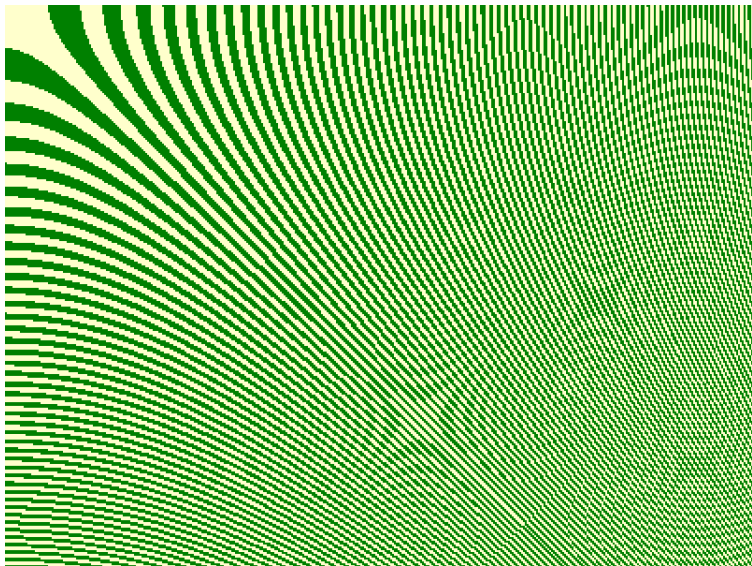


$$x^2 - y^2$$

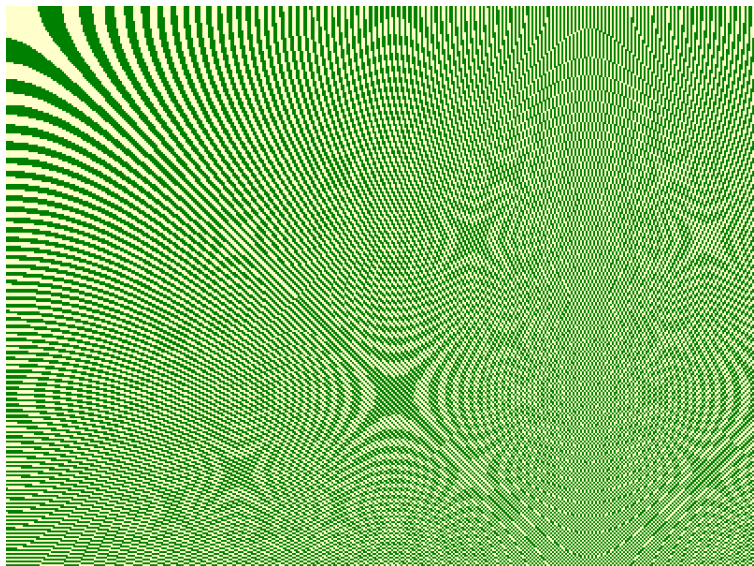
0.02 0.02 2



$$x^2 - y^2 \quad 0.03 \quad 0.03 \quad 2$$

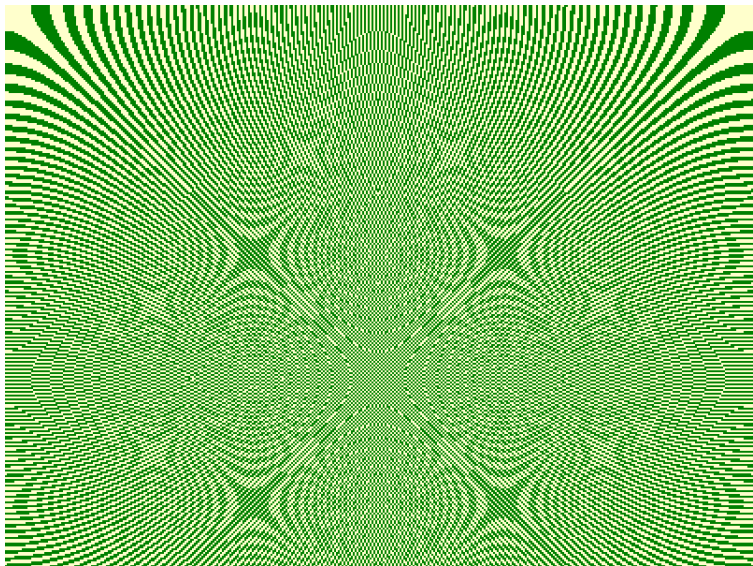


$$x^2 - y^2 \quad 0.04 \quad 0.04 \quad 2$$



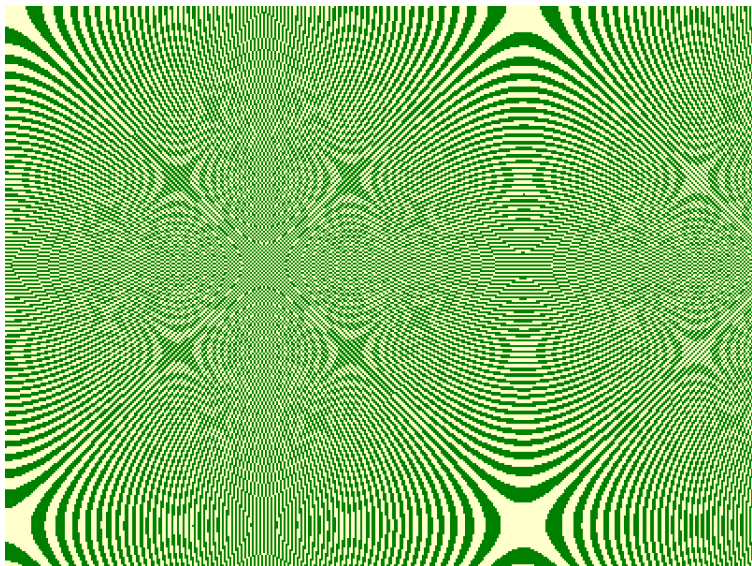
$$x^2 - y^2$$

0.05 0.05 2



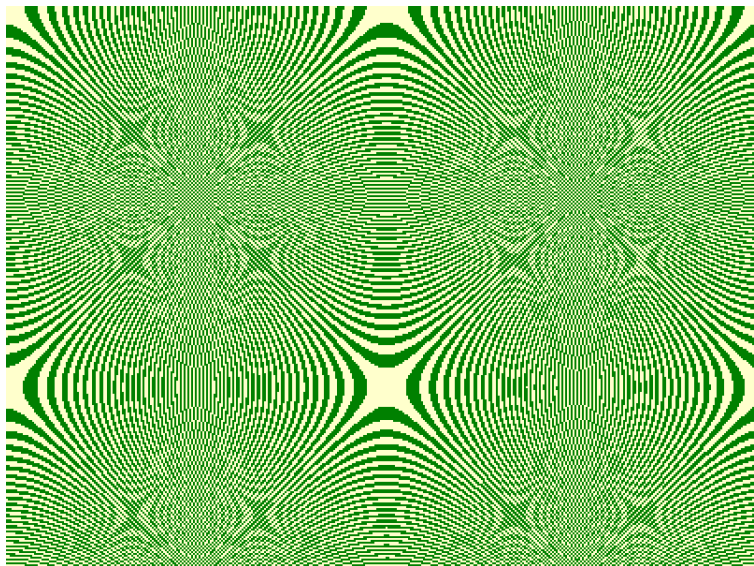
$$x^2 - y^2$$

0.06 0.06 2



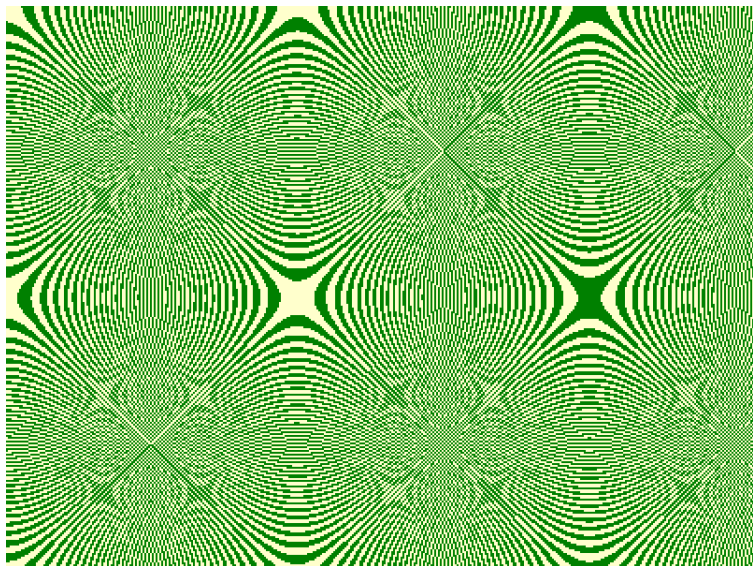
$$x^2 - y^2$$

0.07 0.07 2



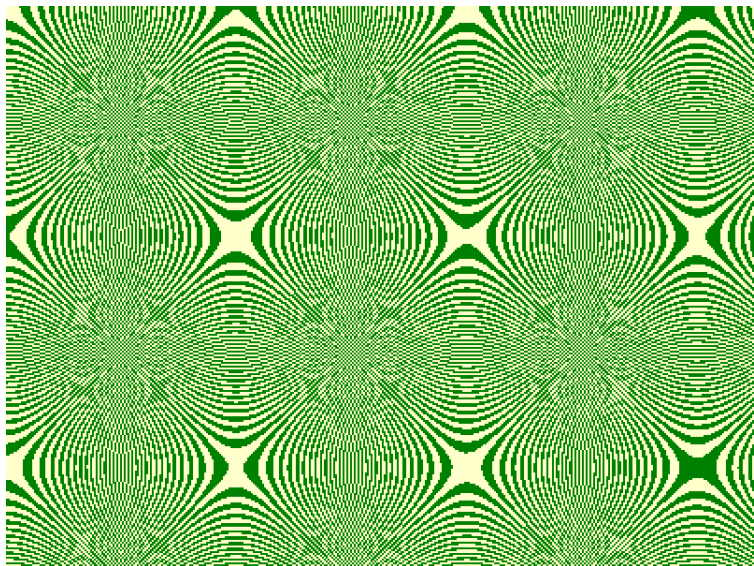
$$x^2 - y^2$$

0.08 0.08 2



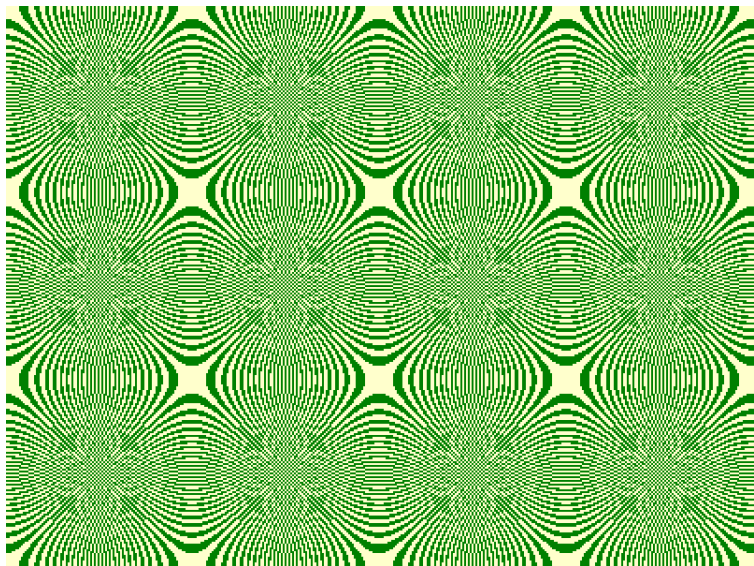
$$x^2 - y^2$$

0.09 0.09 2



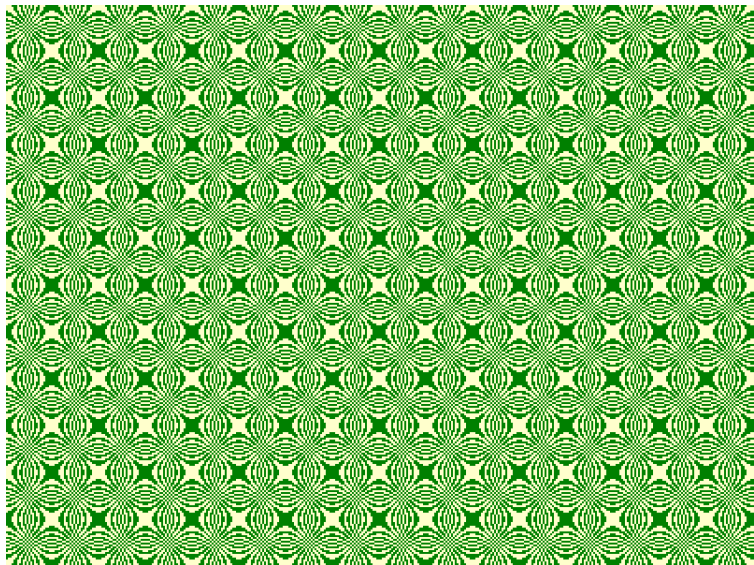
$$x^2 - y^2$$

0.1 0.1 2



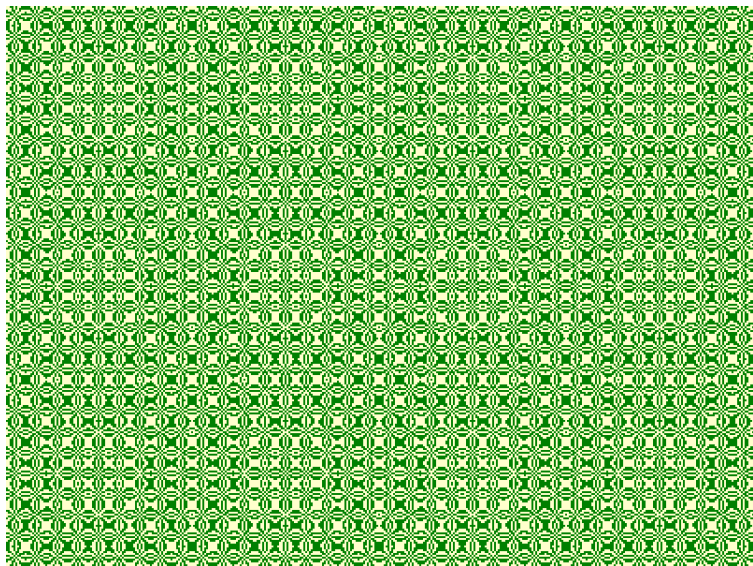
$$x^2 - y^2$$

