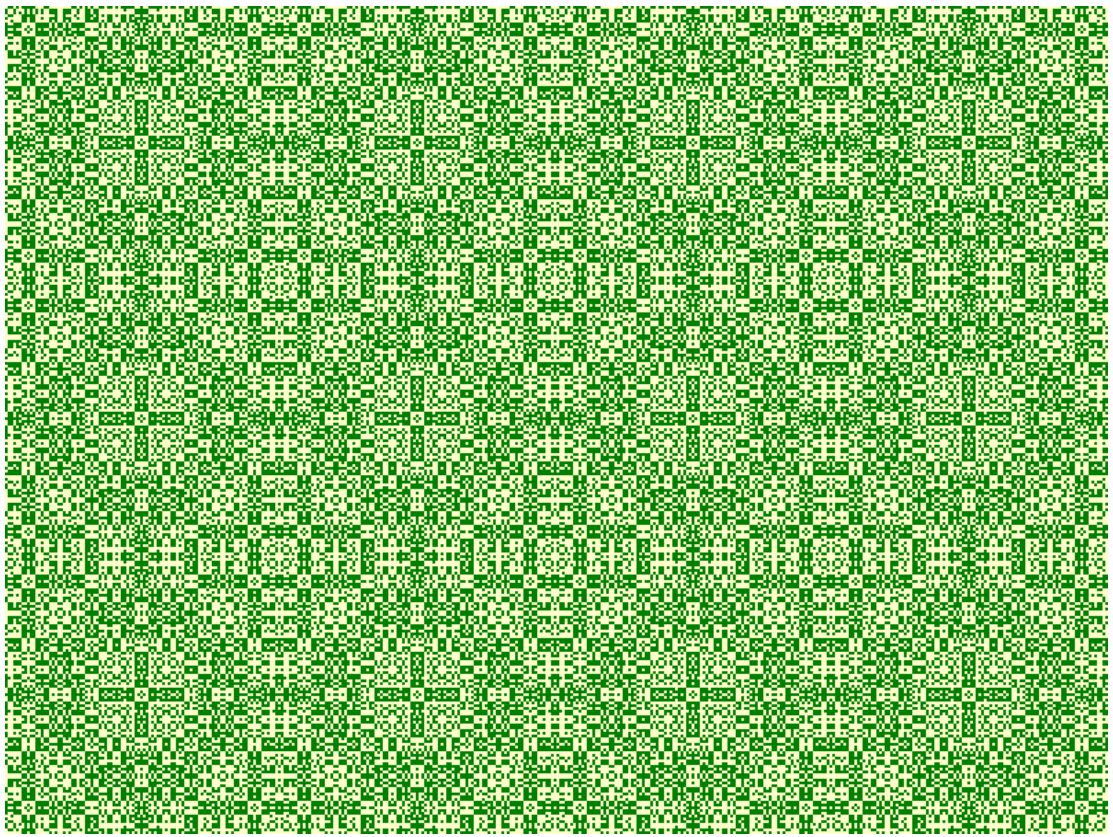
Hidden Patterns in Functions

Brian Heinold
Mount St. Mary's University

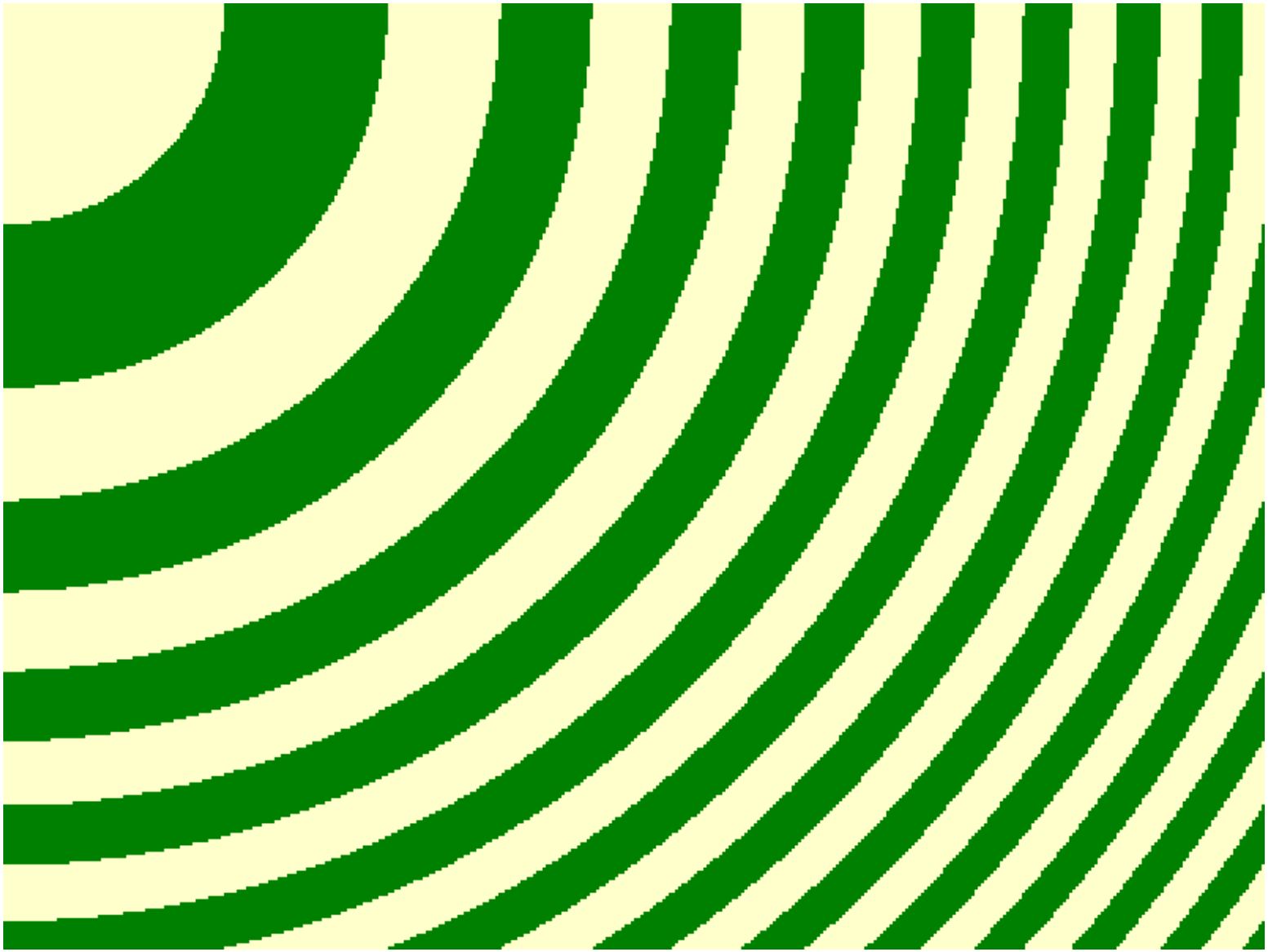


Definition

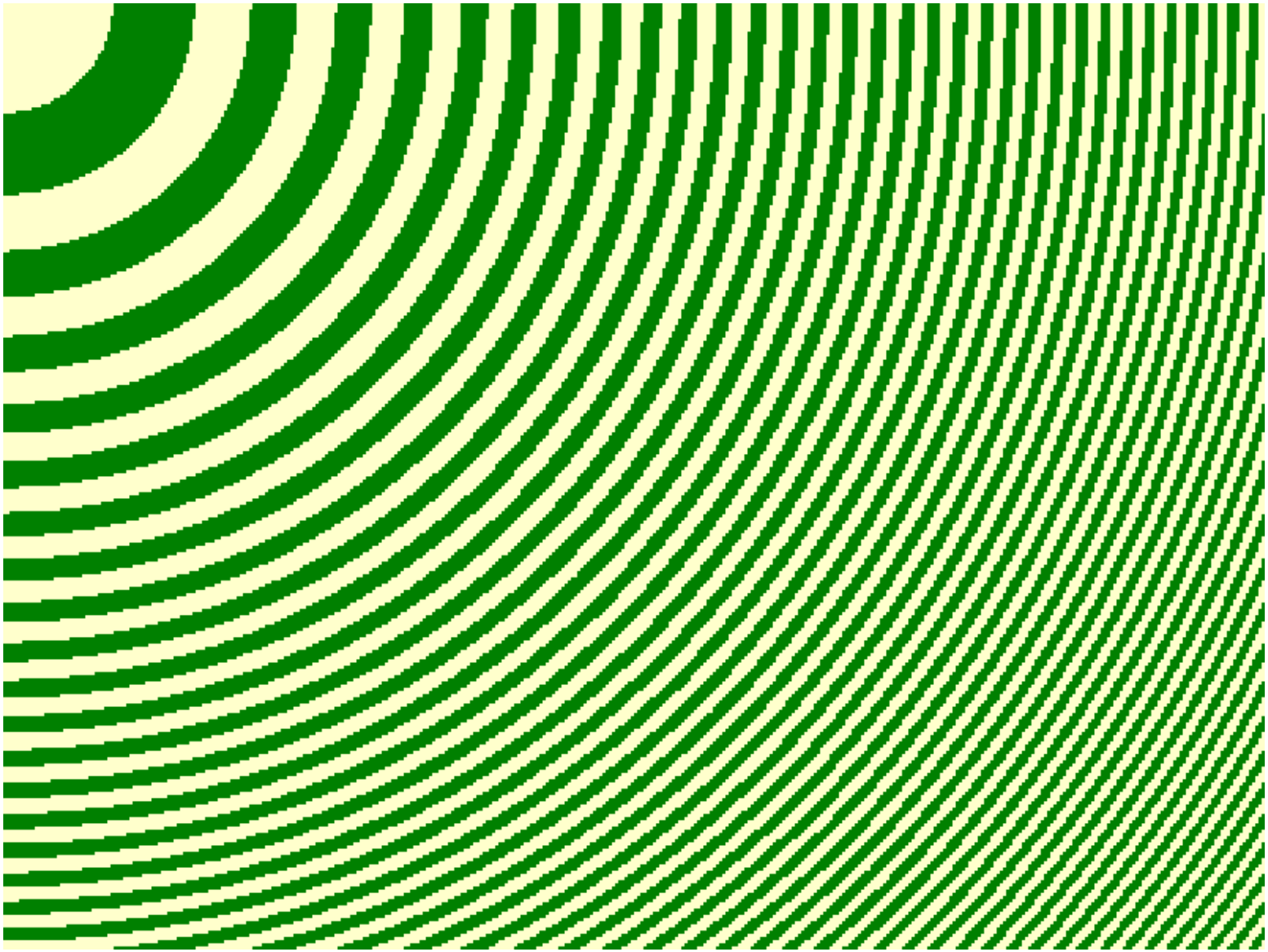
Given a function $f(x, y)$
Fix $a, b \in \mathbb{R}$. Lattice of points $\{(ja, kb) : j, k \in \mathbb{Z}^+\}$