0.2 0.2 2



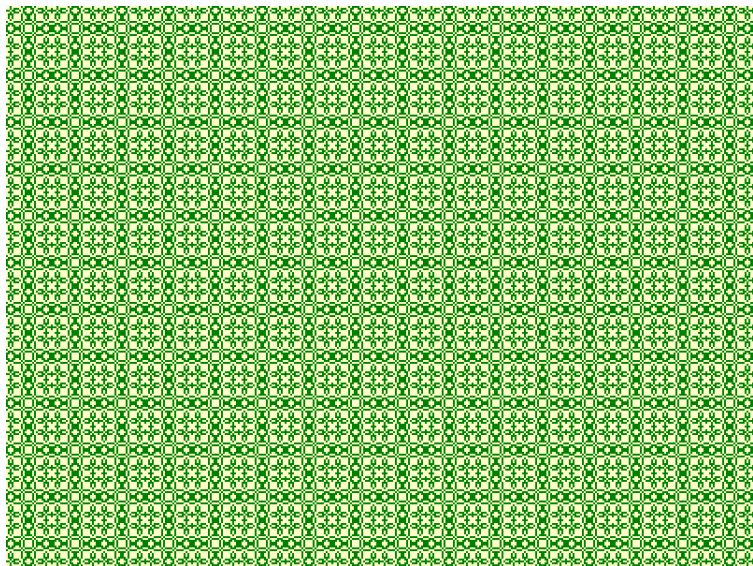
$$x^2 - y^2$$

0.3 0.3 2



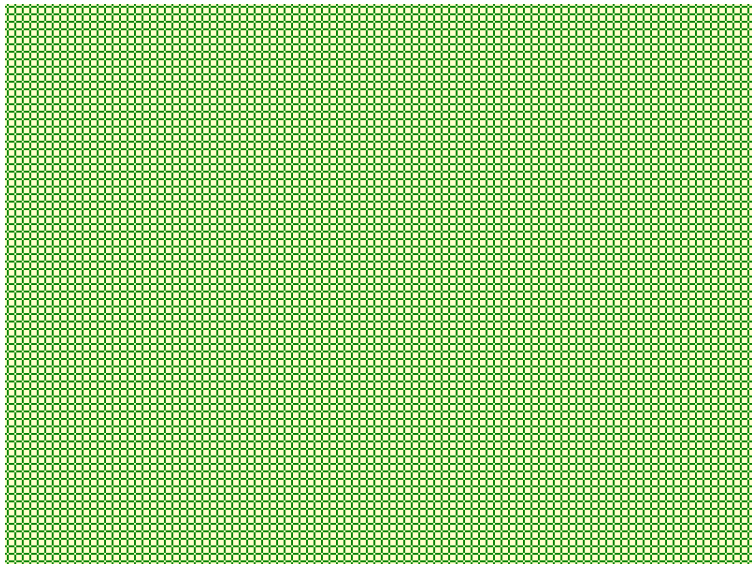
$$x^2 - y^2$$

0.4 0.4 2



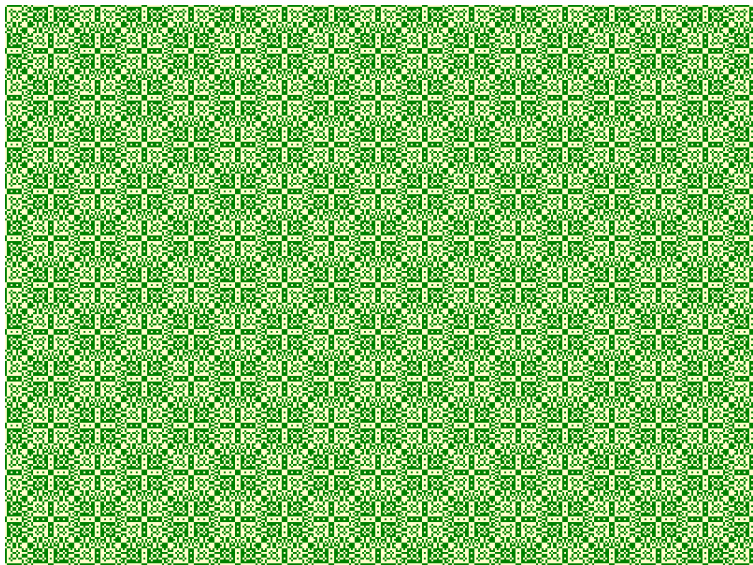
$$x^2 - y^2$$

0.5 0.5 2

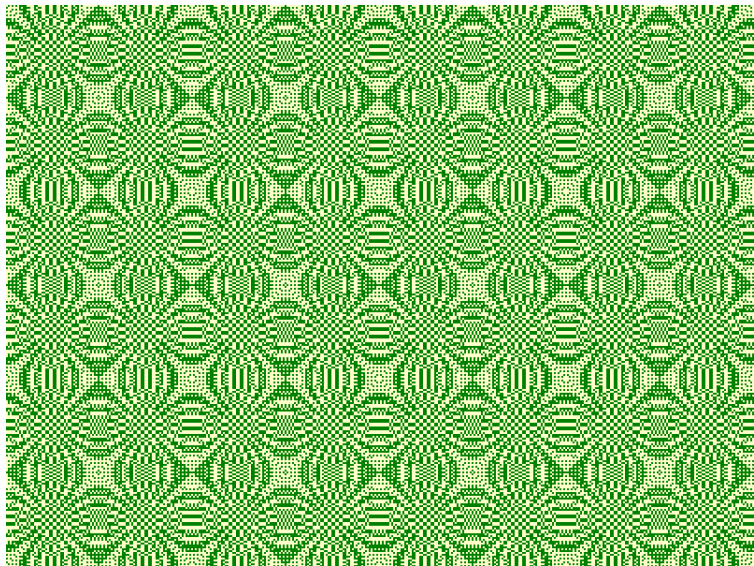


$$x^2 - y^2$$

0.6 0.6 2

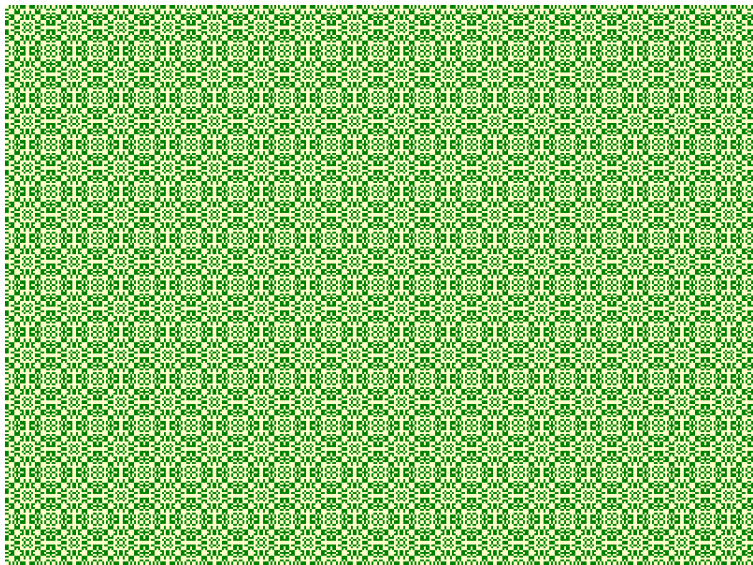


$$x^2 - y^2 \quad 0.7 \ 0.7 \ 2$$



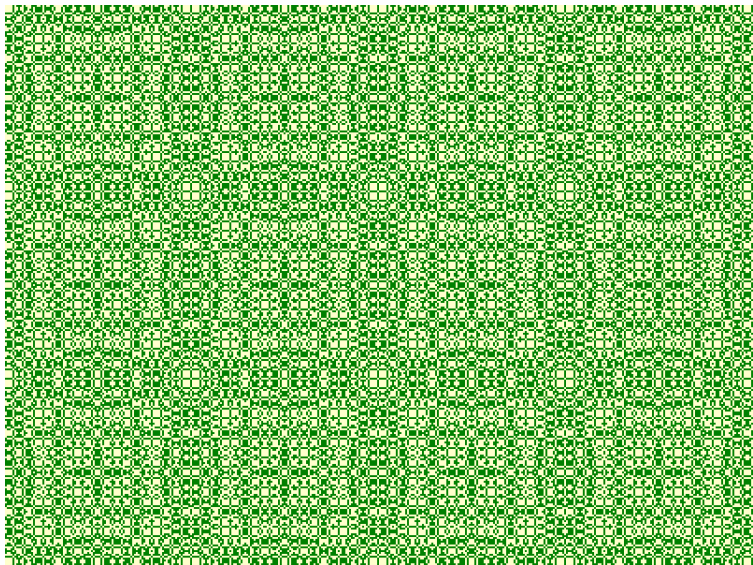
$$x^2 - y^2$$

0.8 0.8 2



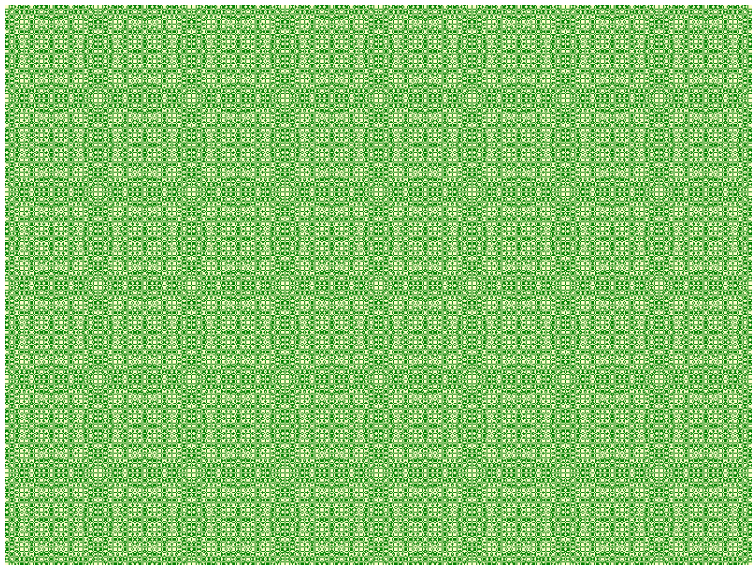
$$x^2 - y^2$$

0.9 0.9 2



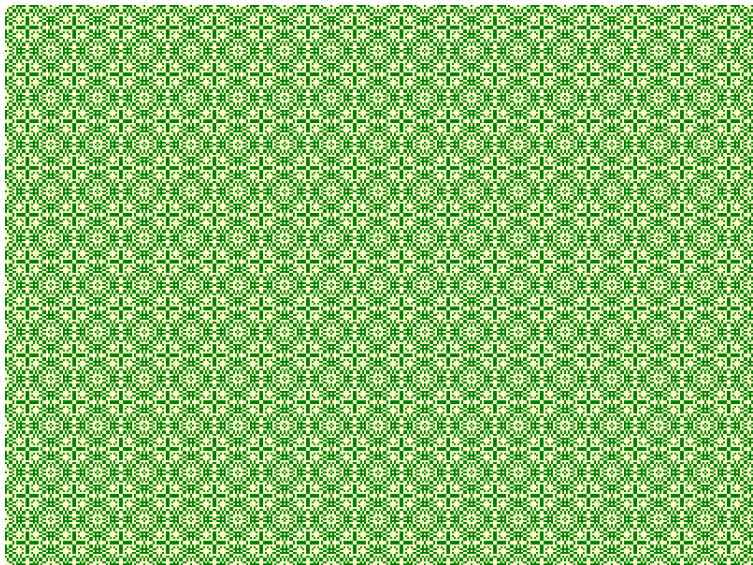
$$x^2 - y^2$$

0.9 0.9 1



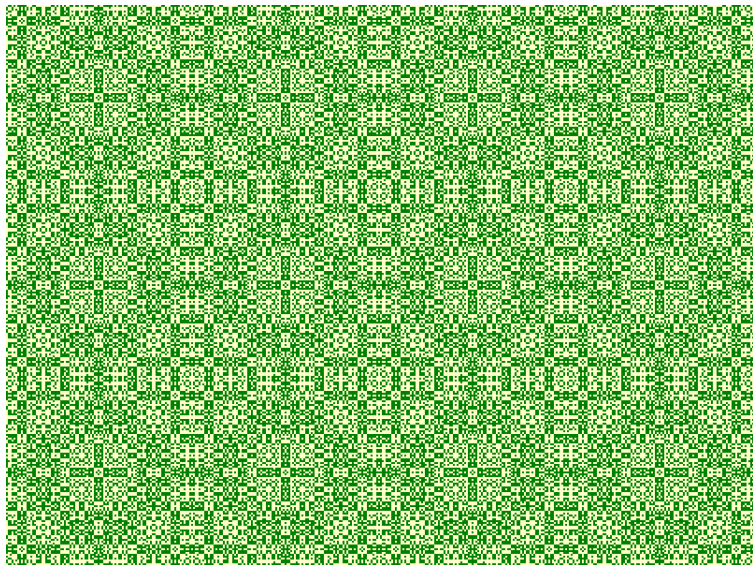
$$x^2 - y^2$$

1.2 1.2 2



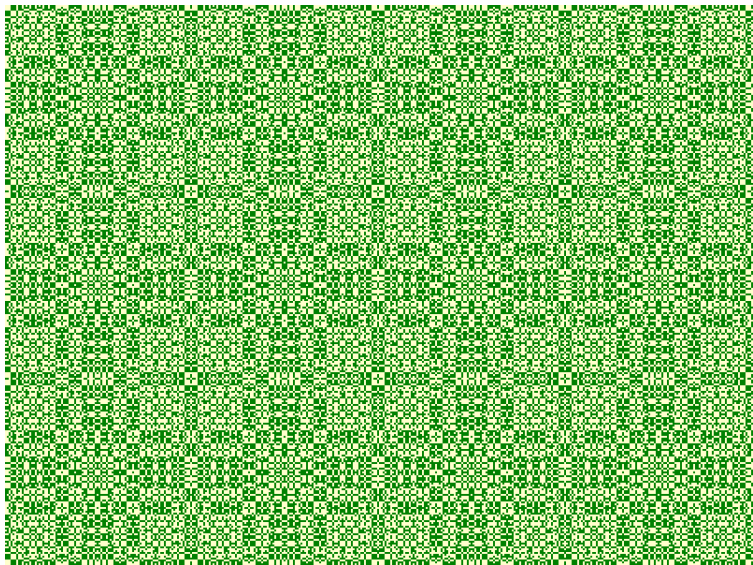
$$x^2 - y^2$$

1.9 1.9 2



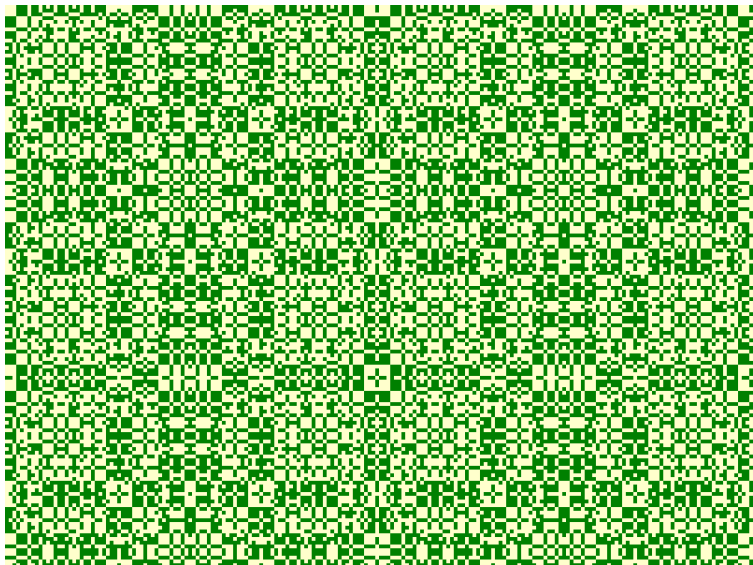