1. Compute $f(x, y)$ at each lattice point
2. Round the result to the nearest integer
3. Mod this result by 2

$x^2 + y^2$ 0.01 0.01 2



$x^2 + y^2$ 0.02 0.02 2



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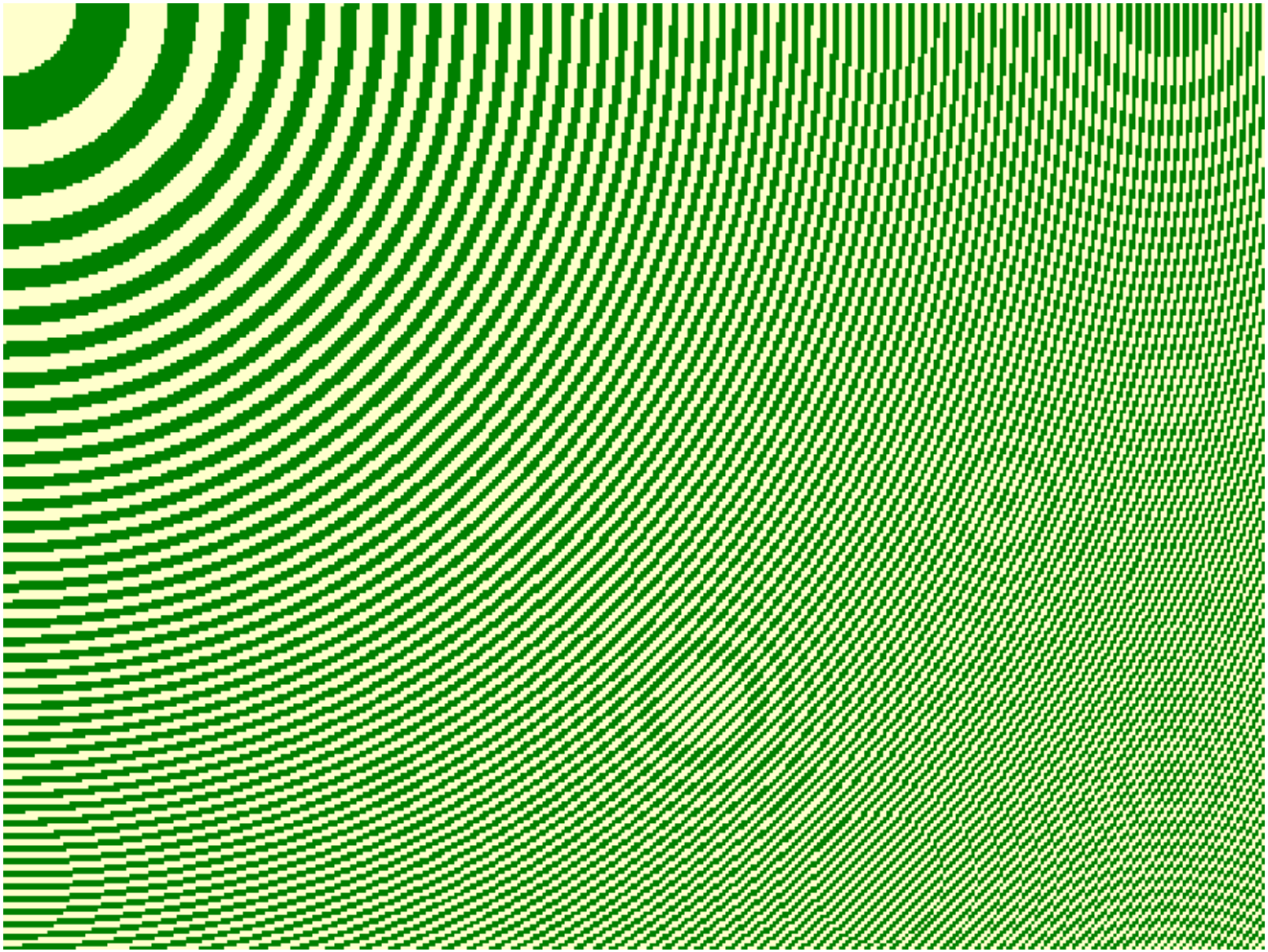

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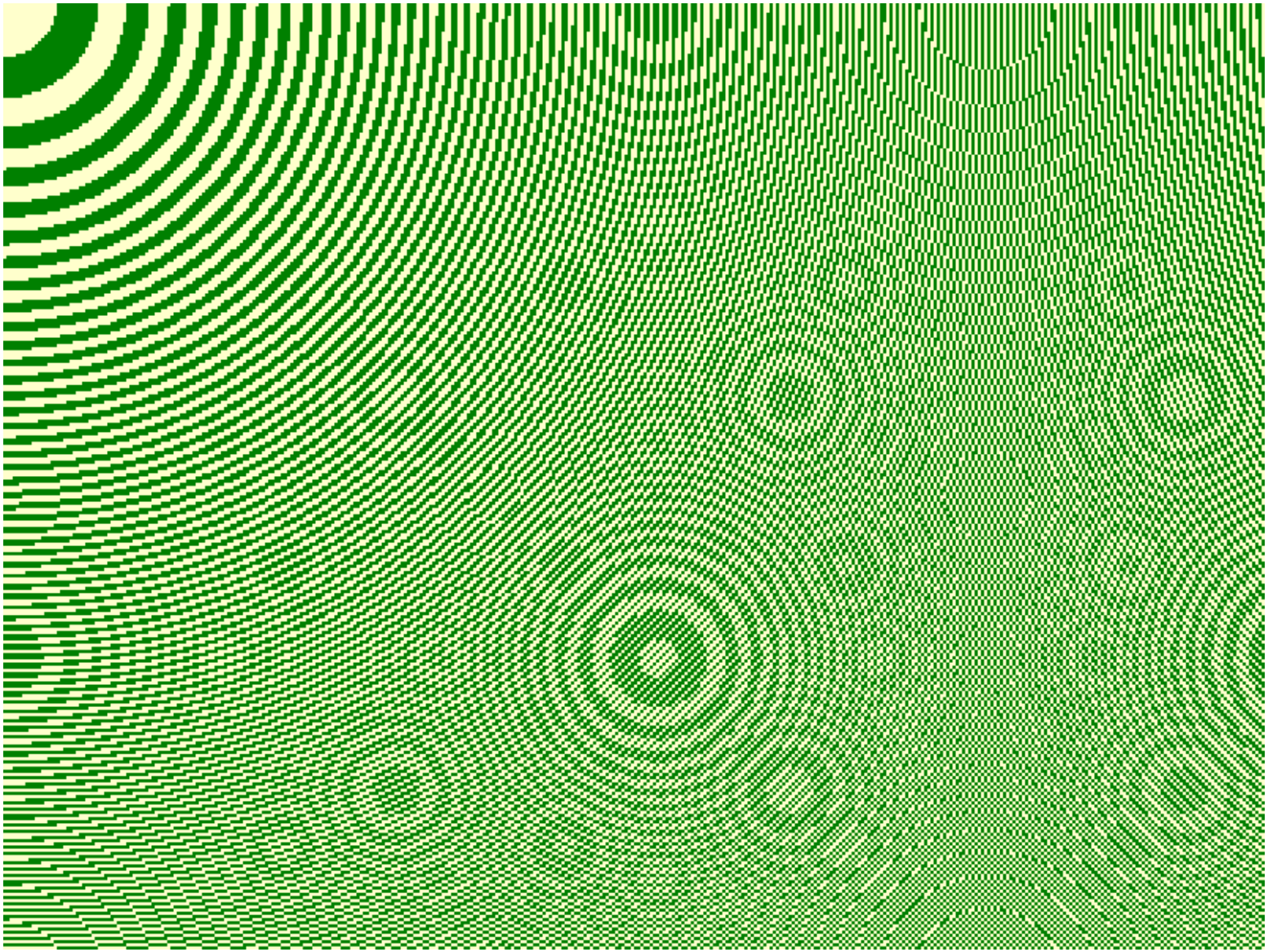

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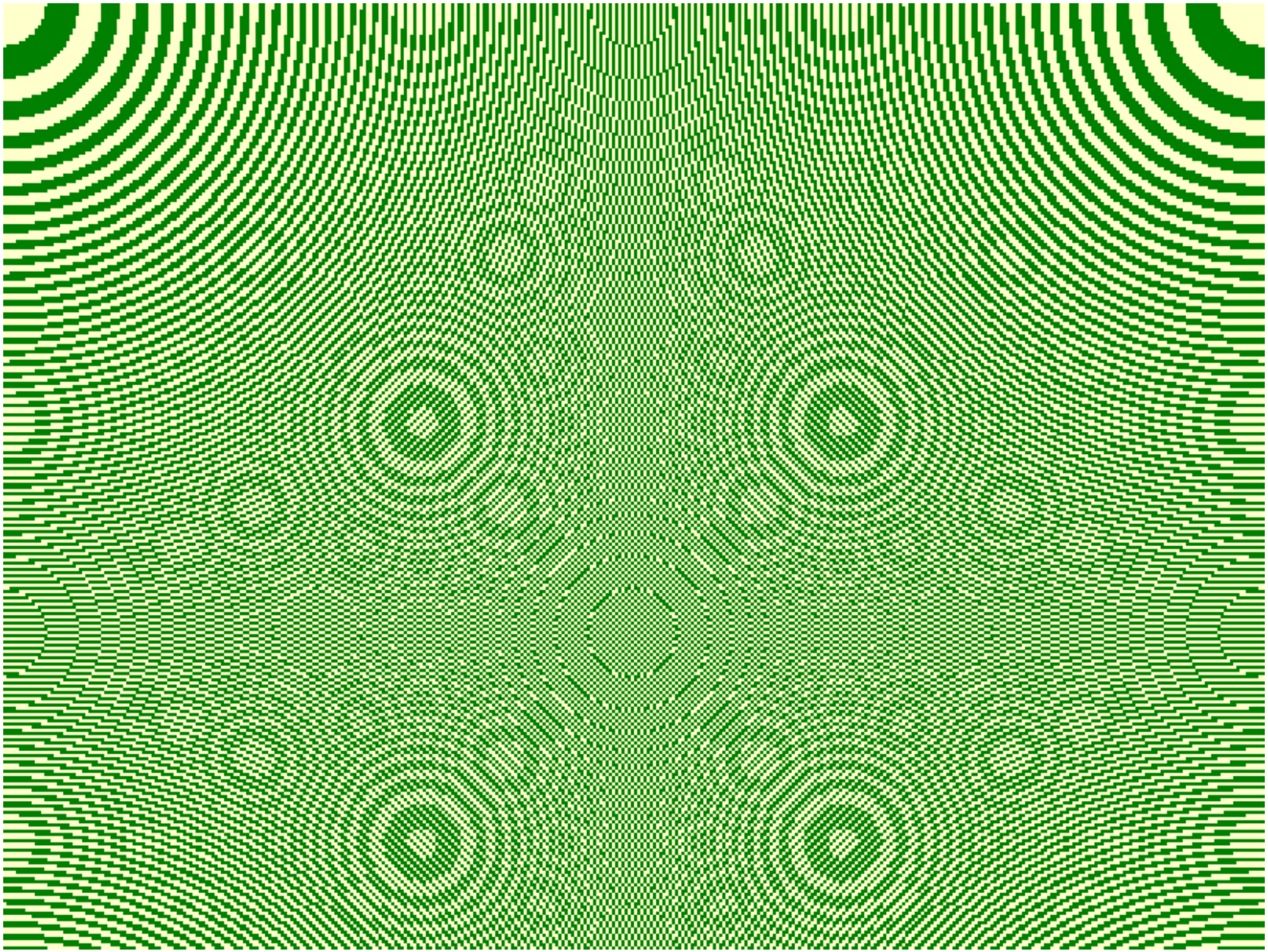
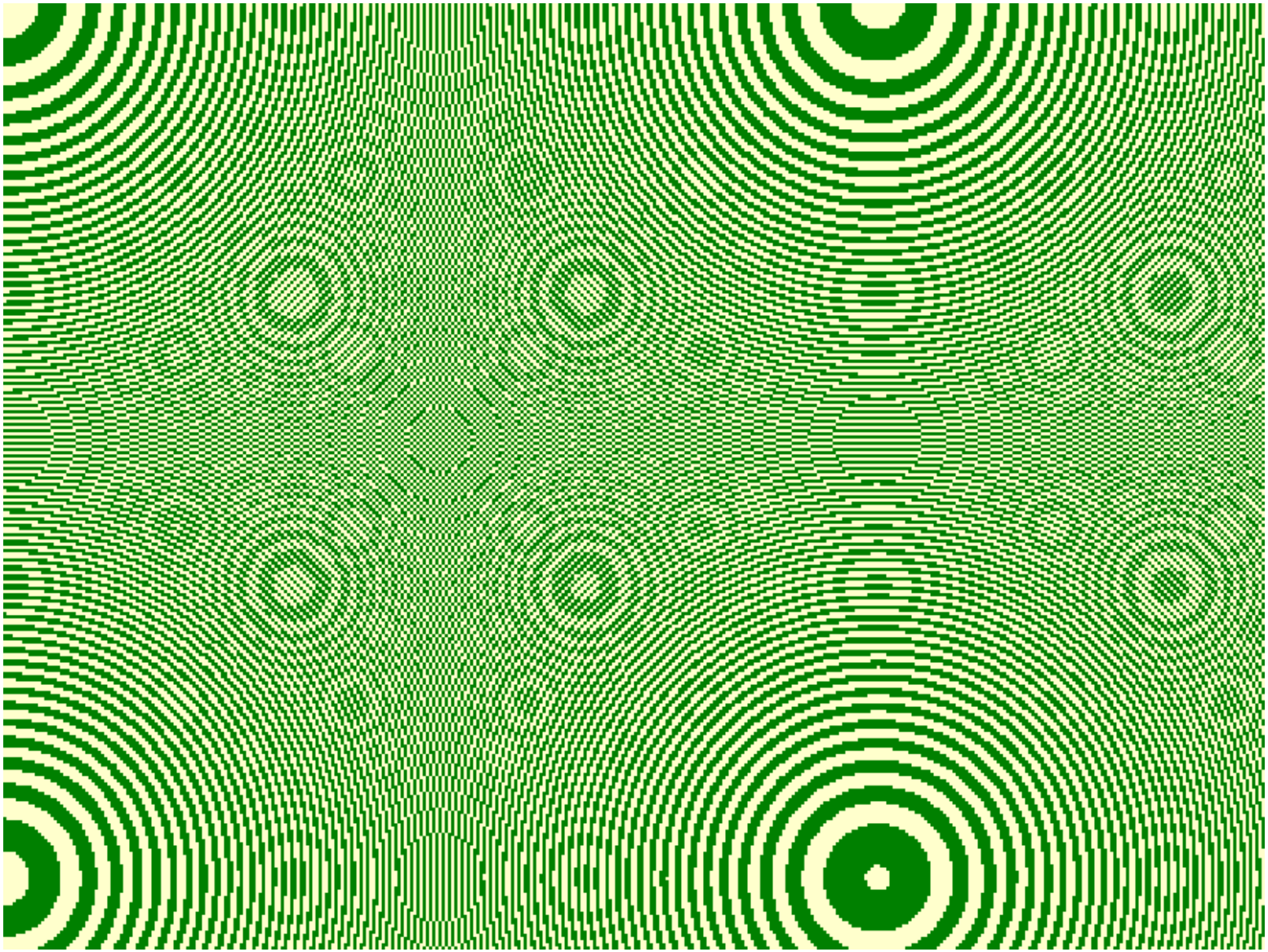