$$x^2 - y^2$$

2.3 2.3 2



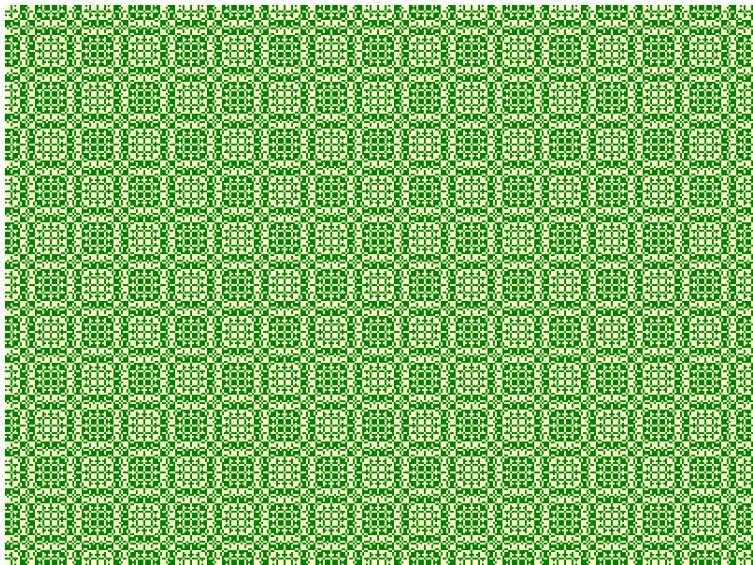
$$x^2 - y^2$$

2.3 2.3 4



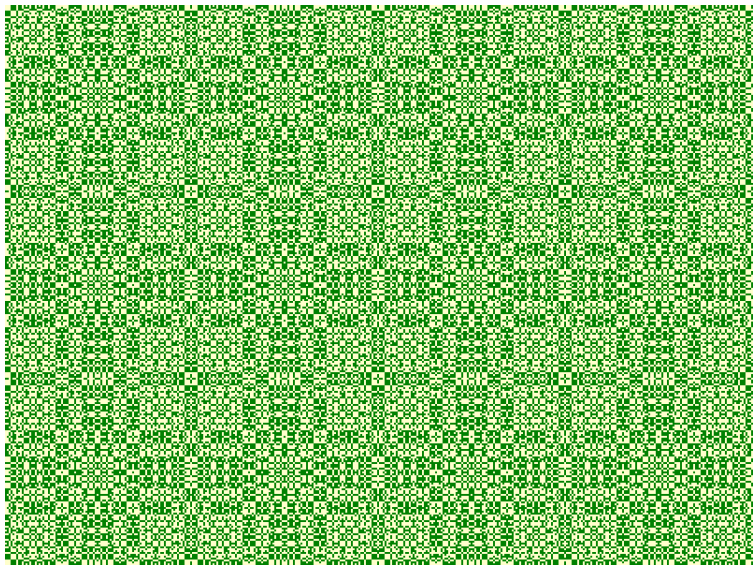
$$x^2 - y^2$$

2.6 2.6 2



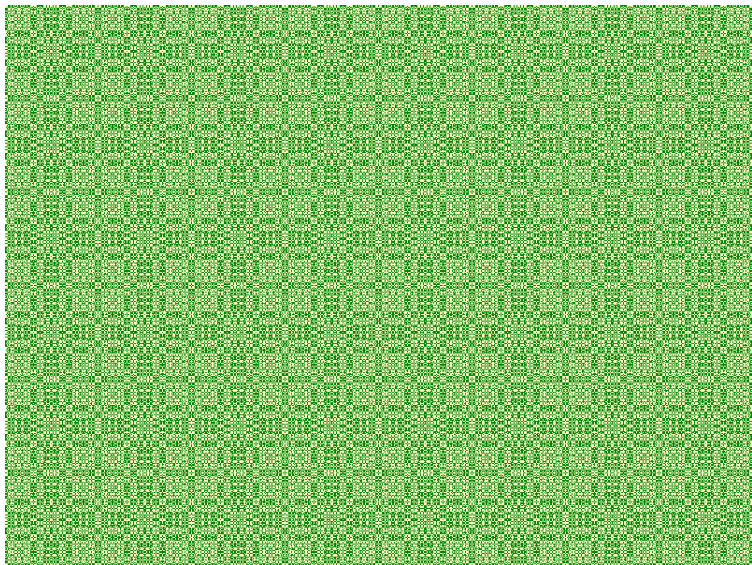
$$x^2 - y^2$$

2.7 2.7 2



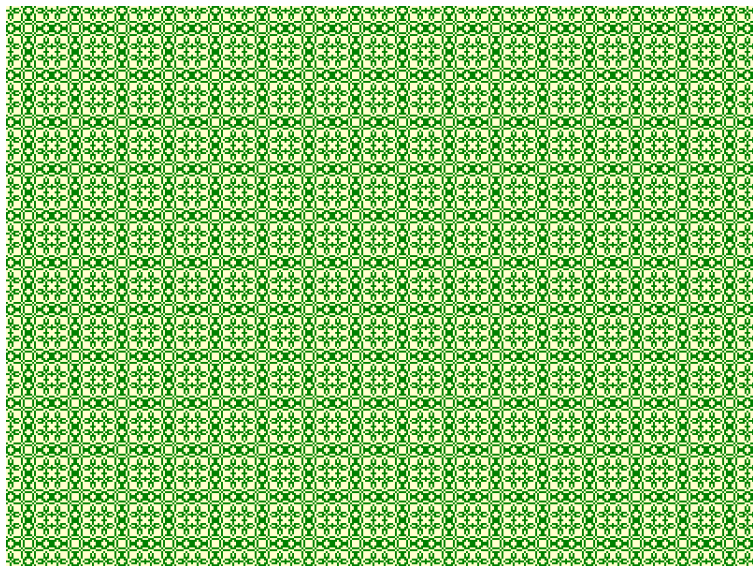
$$x^2 - y^2$$

2.7 2.7 1

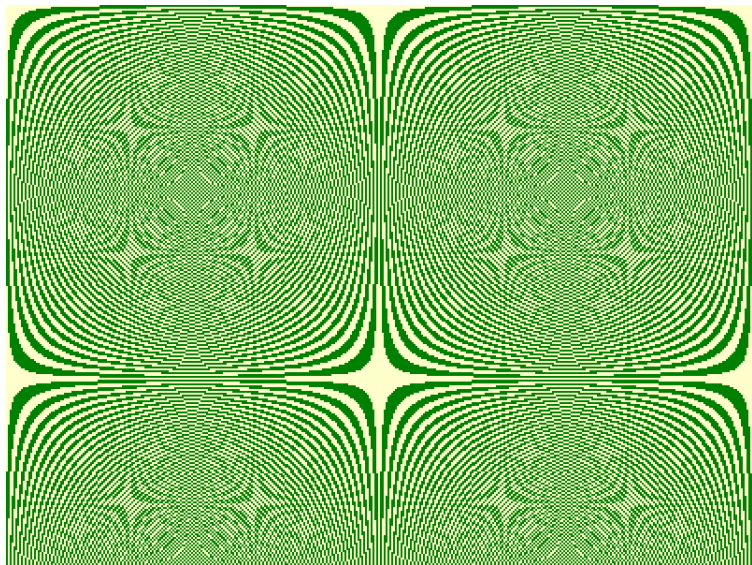


$$x^2 - y^2$$

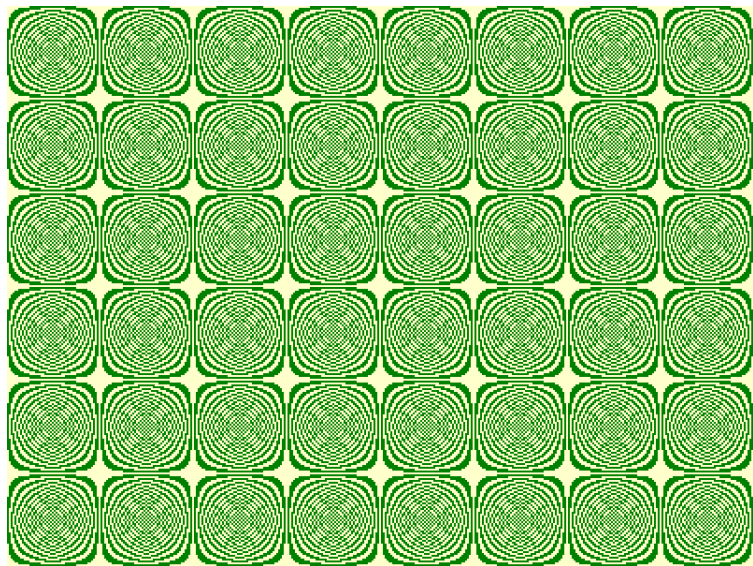
2.8 2.8 2

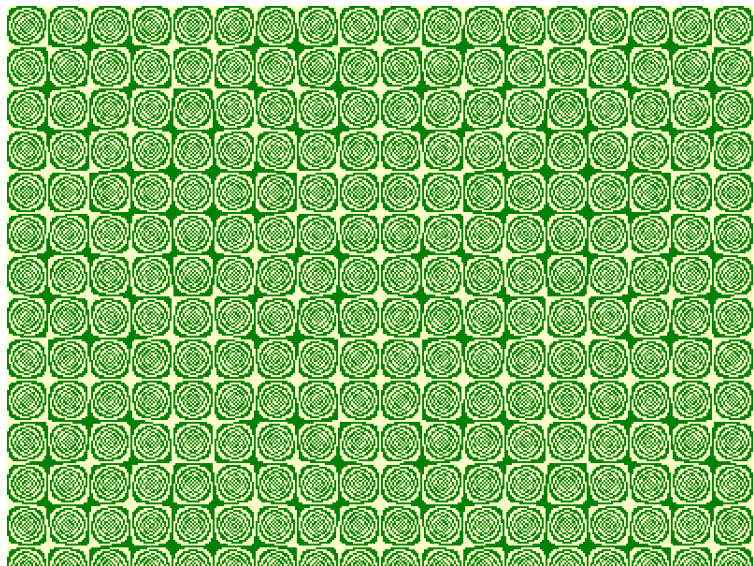


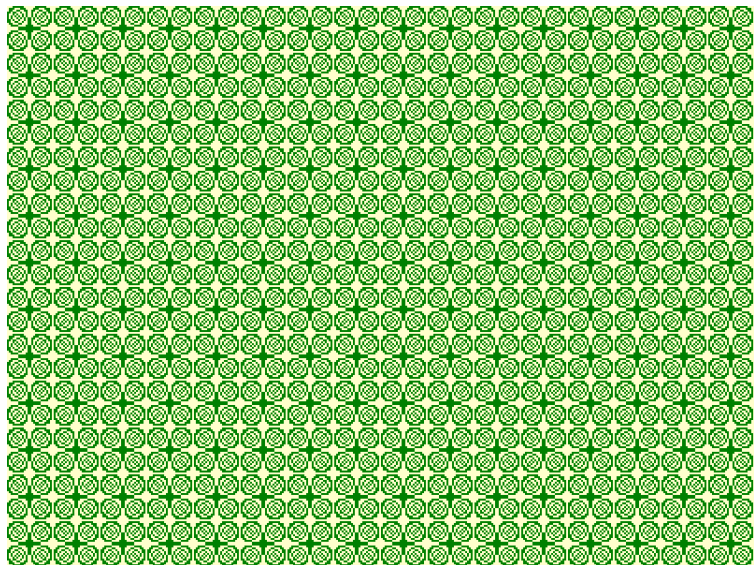
xy 0.1 0.1 2



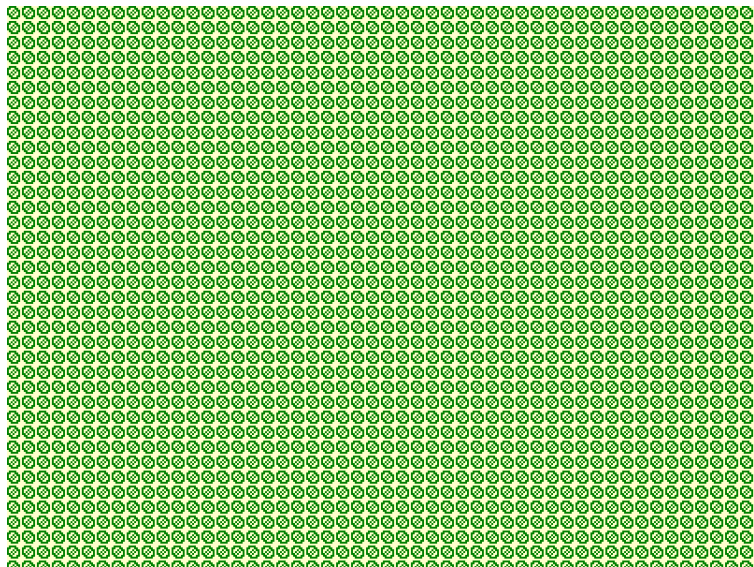
xy 0.2 0.2 2

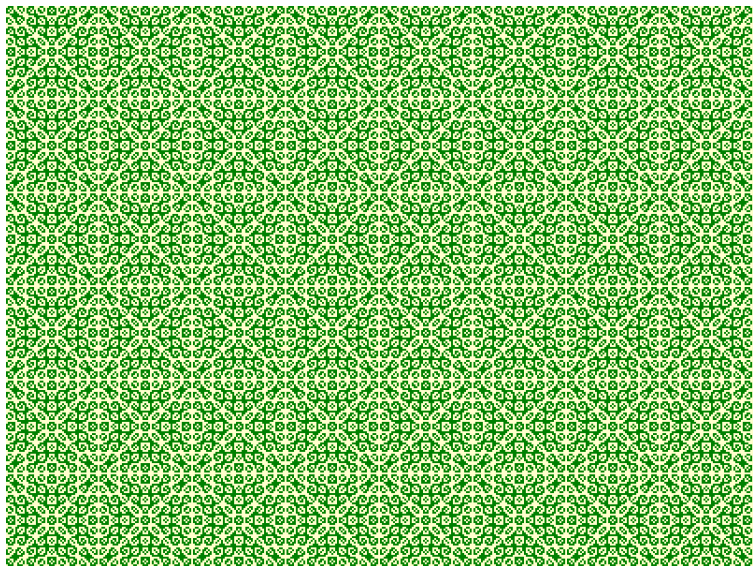




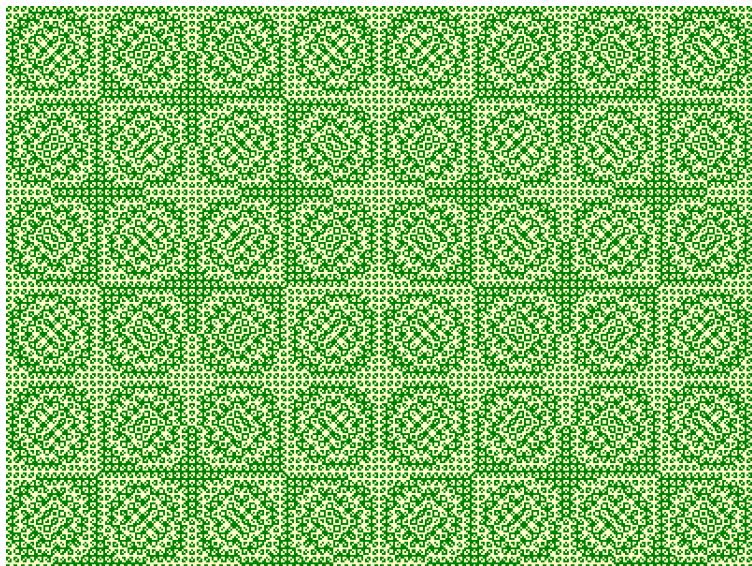


xy 0.5 0.5 2

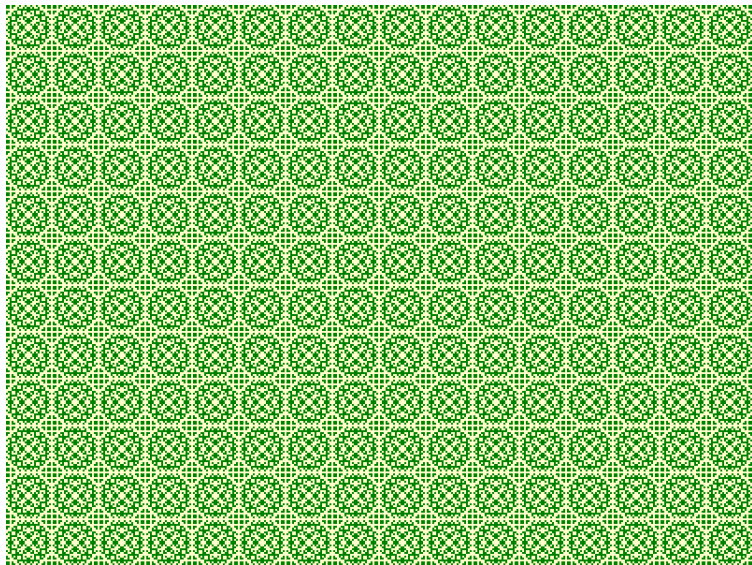