$x^2 + y^2$ 0.06 0.06 2



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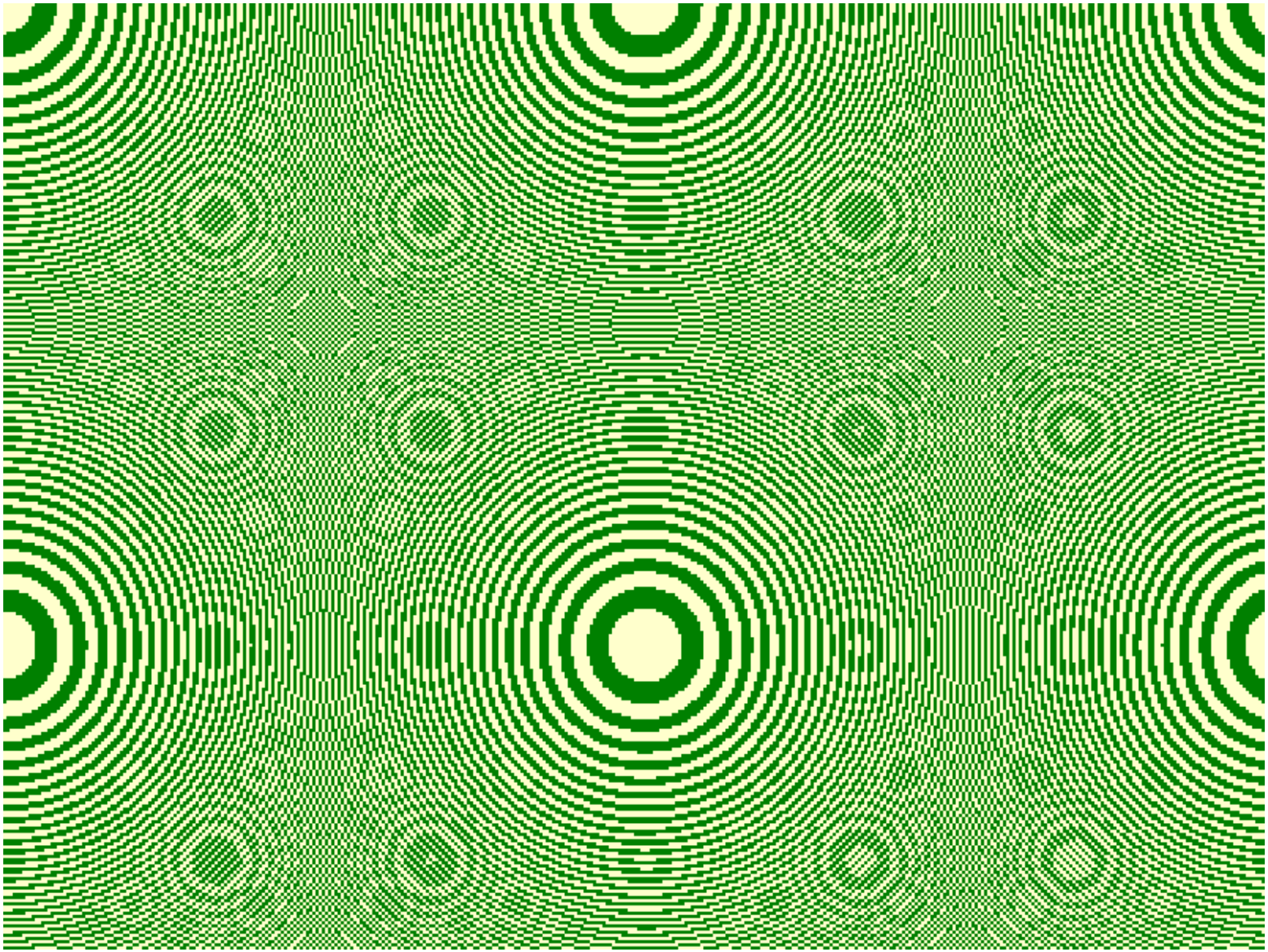
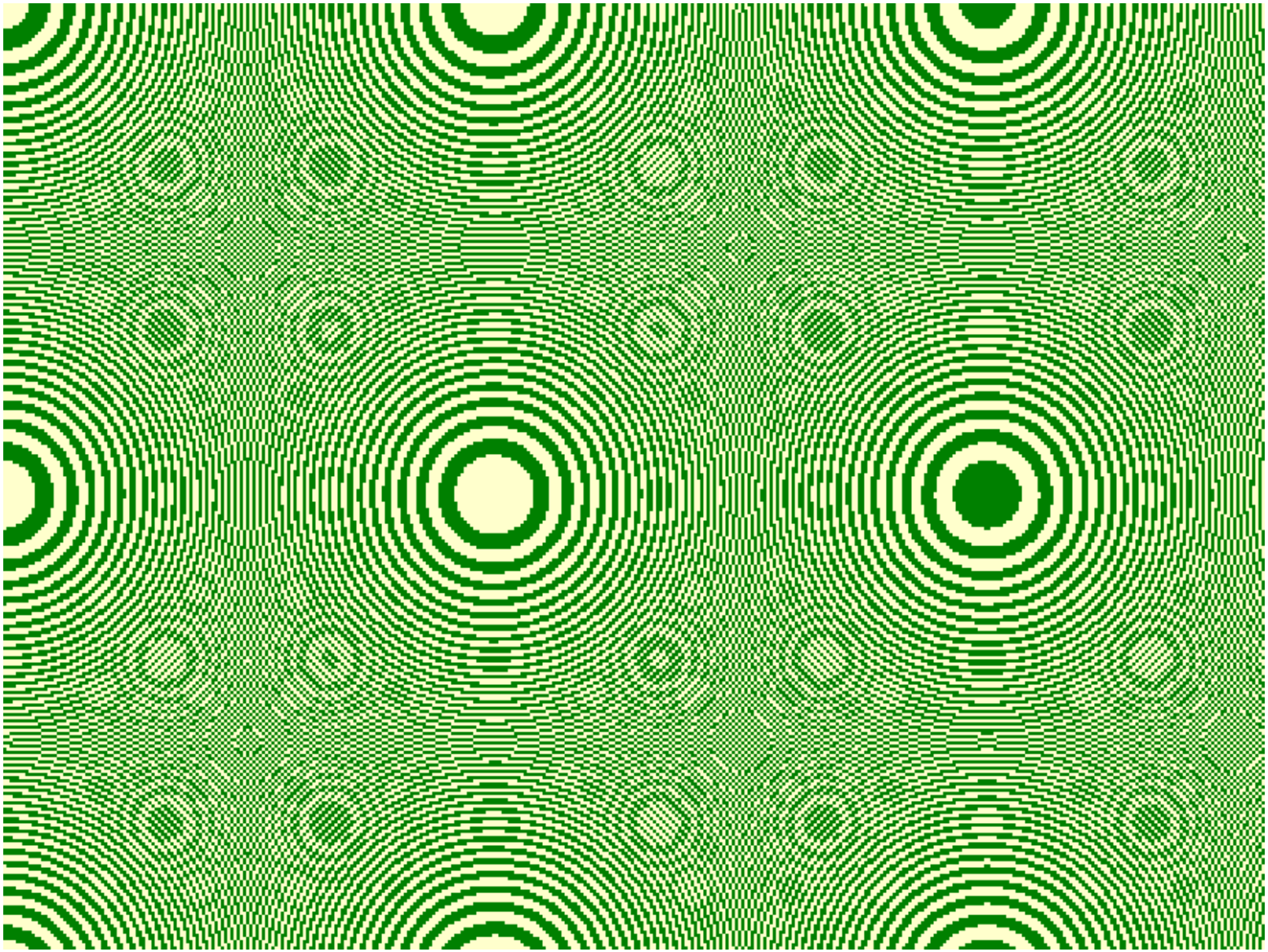
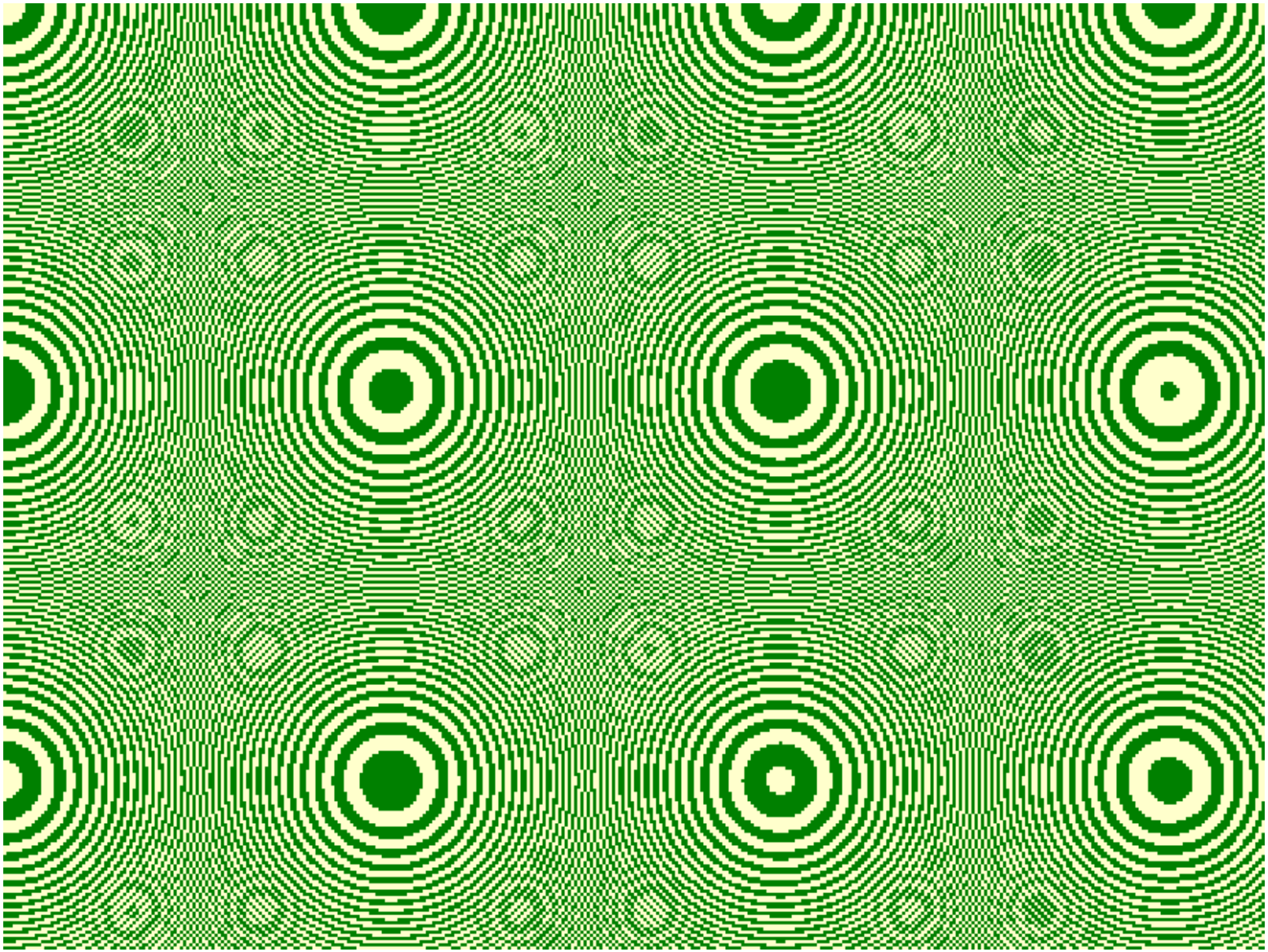