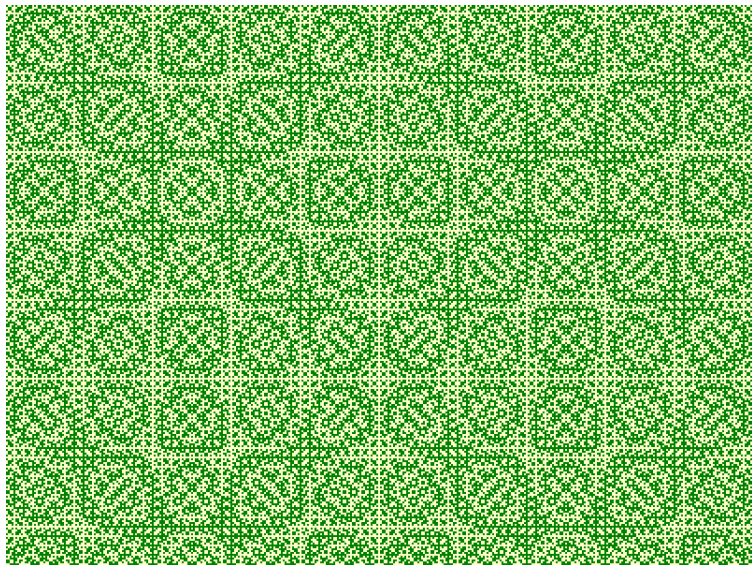
xy 0.7 0.7 2



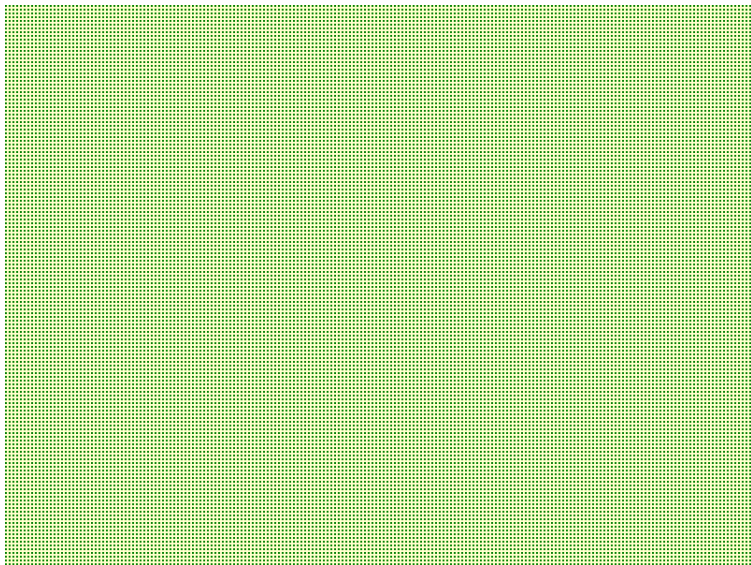
xy 0.8 0.8 2

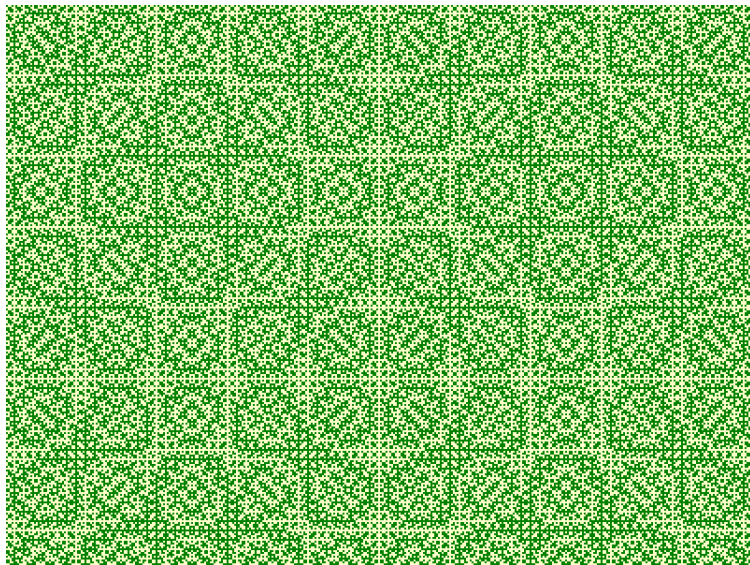


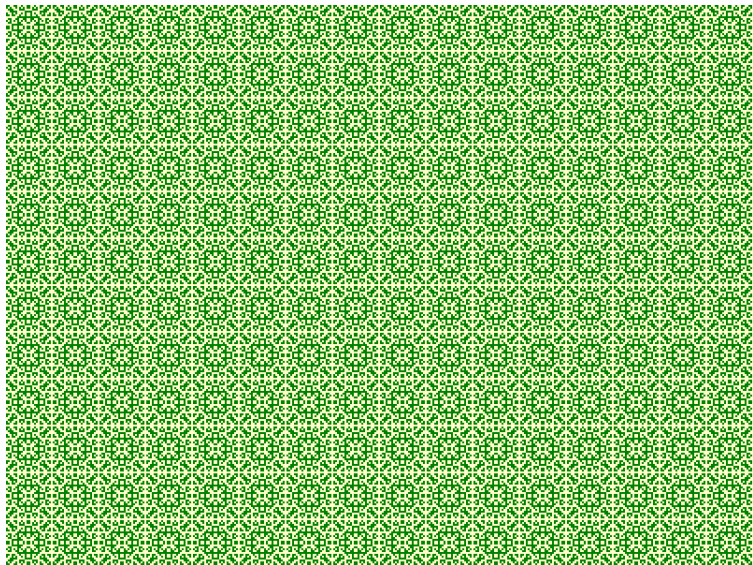
xy 0.9 0.9 2

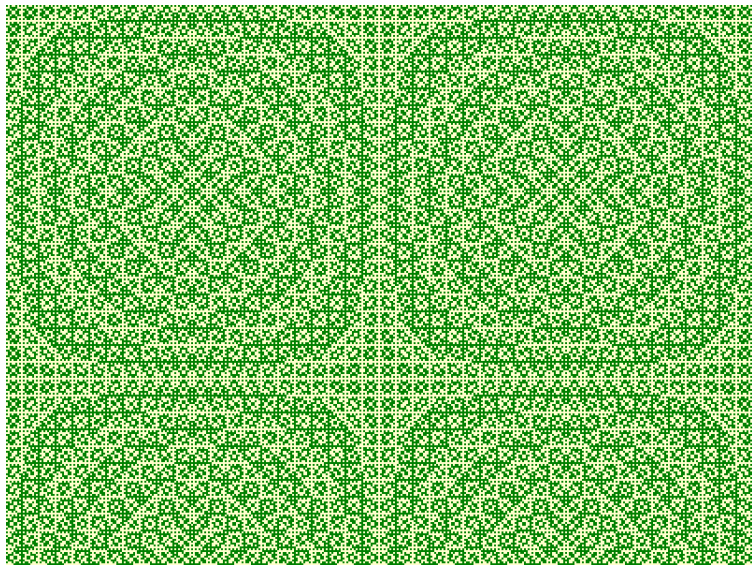


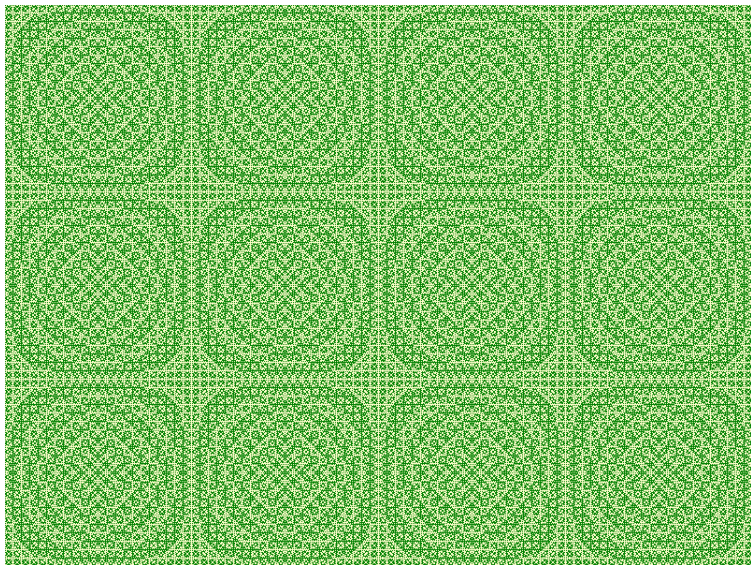
xy 1.0 1.0 2

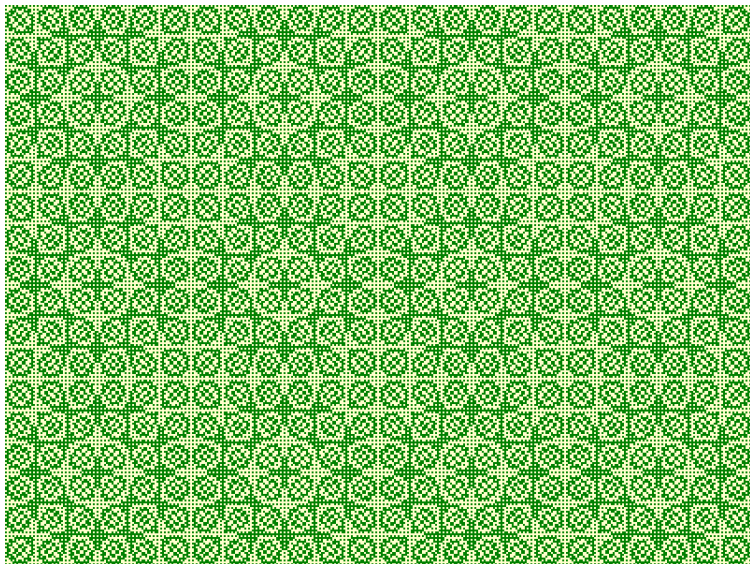


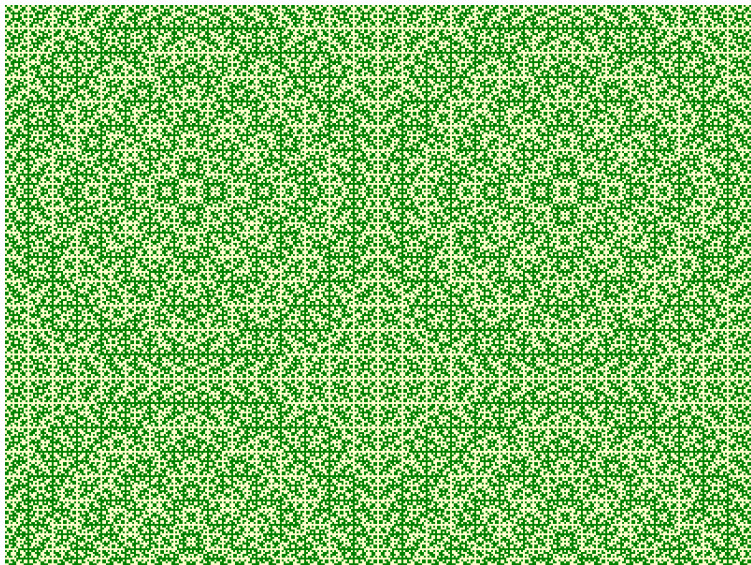


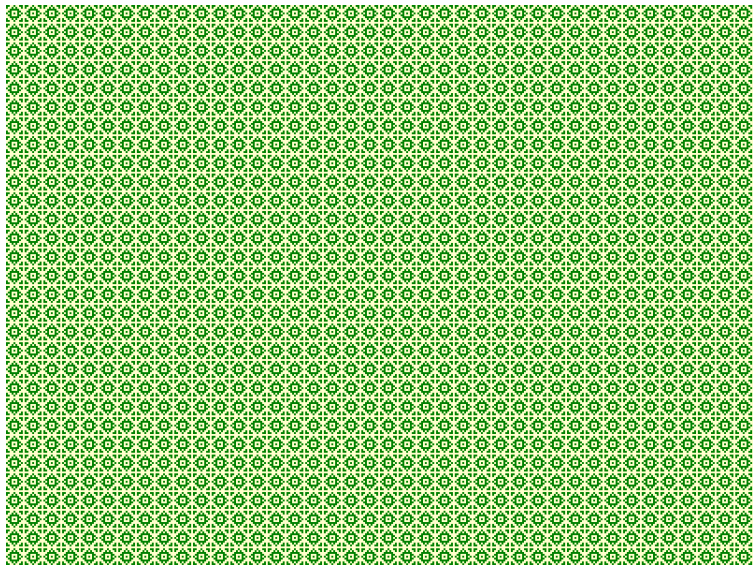


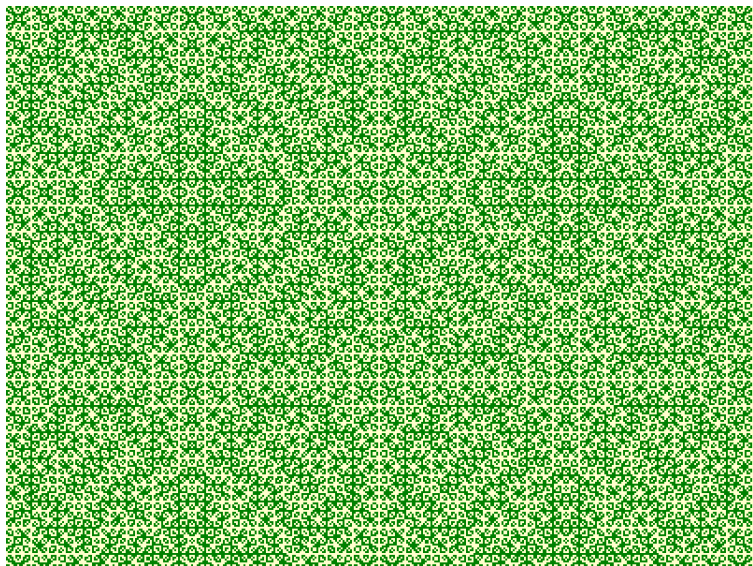


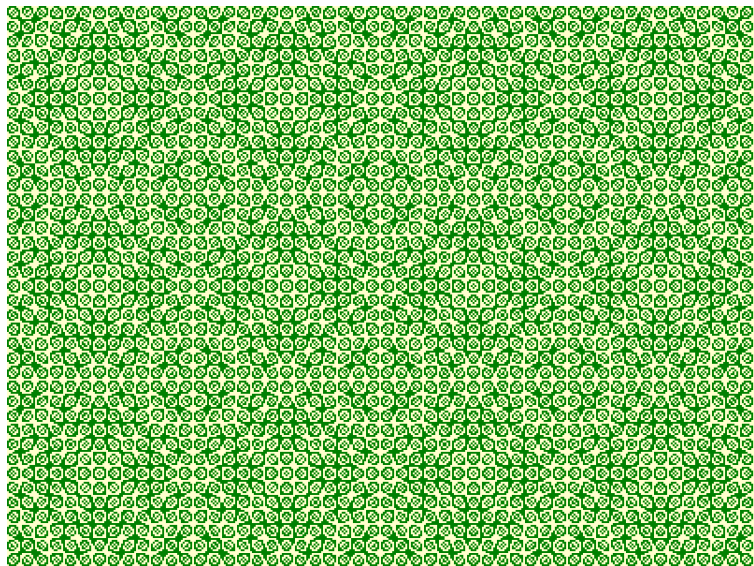


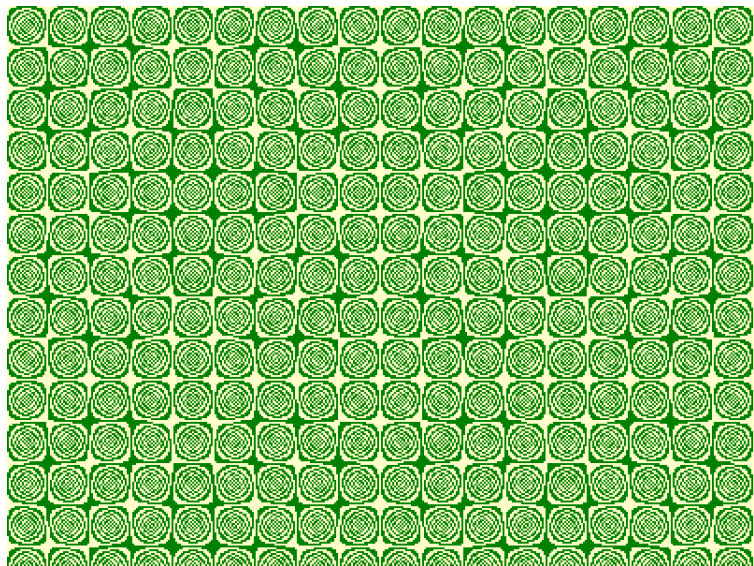


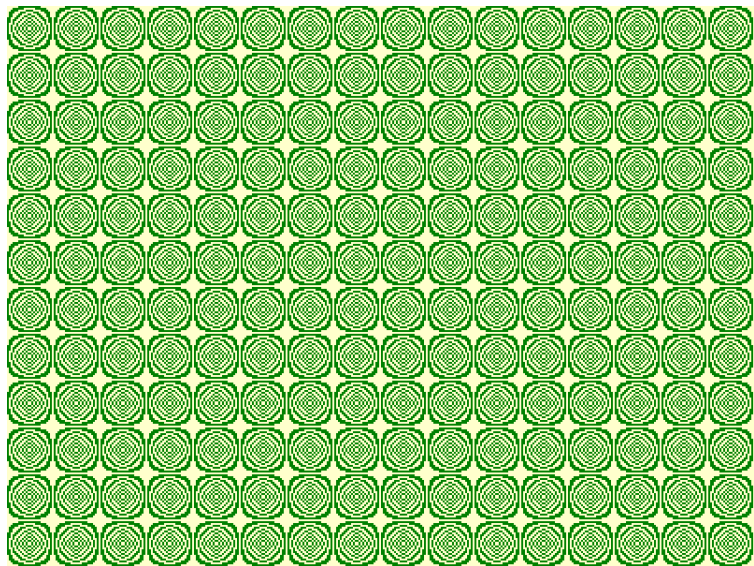


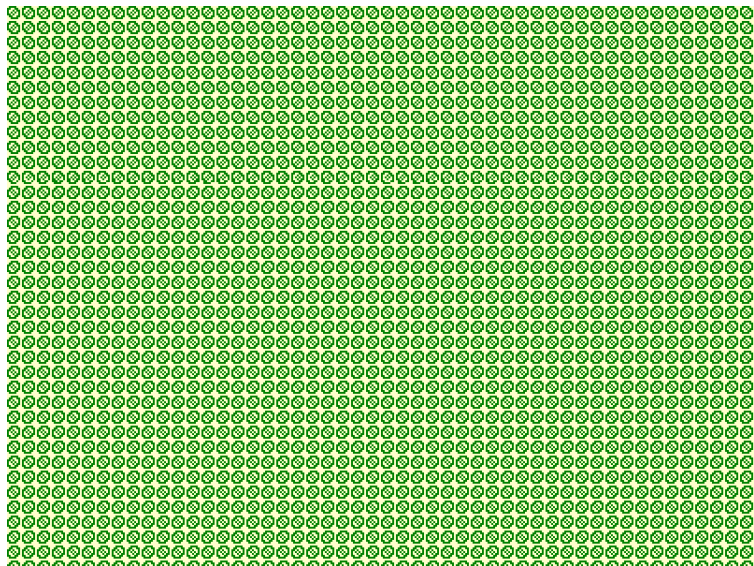


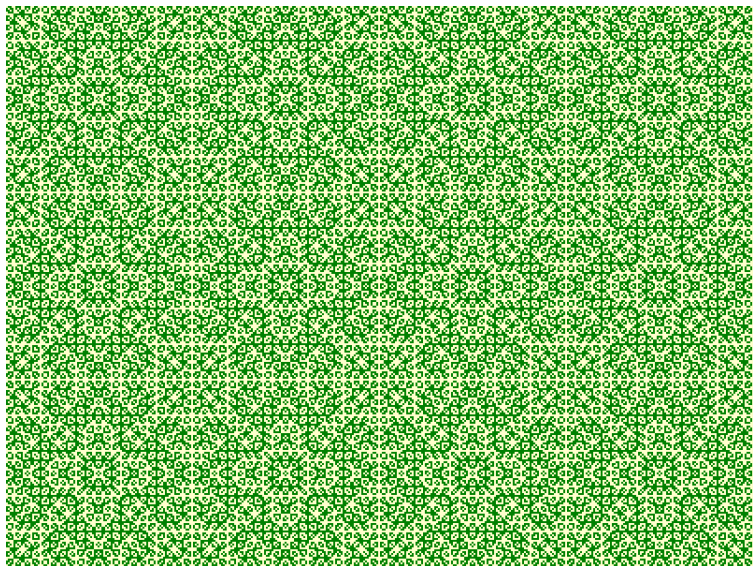


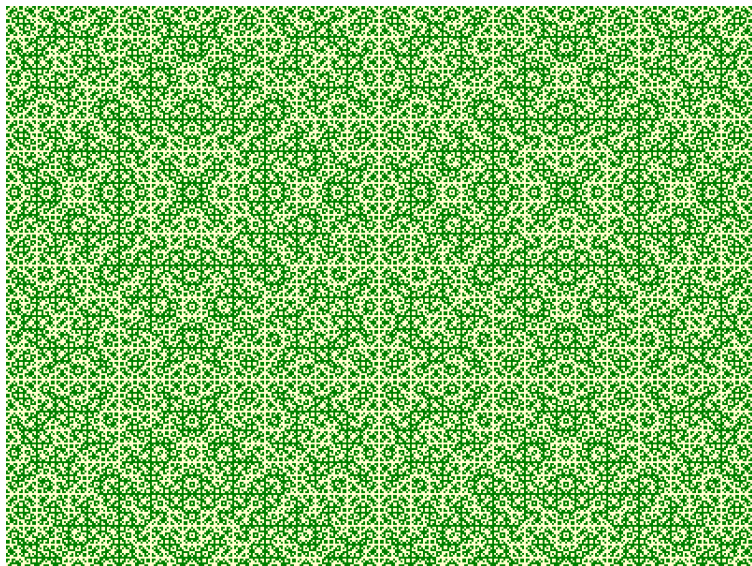