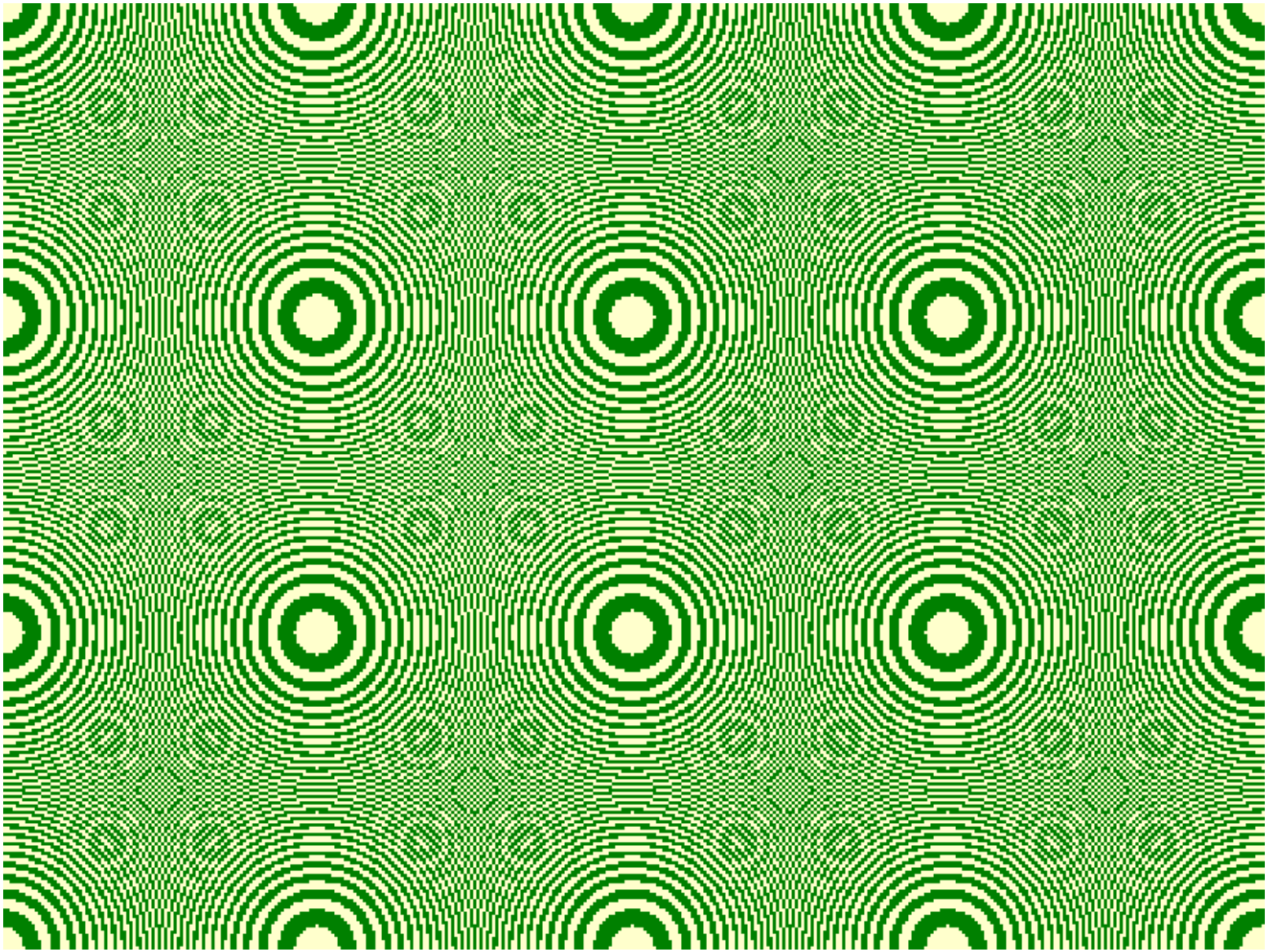
$x^2 + y^2$ 0.08 0.08 2



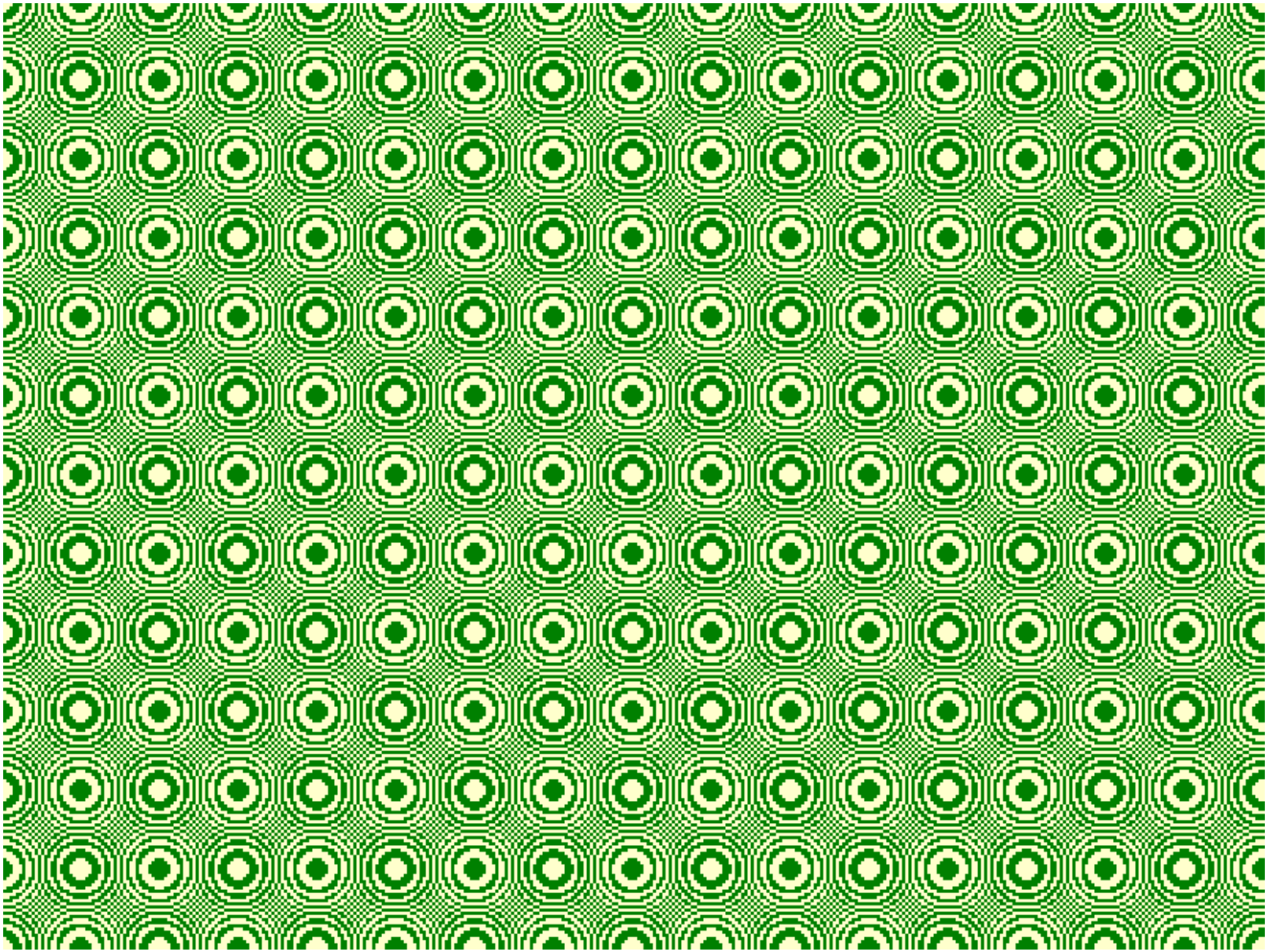
$x^2 + y^2$ 0.09 0.09 2



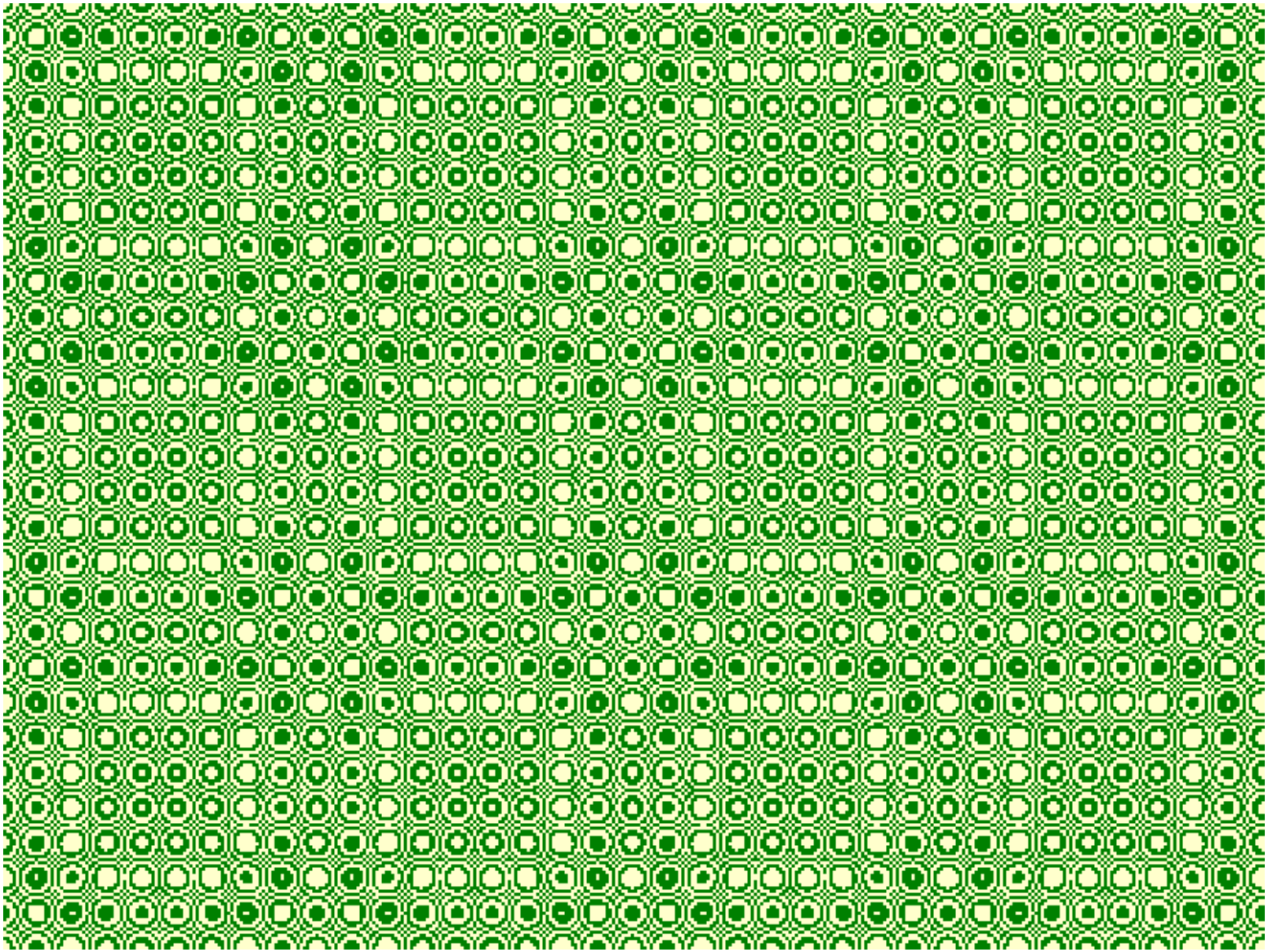