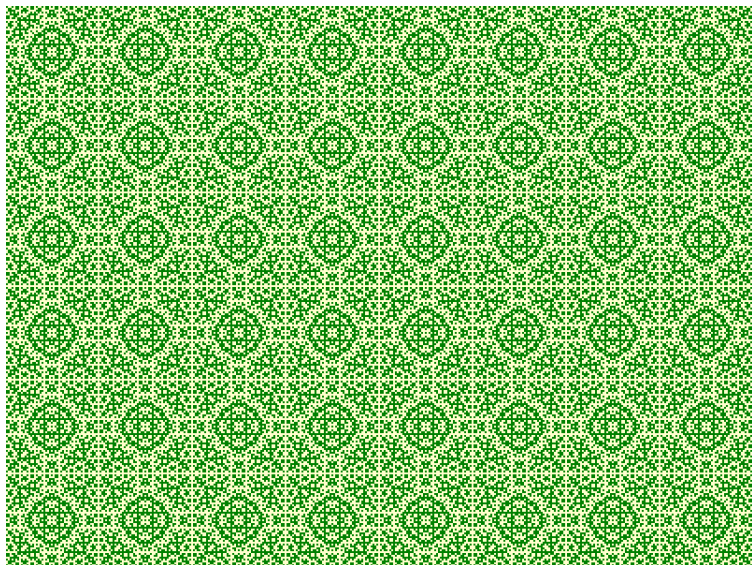


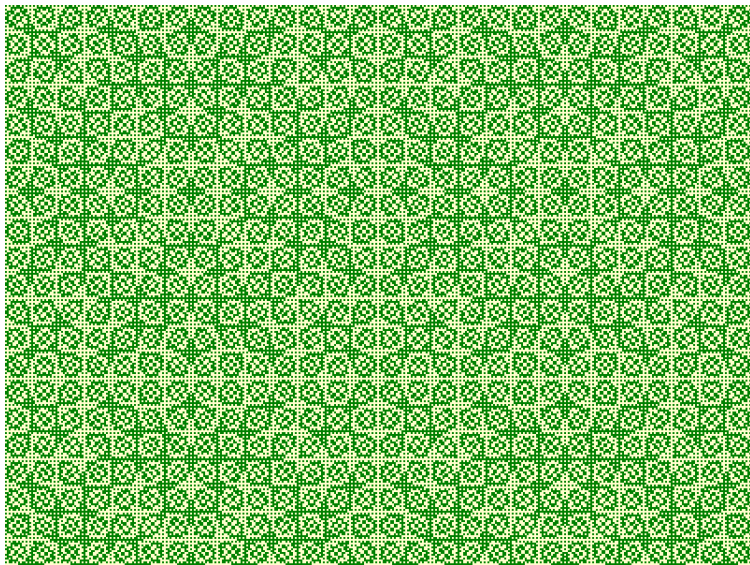


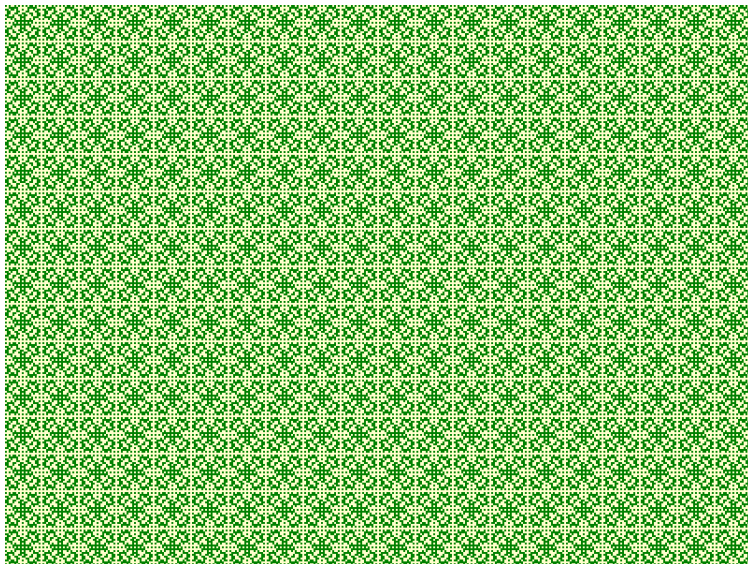




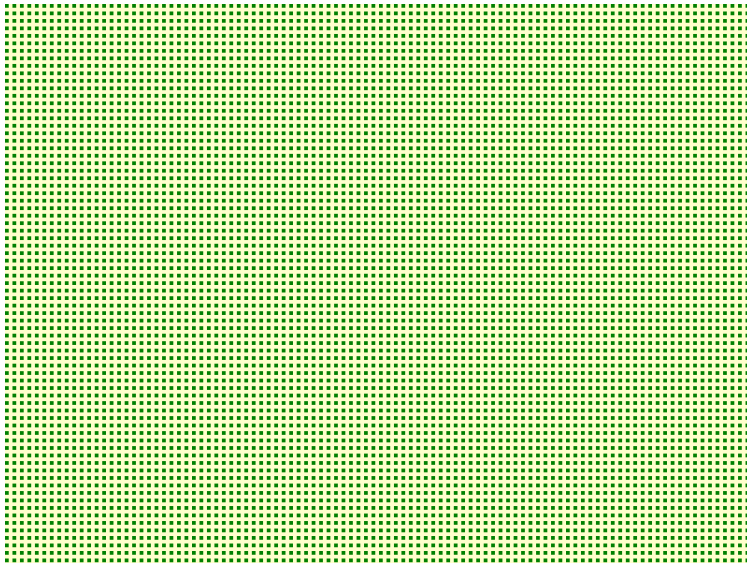


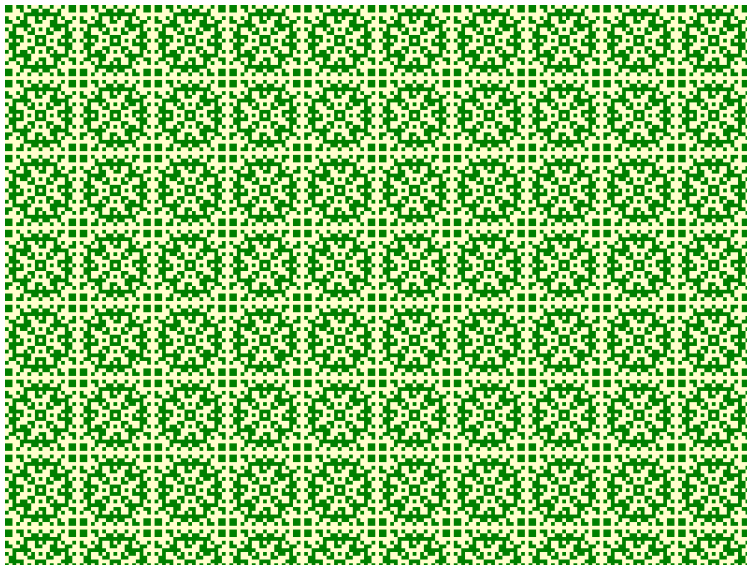


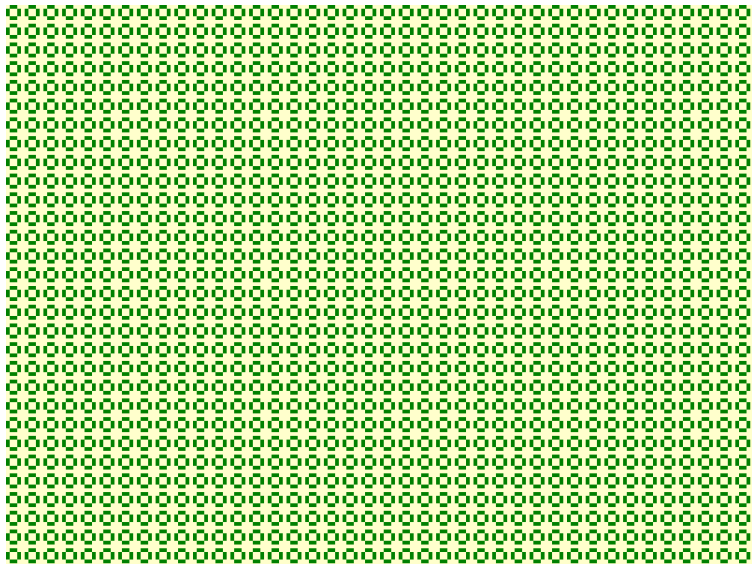


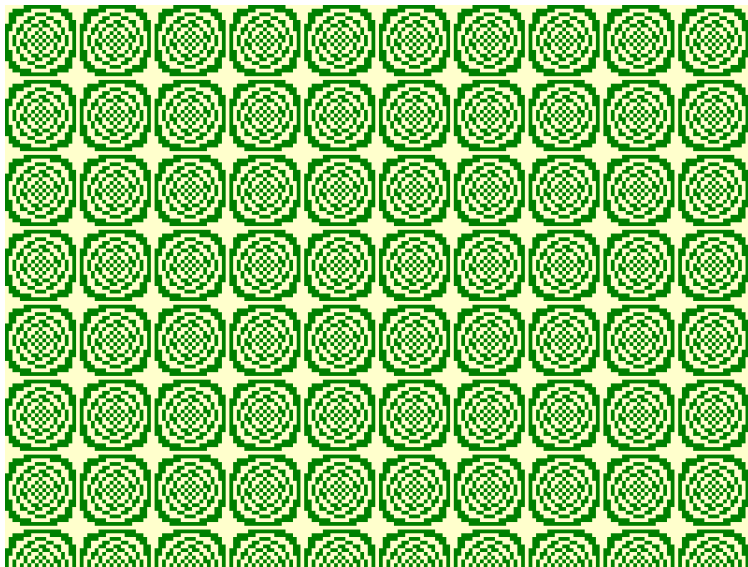


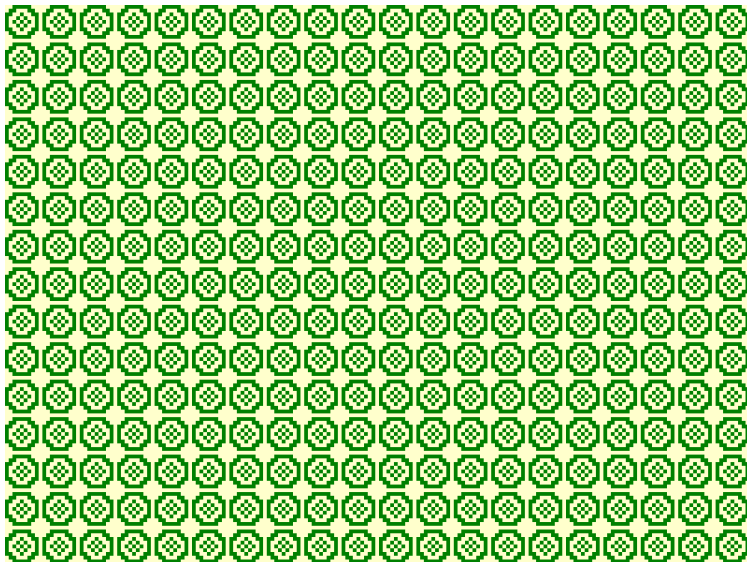
xy 3.0 3.0 4



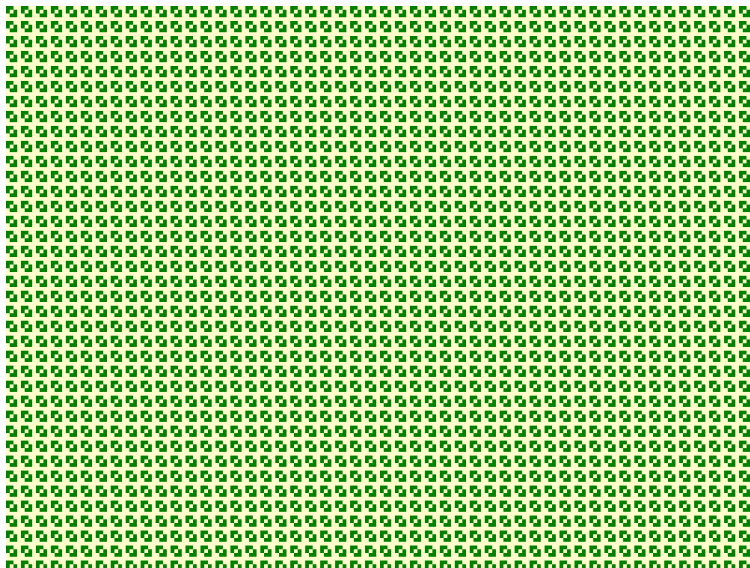






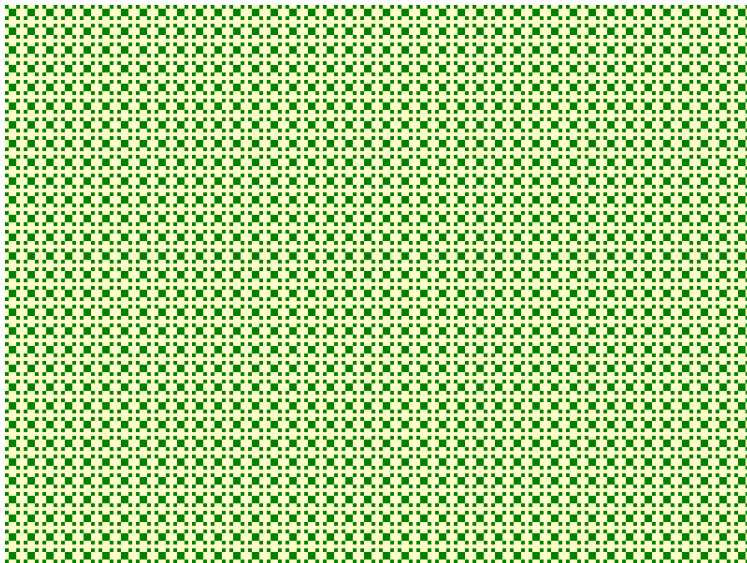


xy 3.0 3.5 4



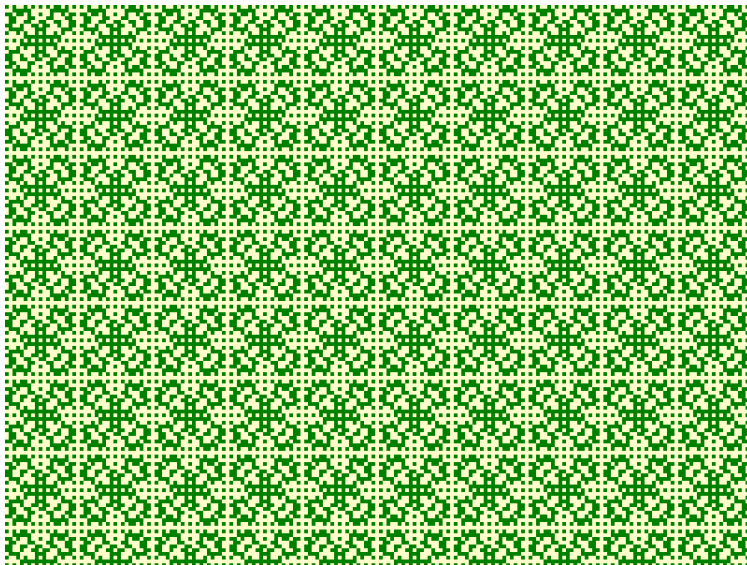
xy

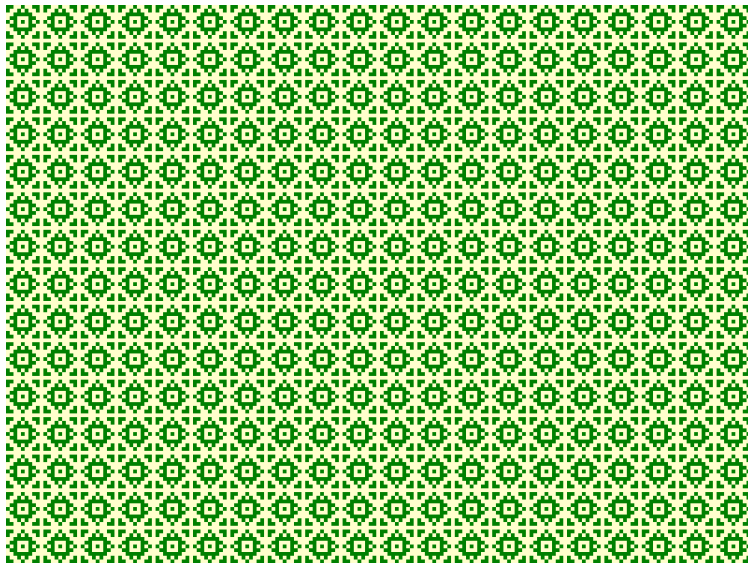
3.0 3.6 4

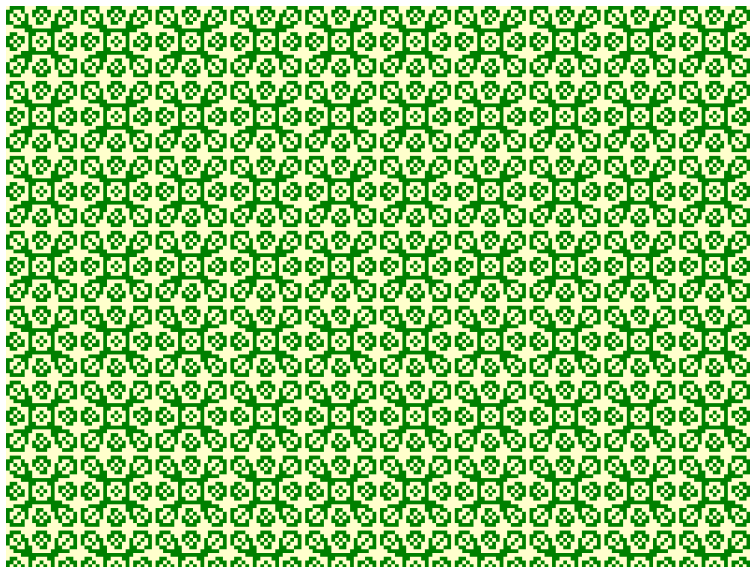


xy

3.0 3.7 4

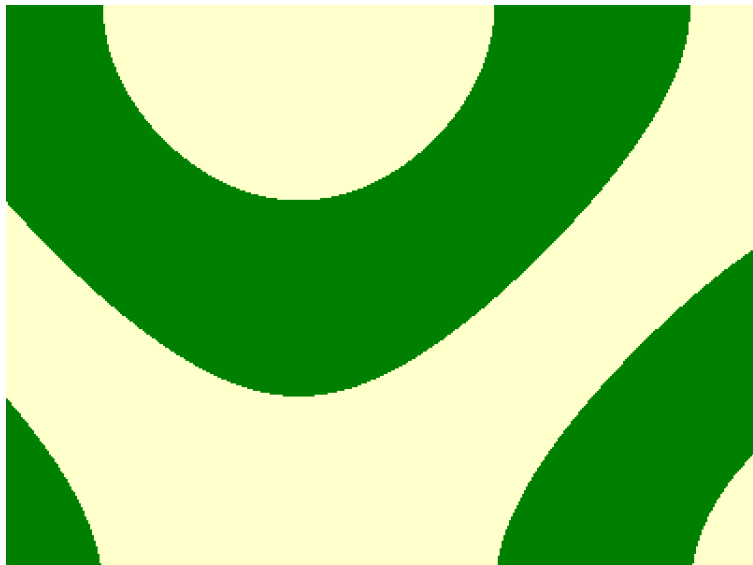






$$\sin x + \cos y$$

0.01 0.01 2



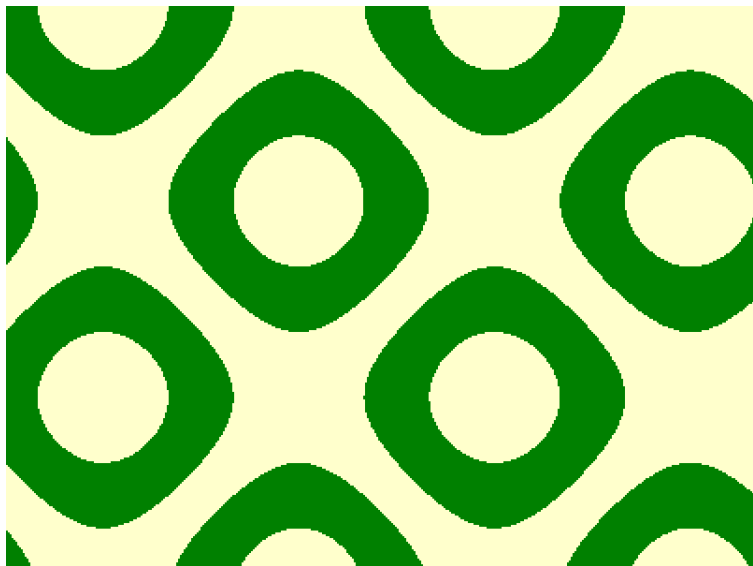
$\sin x + \cos y$

0.02 0.02 2



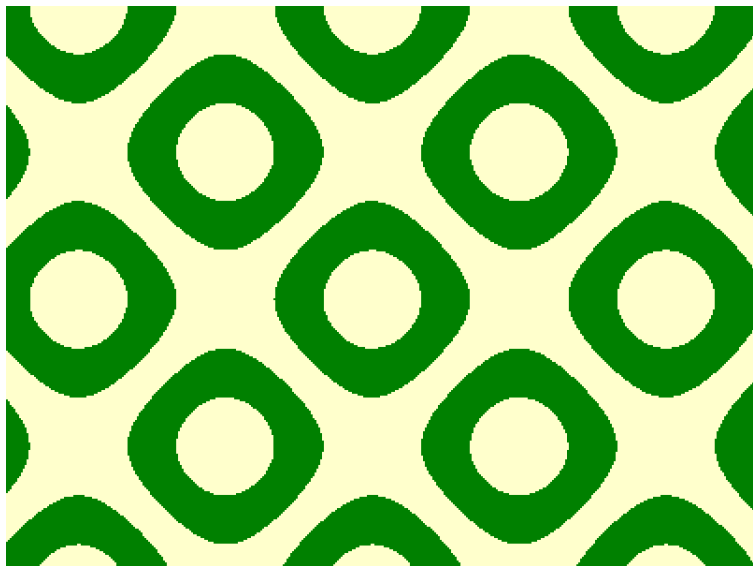
$\sin x + \cos y$

0.03 0.03 2



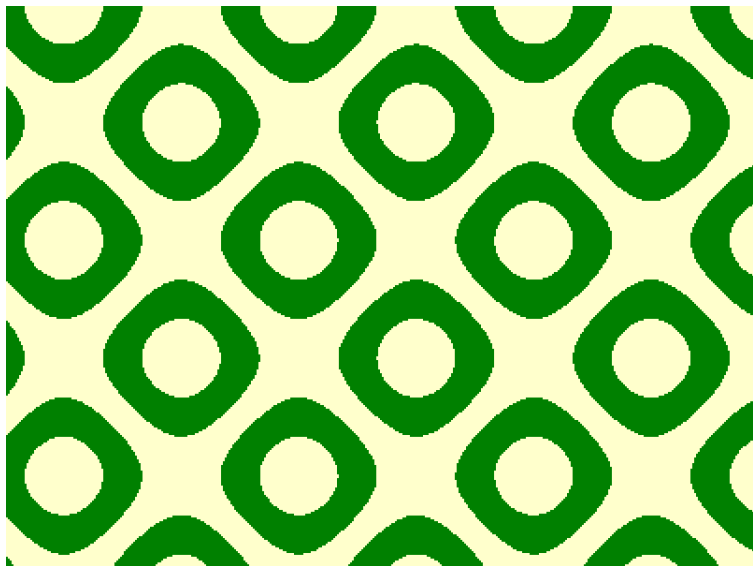
$\sin x + \cos y$

0.04 0.04 2



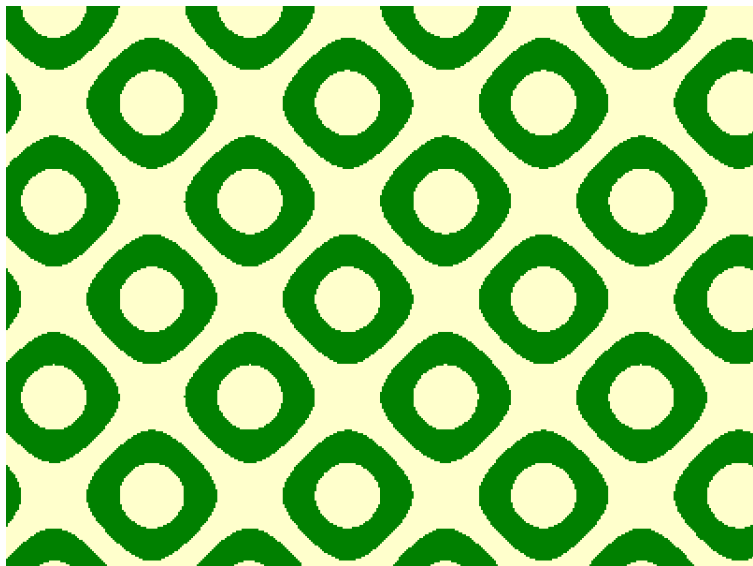
$\sin x + \cos y$

0.05 0.05 2



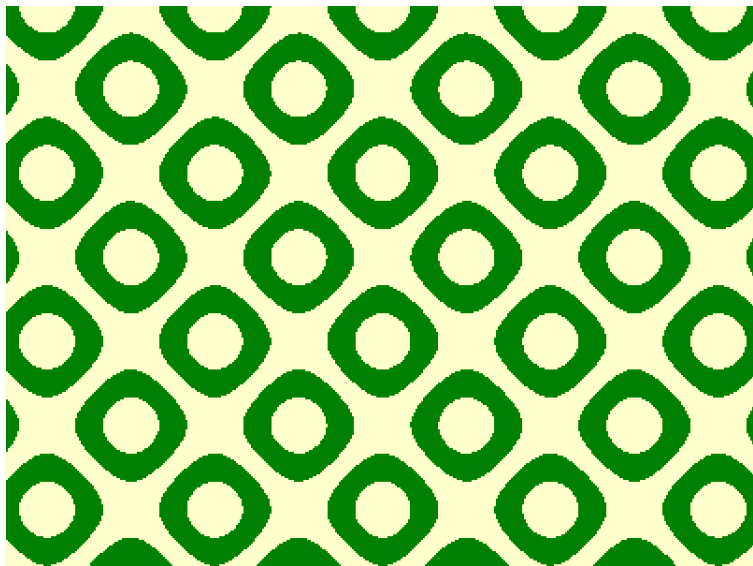
$\sin x + \cos y$

0.06 0.06 2



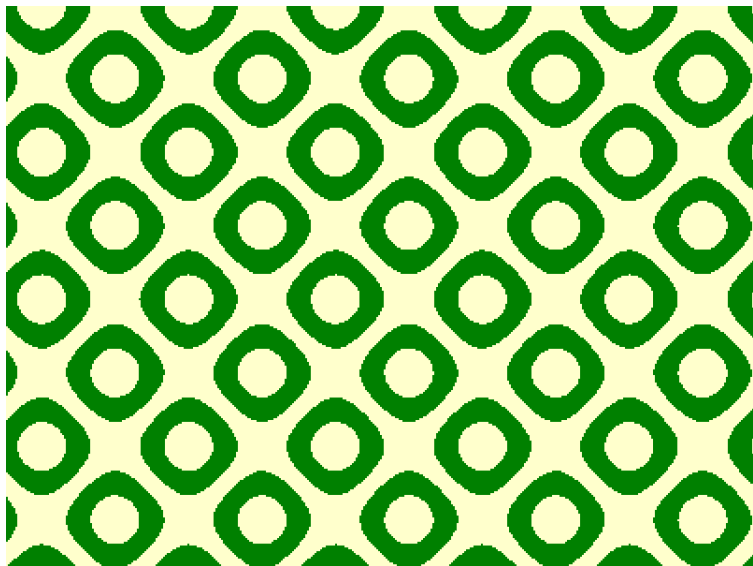
$\sin x + \cos y$

0.07 0.07 2

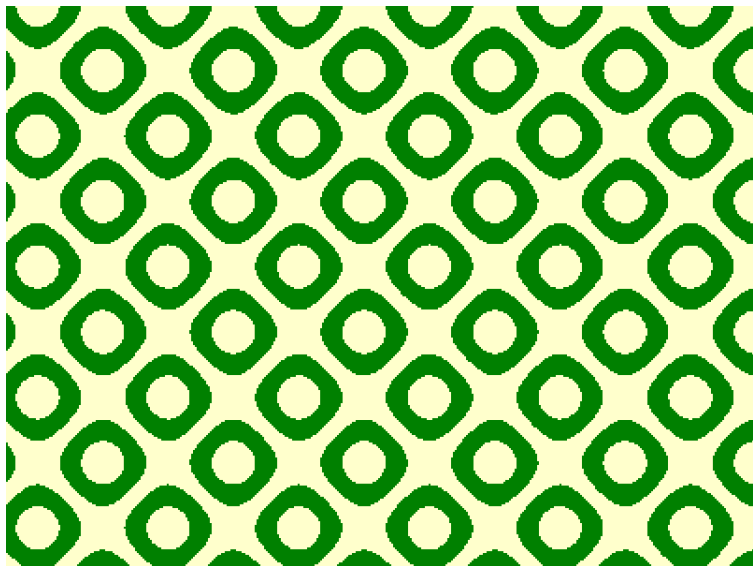


$\sin x + \cos y$

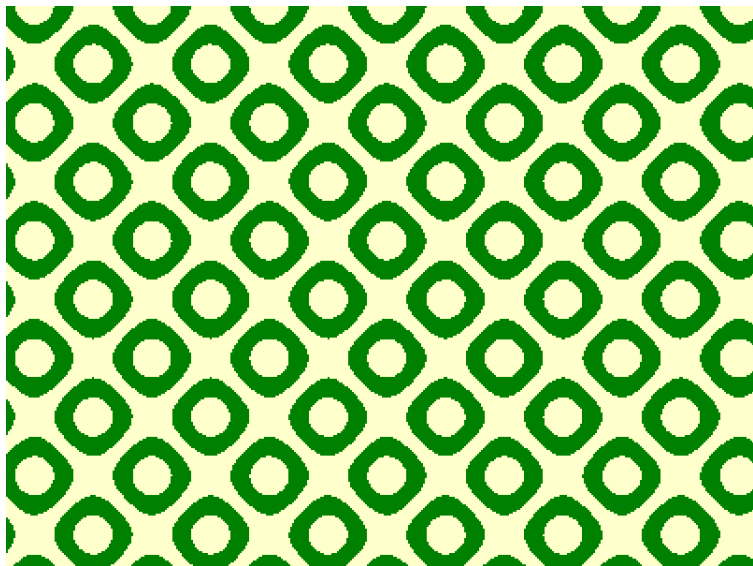
0.08 0.08 2



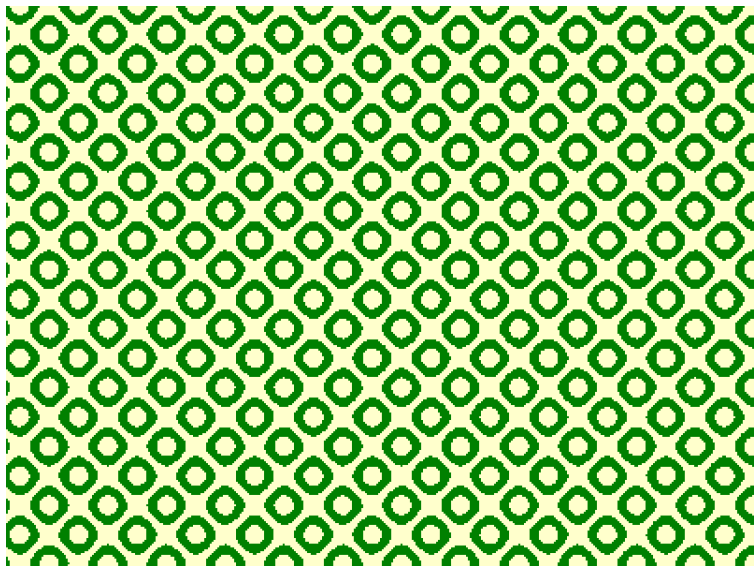
$$\sin x + \cos y \quad 0.09 \quad 0.09 \quad 2$$



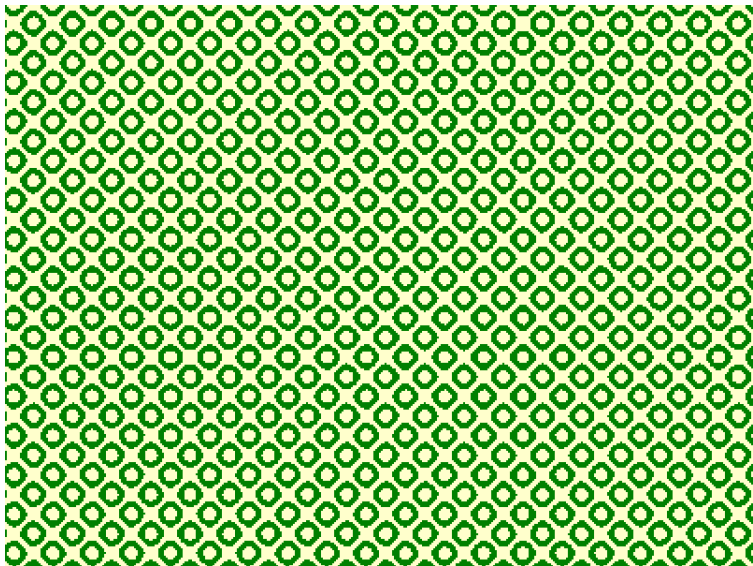
$$\sin x + \cos y \quad 0.1 \quad 0.1 \quad 2$$