$x^2 + y^2$ 0.1 0.1 2



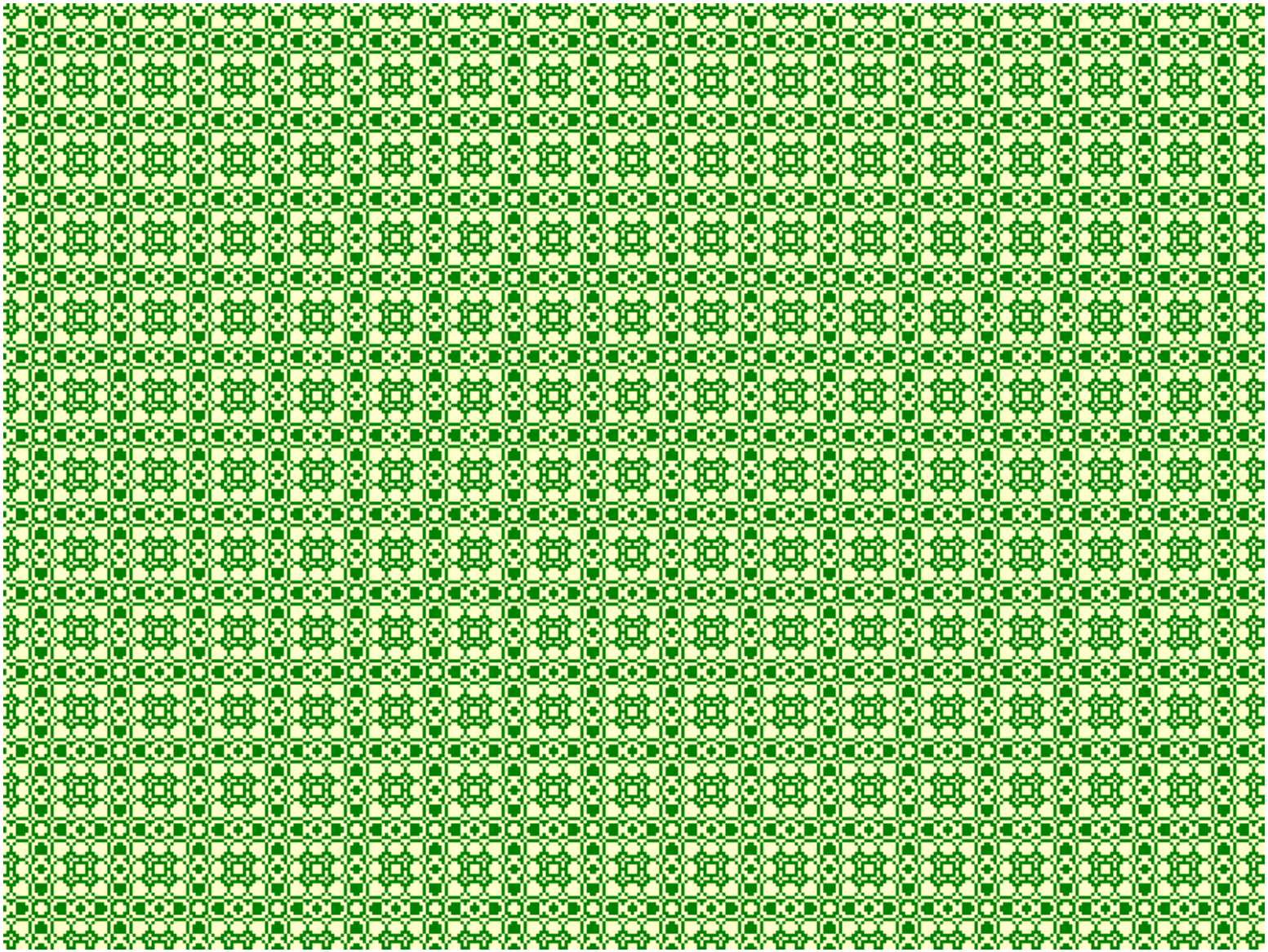
$x^2 + y^2$ 0.2 0.2 2



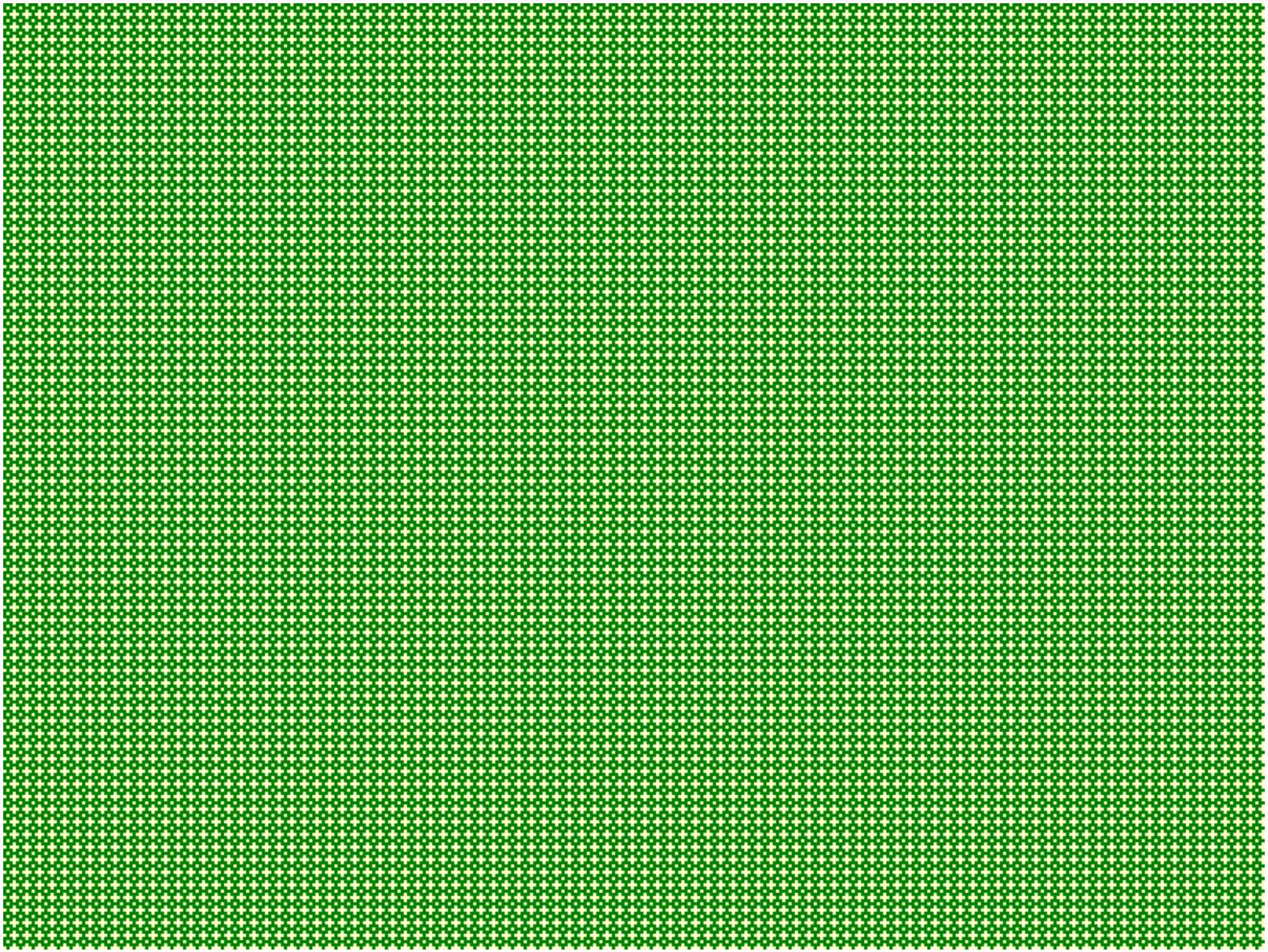
$x^2 + y^2$ 0.3 0.3 2



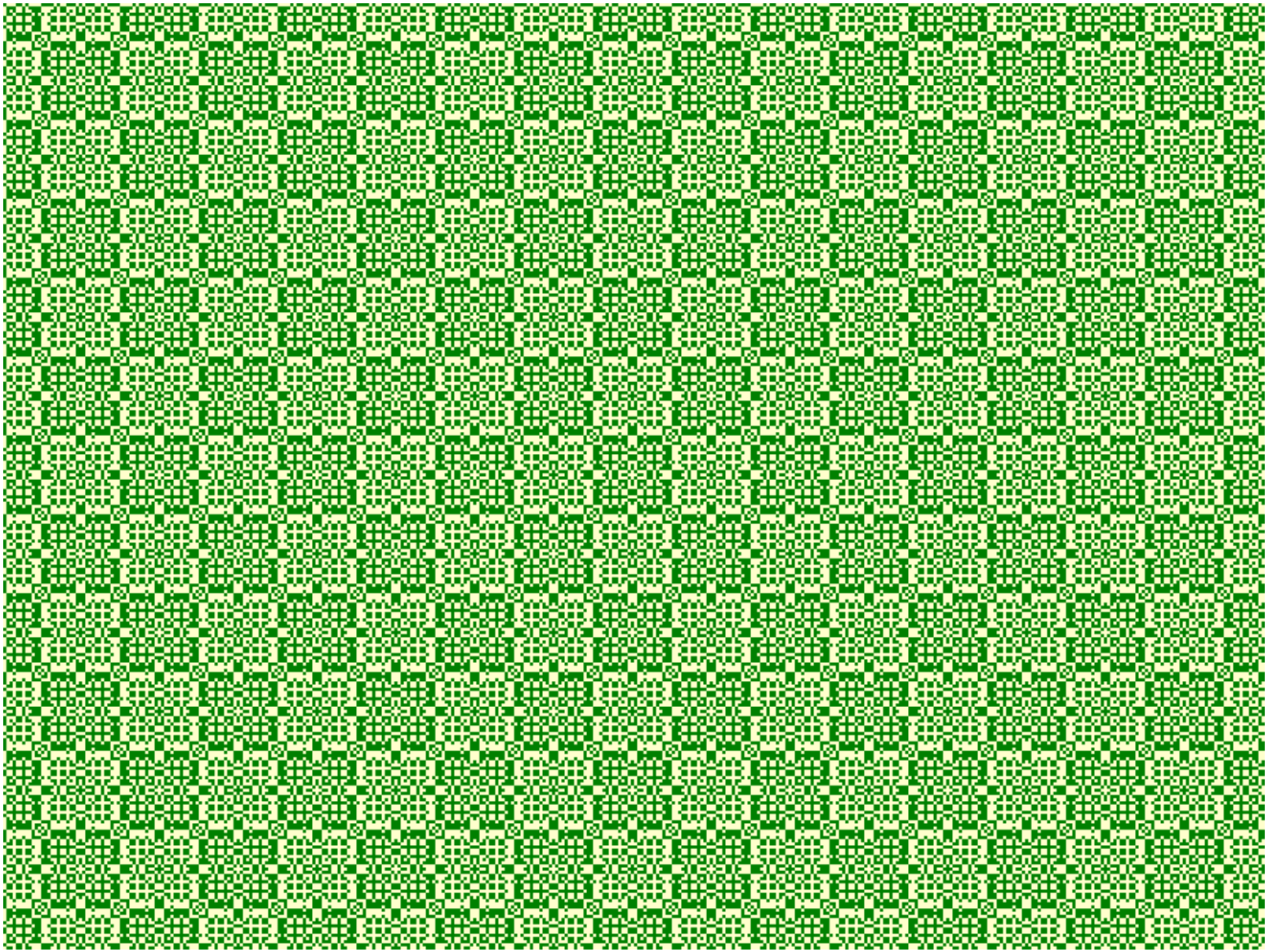
$x^2 + y^2$ 0.4 0.4 2



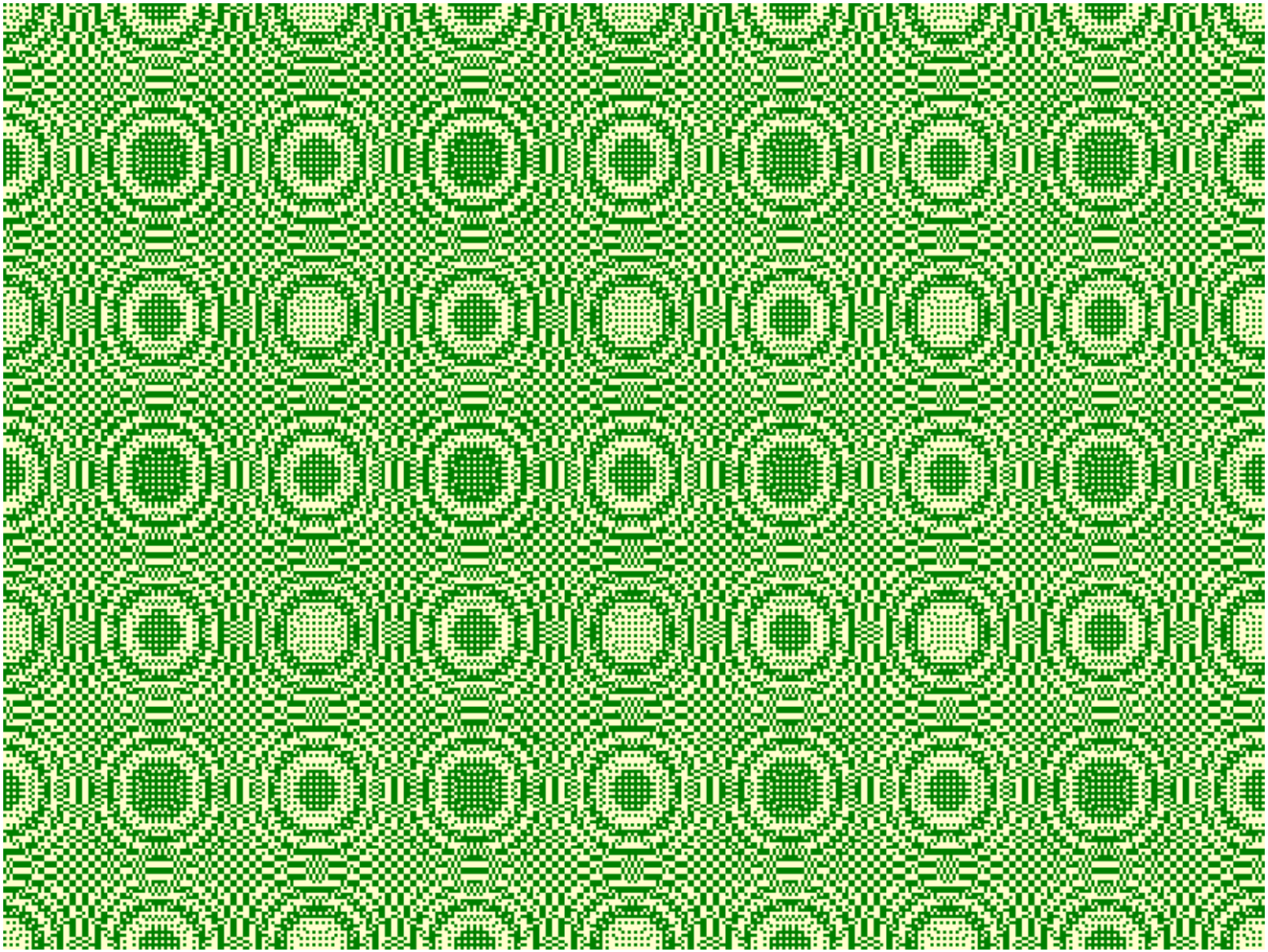
$x^2 + y^2$ 0.5 0.5 2



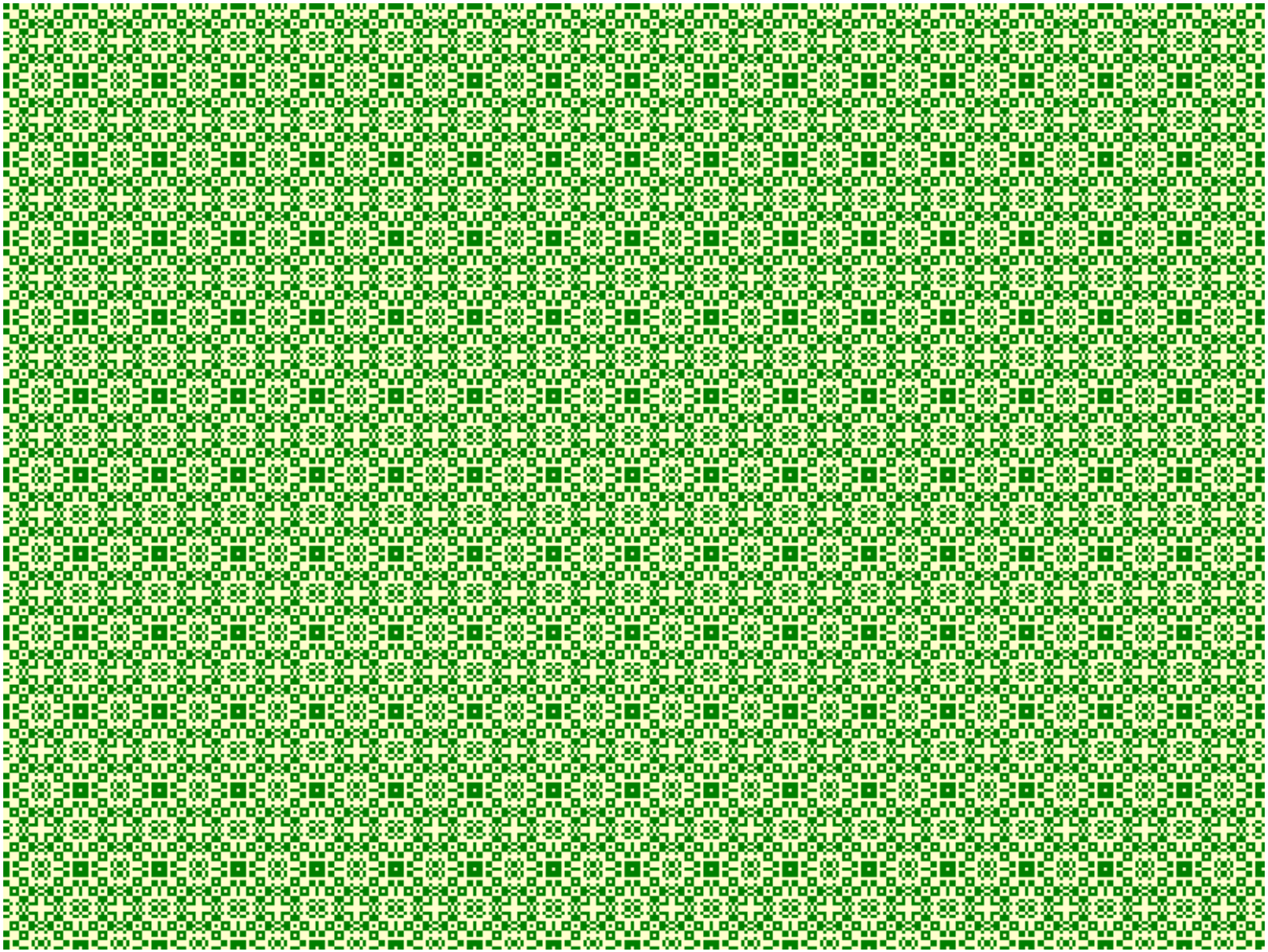
$x^2 + y^2$ 0.6 0.6 2