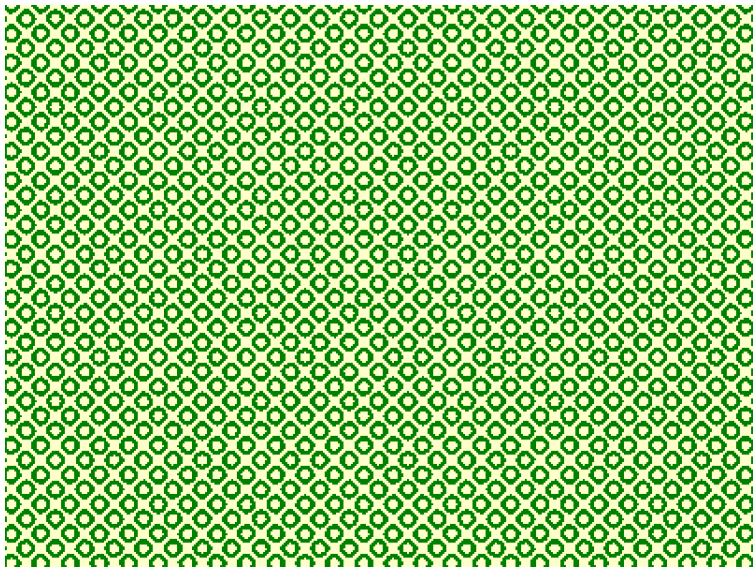
$$\sin x + \cos y \quad 0.2 \quad 0.2 \quad 2$$



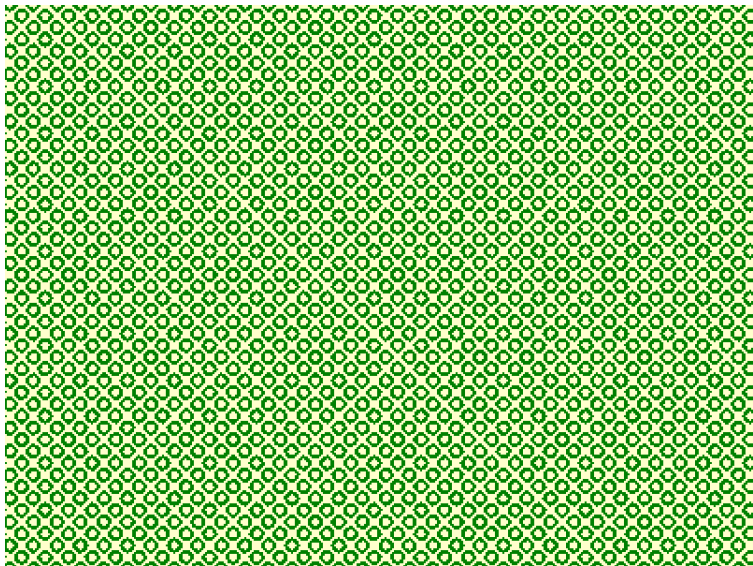
$$\sin x + \cos y \quad 0.3 \ 0.3 \ 2$$



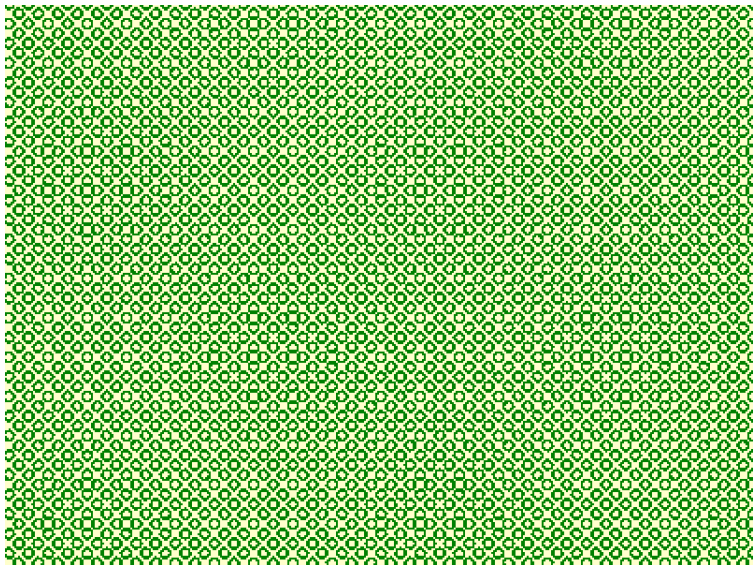
$$\sin x + \cos y \quad 0.4 \ 0.4 \ 2$$



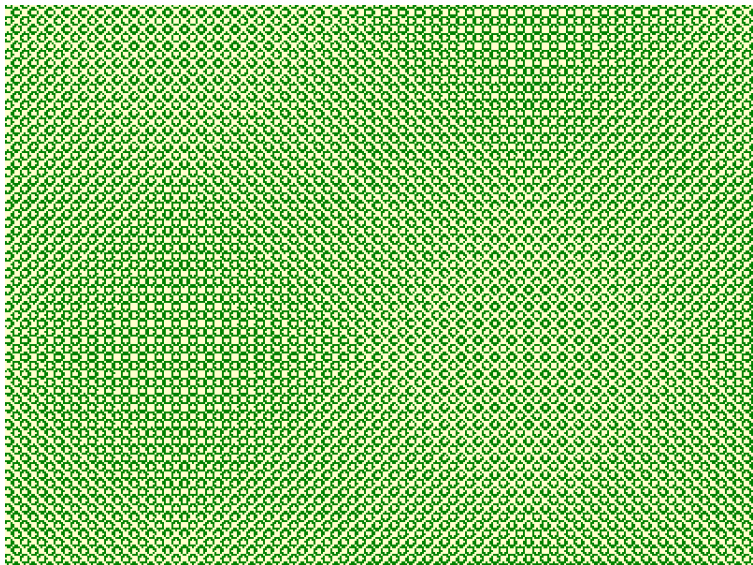
$$\sin x + \cos y \quad 0.5 \quad 0.5 \quad 2$$



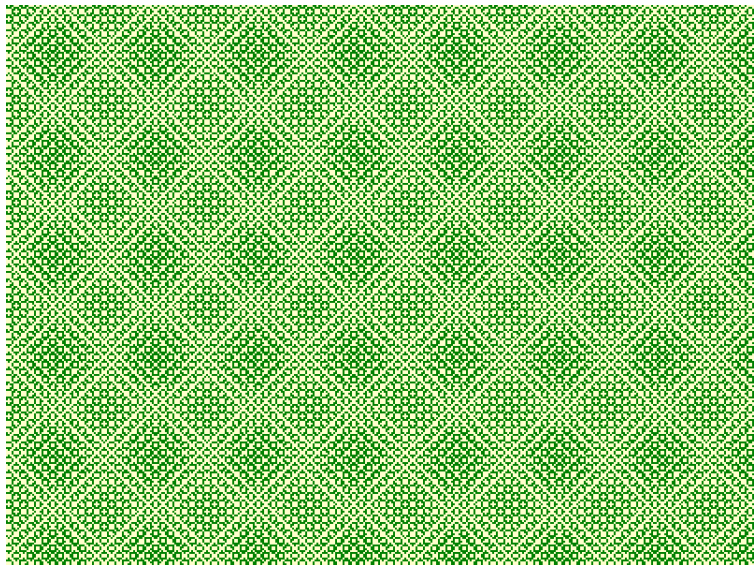
$$\sin x + \cos y \quad 0.6 \quad 0.6 \quad 2$$



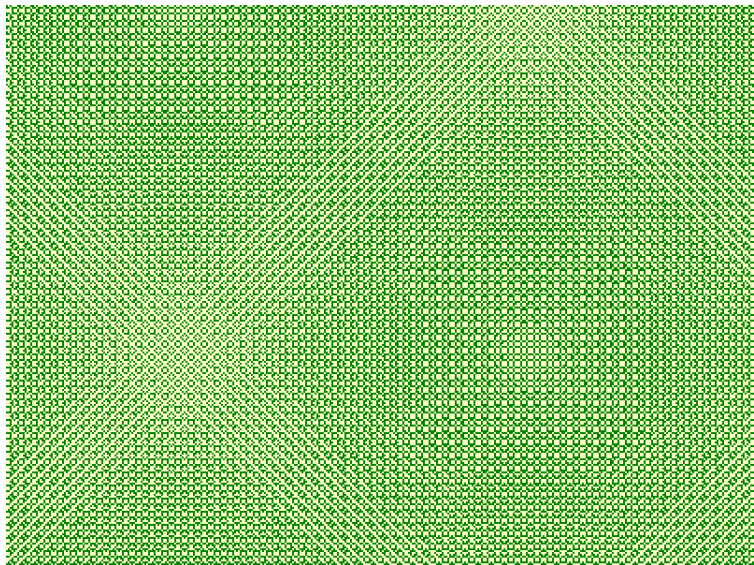
$$\sin x + \cos y \quad 0.7 \quad 0.7 \quad 2$$



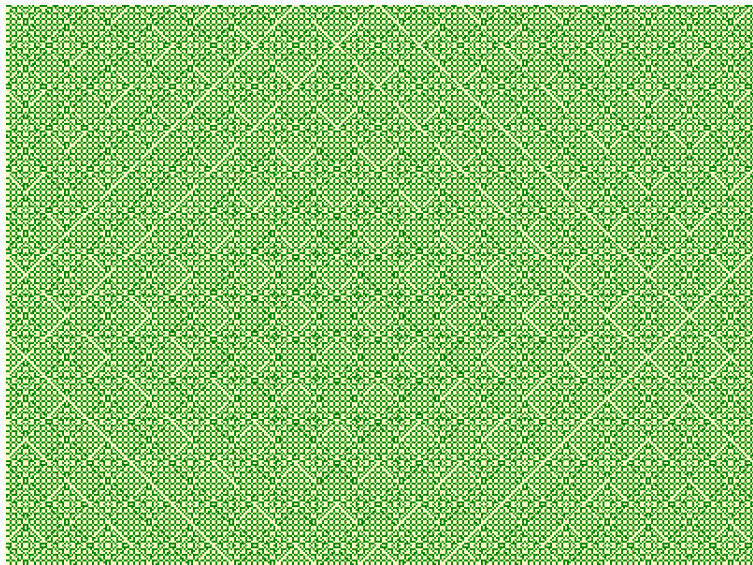
$$\sin x + \cos y \quad 0.8 \quad 0.8 \quad 2$$



$$\sin x + \cos y \quad 0.9 \quad 0.9 \quad 2$$

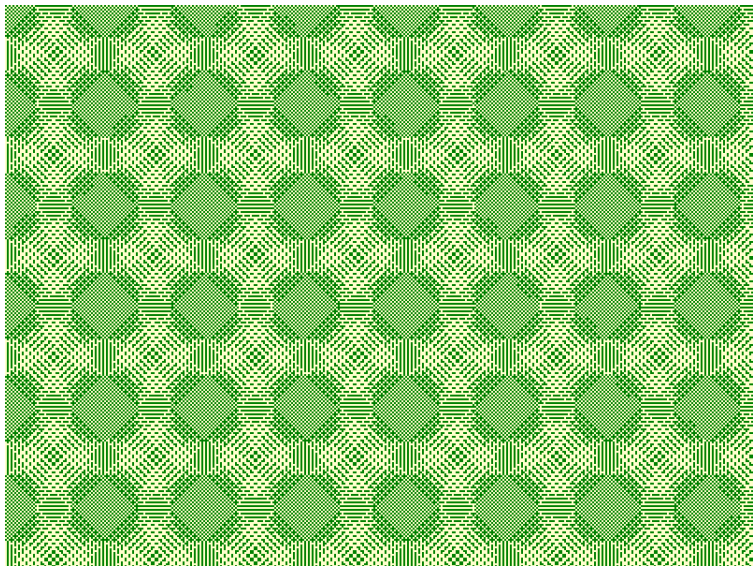


$$\sin x + \cos y \quad 1.0 \quad 1.0 \quad 2$$

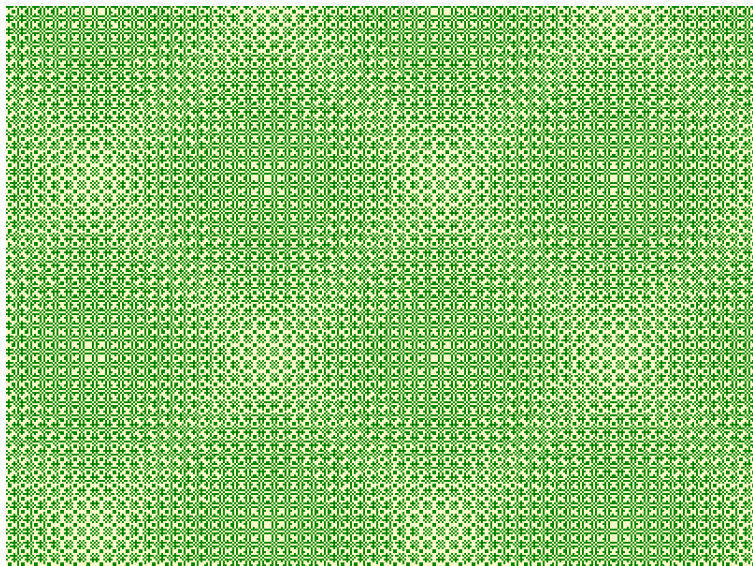


$\sin x + \cos y$

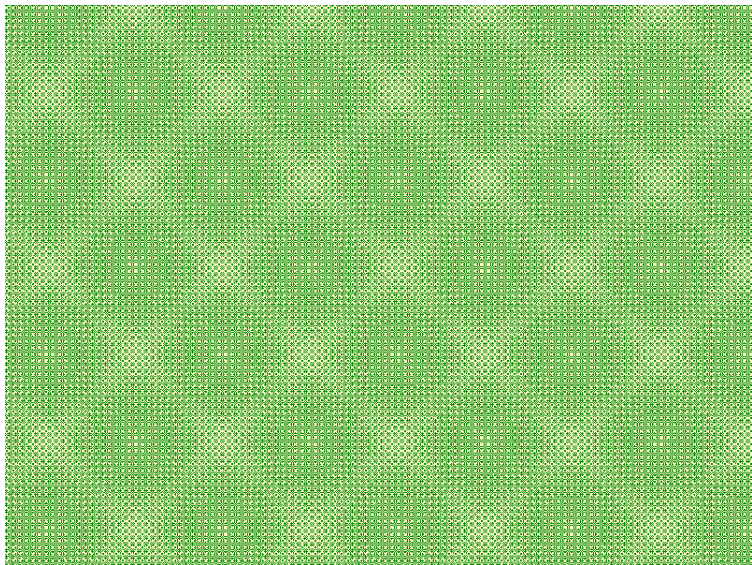
1.6 1.6 2



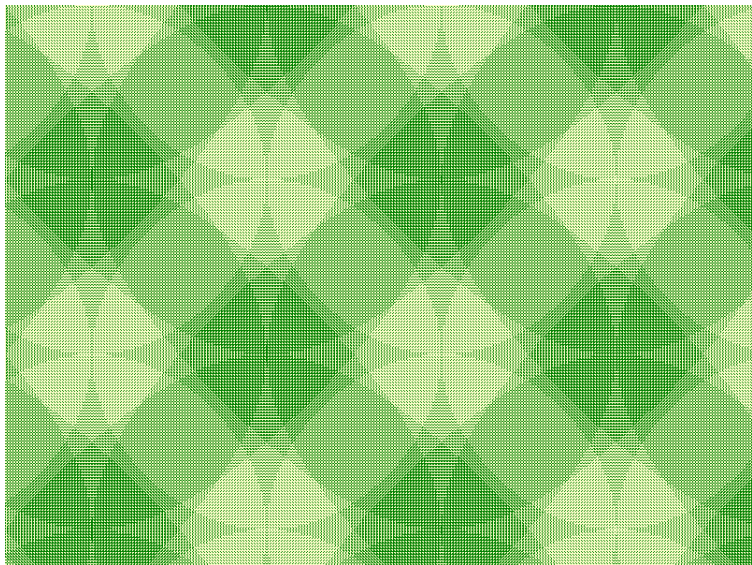
$$\sin x + \cos y \quad 1.8 \quad 1.8 \quad 2$$