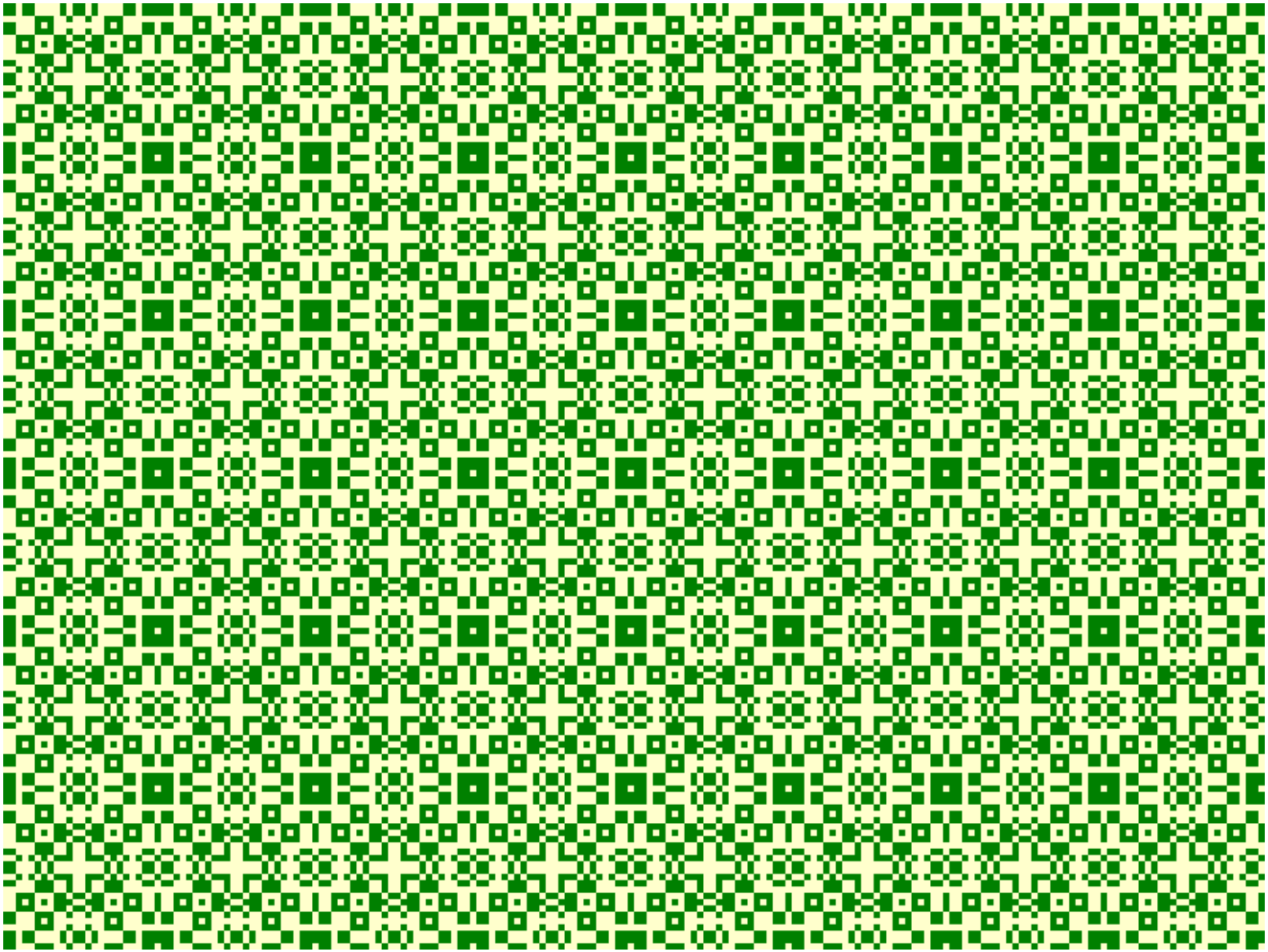
$x^2 + y^2$ 0.7 0.7 2



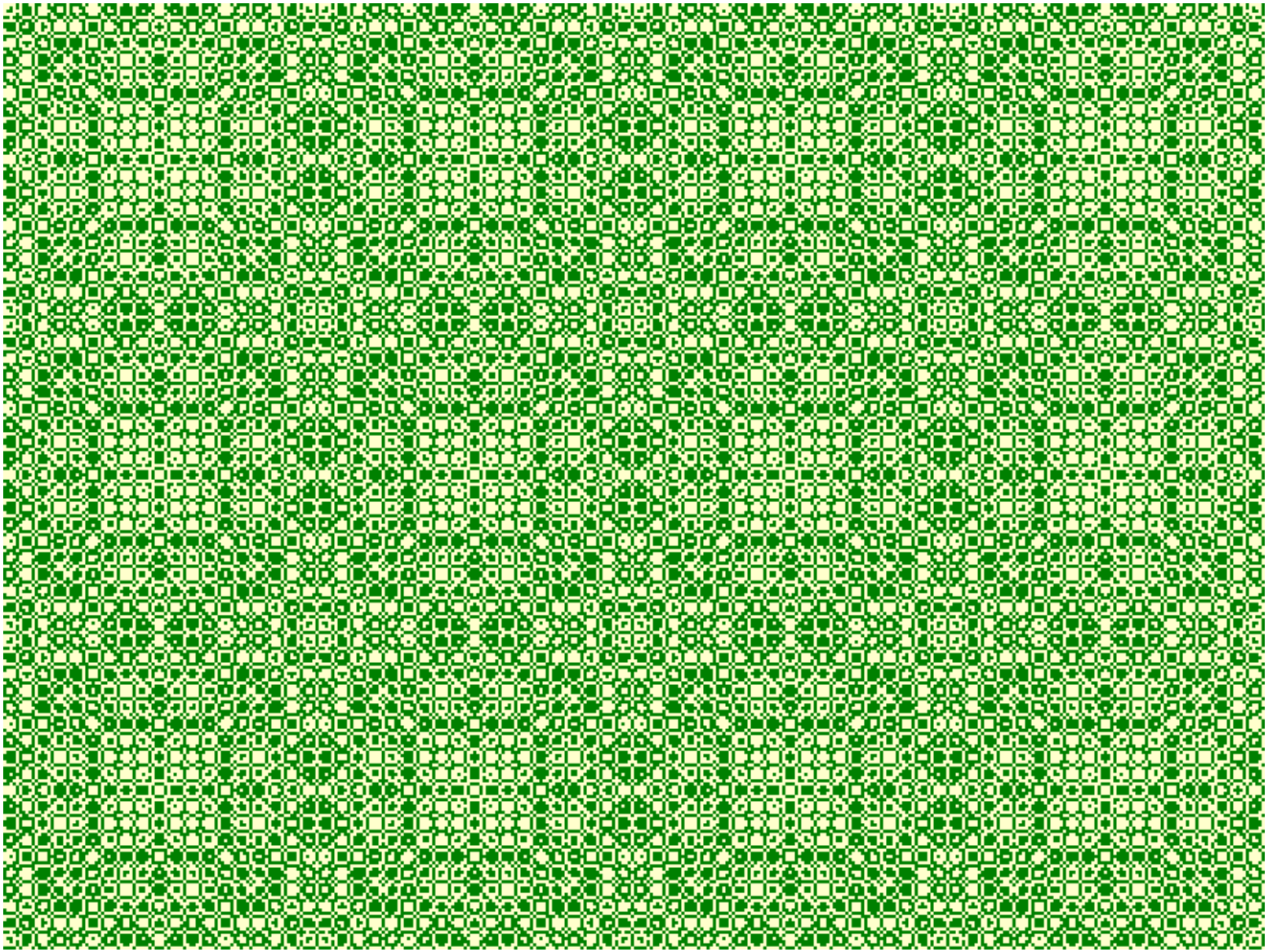
$x^2 + y^2$ 0.8 0.8 2

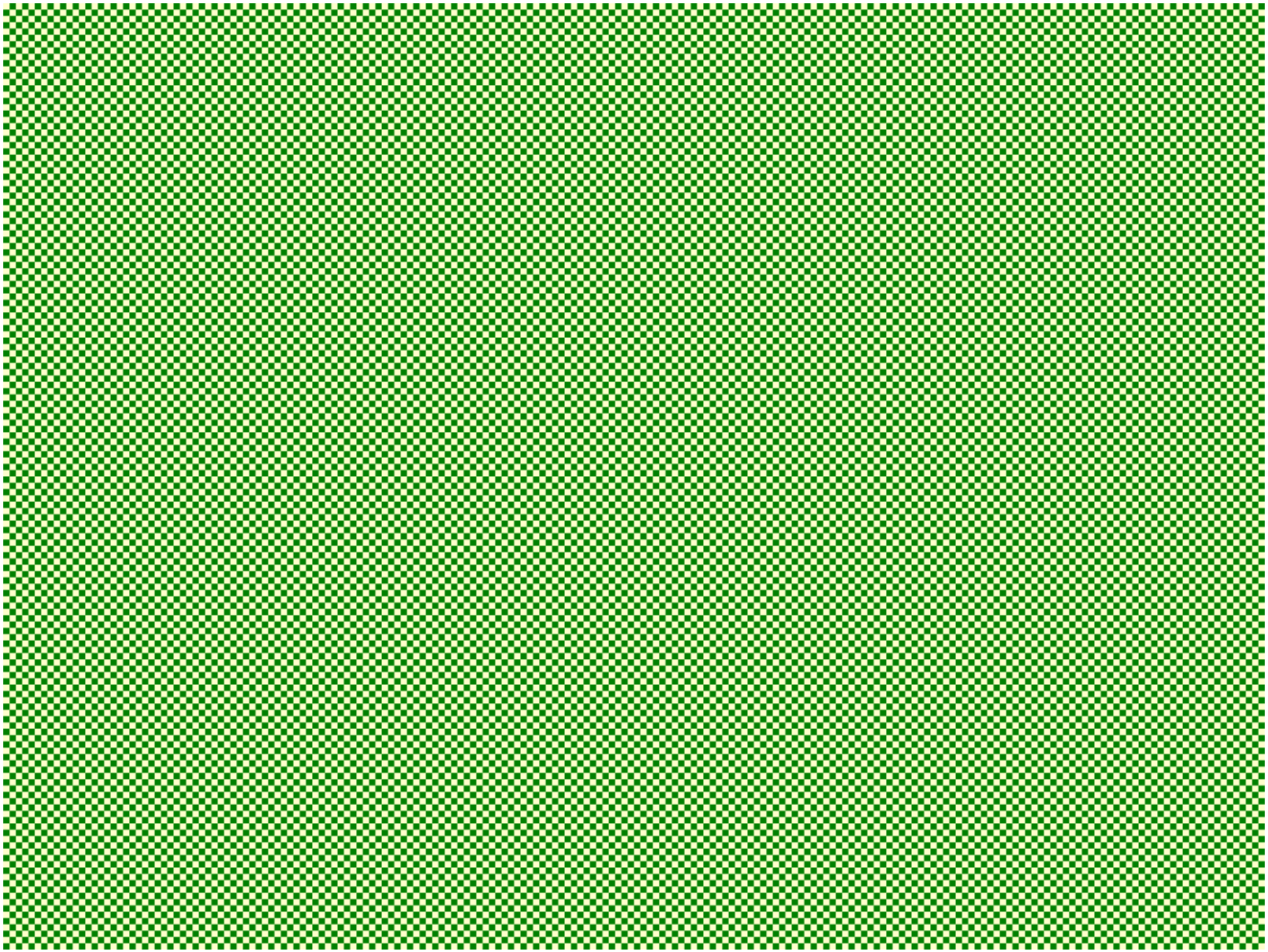