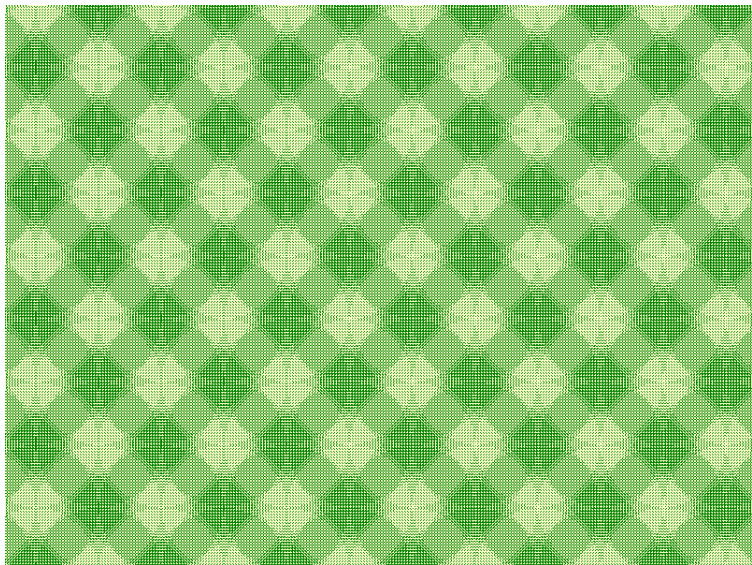
$$\sin x + \cos y \quad 1.8 \quad 1.8 \quad 1$$



$$\sin x + \cos y \quad 2.1 \quad 2.1 \quad 1$$

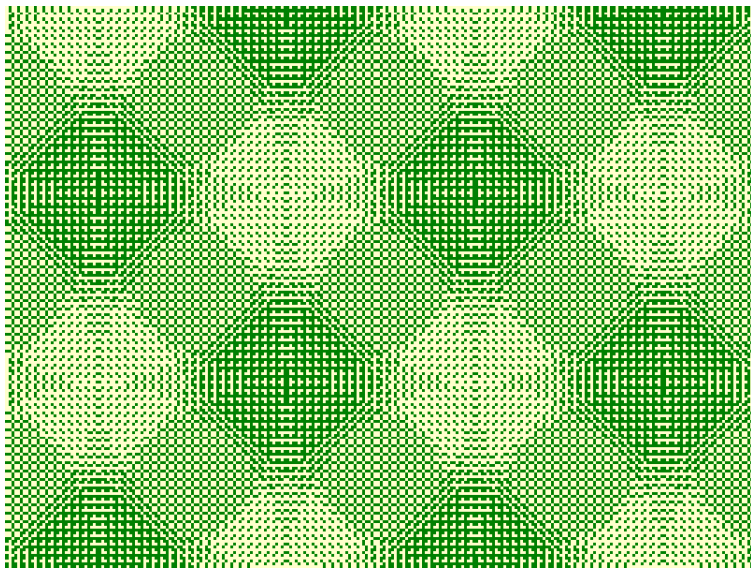


$$\sin x + \cos y \quad 2.11 \quad 2.11 \quad 1$$

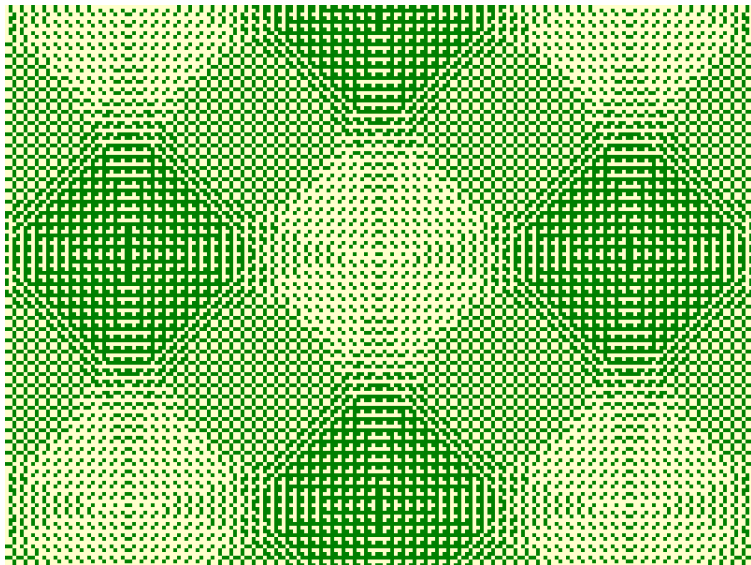


$\sin x + \cos y$

2.11 2.11 3

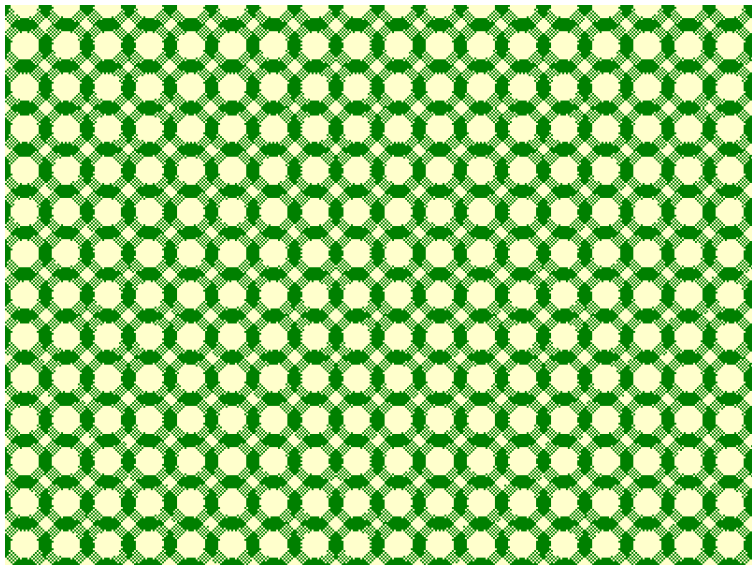


$$\sin x + \cos y \quad 2.11 \quad 2.11 \quad 4$$



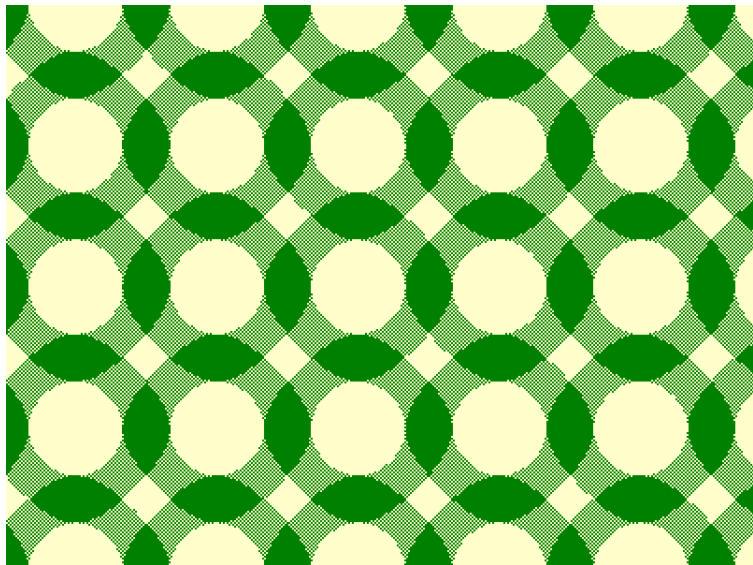
$\sin x + \cos y$

3.0 3.0 2



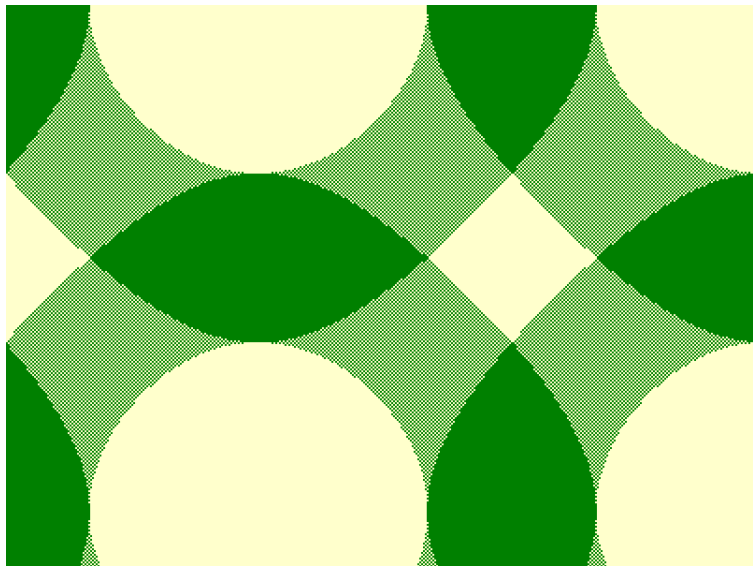
$\sin x + \cos y$

3.1 3.1 2

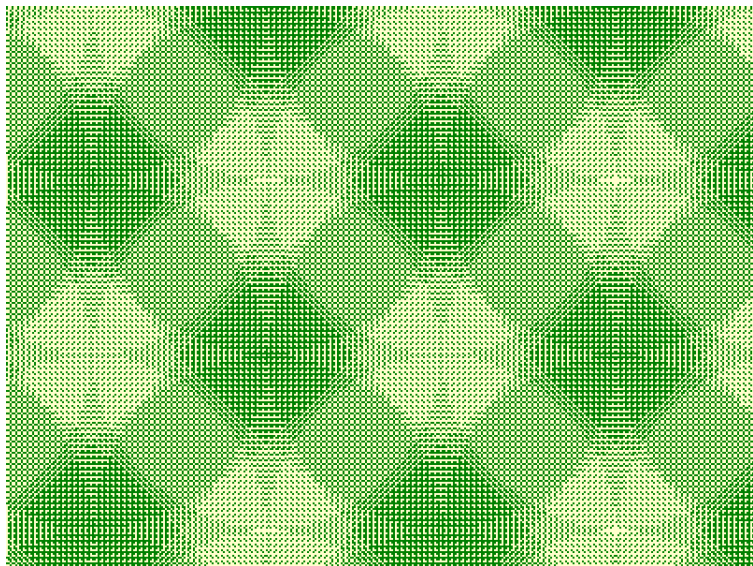


$\sin x + \cos y$

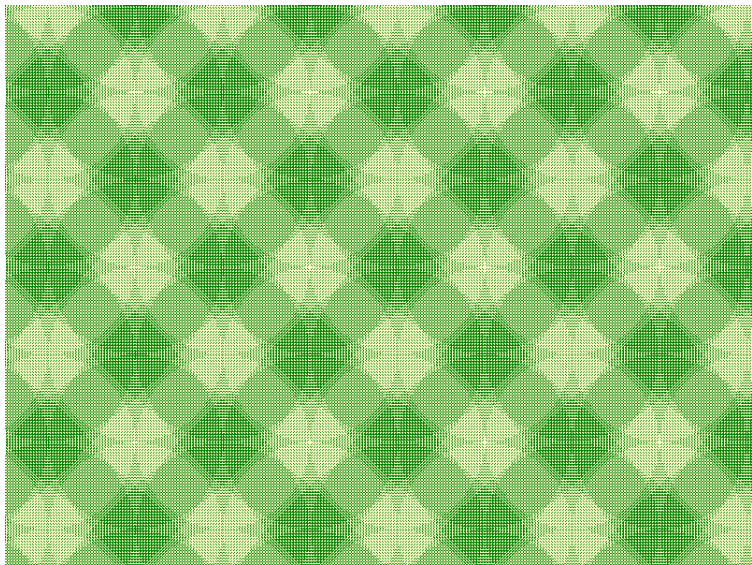
3.13 3.13 2



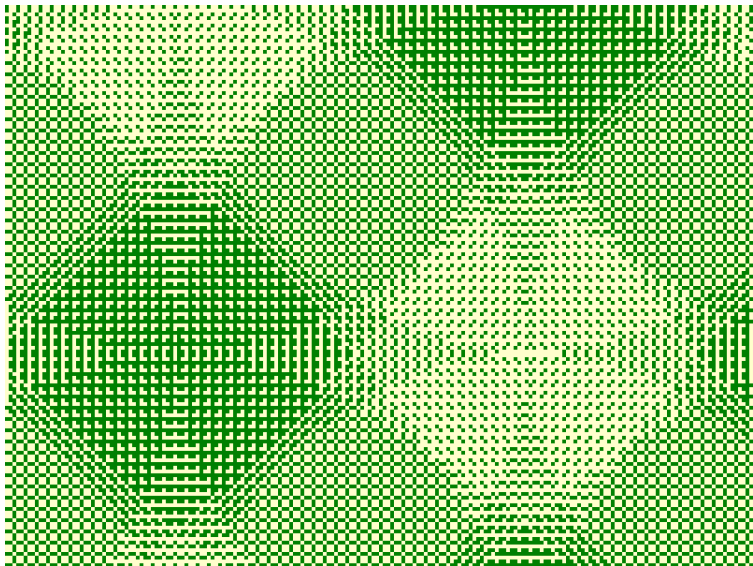
$$\sin x + \cos y \quad 4.2 \quad 4.2 \quad 2$$



$$\sin x + \cos y \quad 4.2 \quad 4.2 \quad 1$$

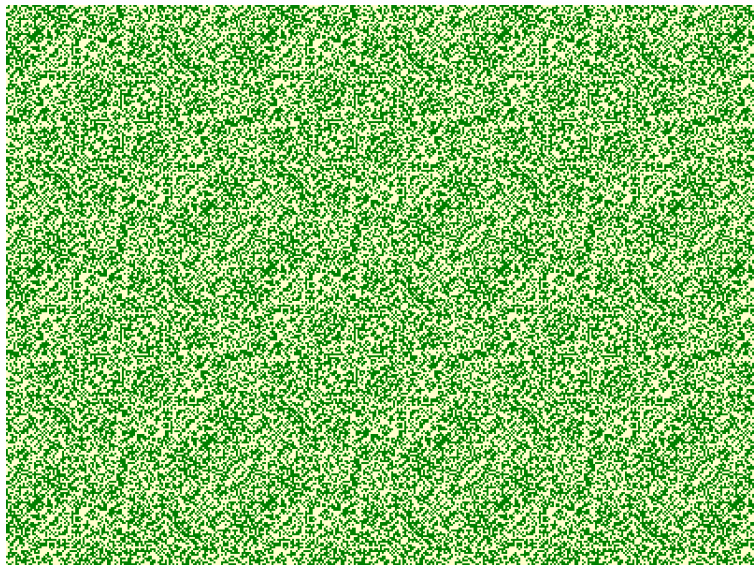


$$\sin x + \cos y \quad 4.2 \quad 4.2 \quad 4$$



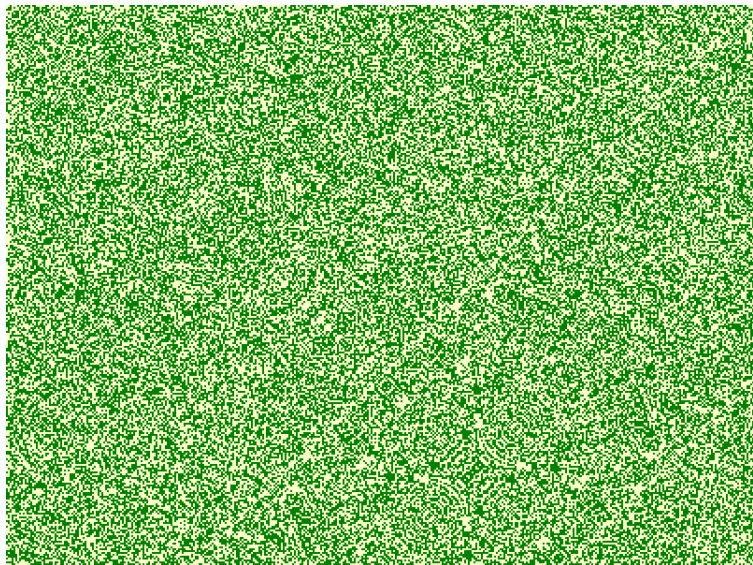
$$x^2 + x^2y + xy^2 + y^3$$

1.6 1.6 2



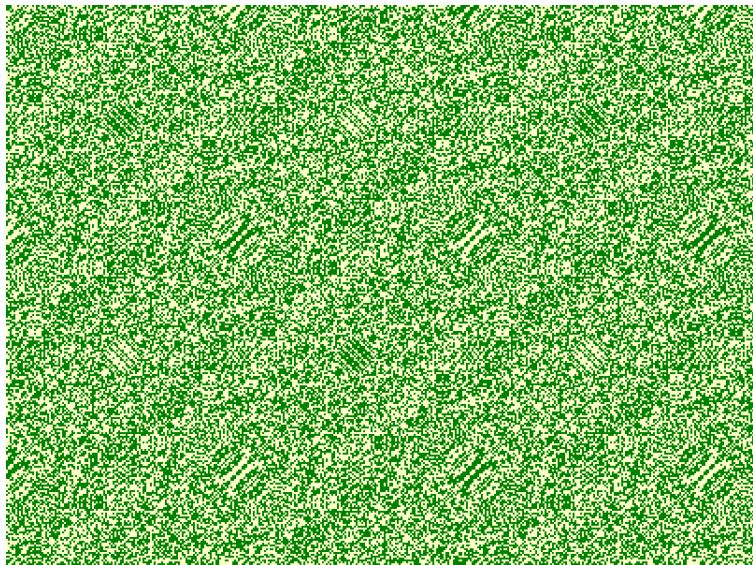
$$x^2 + x^2y + xy^2 + y^3$$

1.7 1.7 2



$$x^2 + x^2y + xy^2 + y^3$$

1.8 1.8 2



$$x^2 + x^2y + xy^2 + y^3$$

1.9 1.9 2