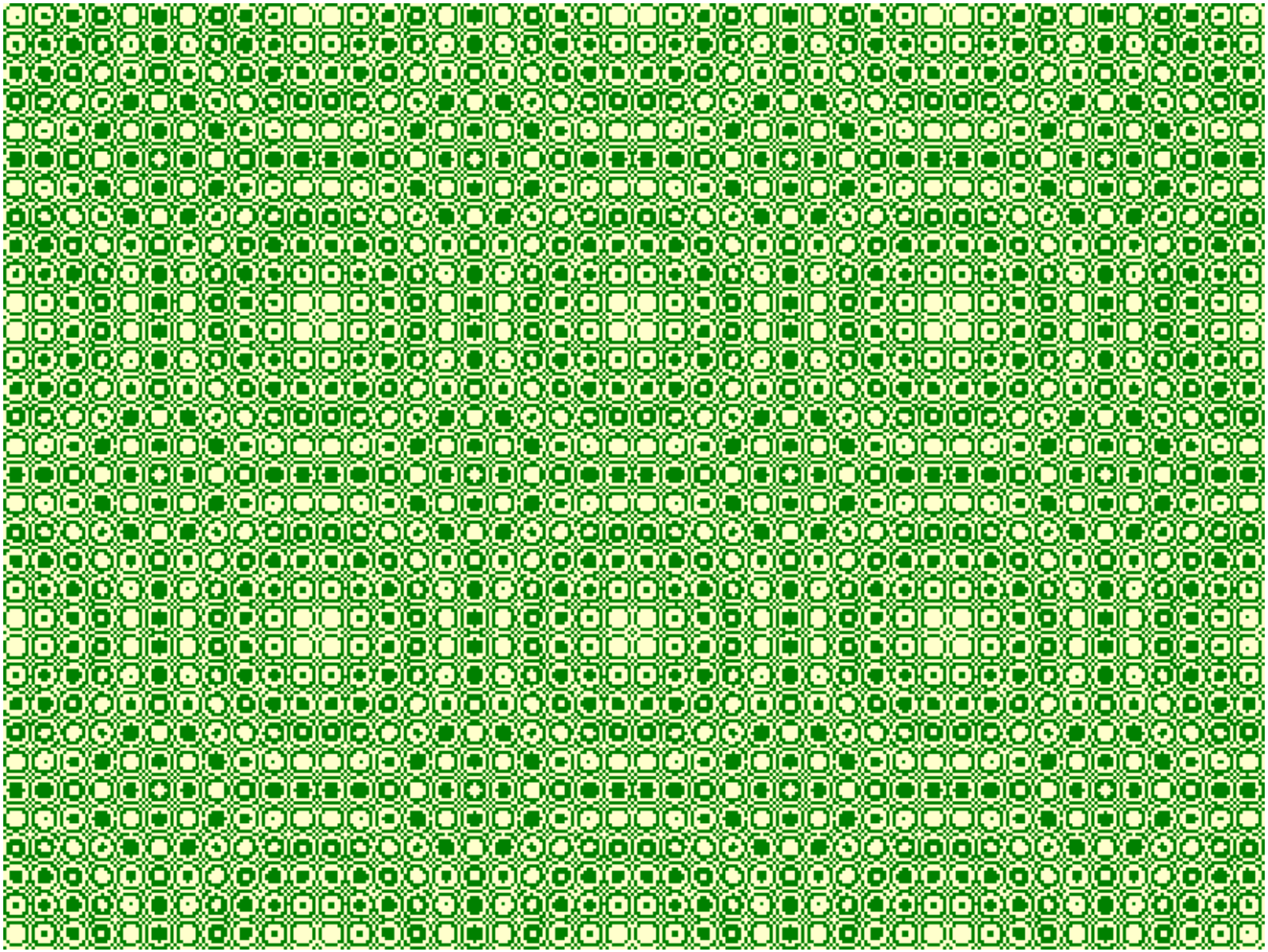
$x^2 + y^2$ 0.8 0.8 4



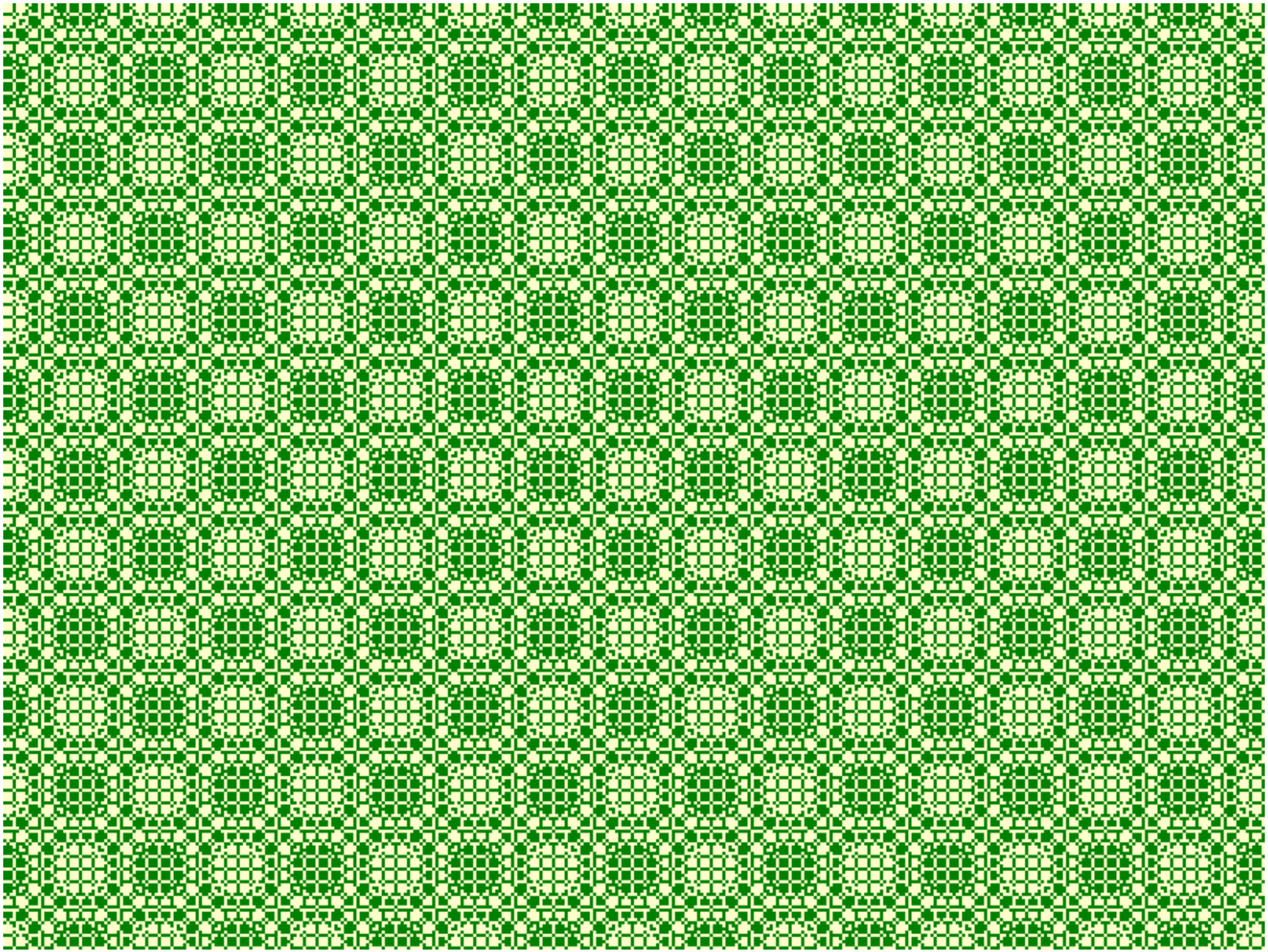
$x^2 + y^2$ 0.9 0.9 2

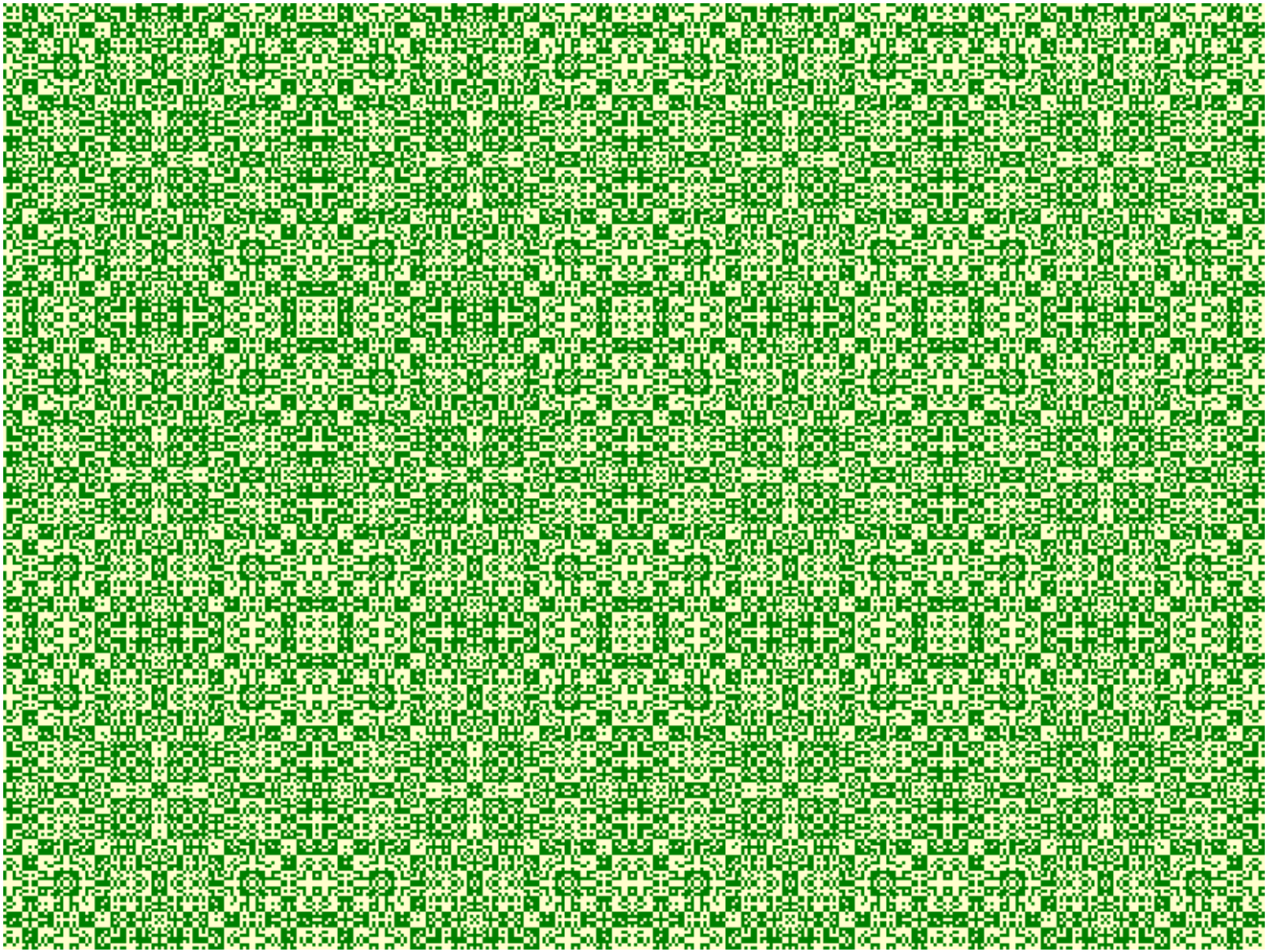


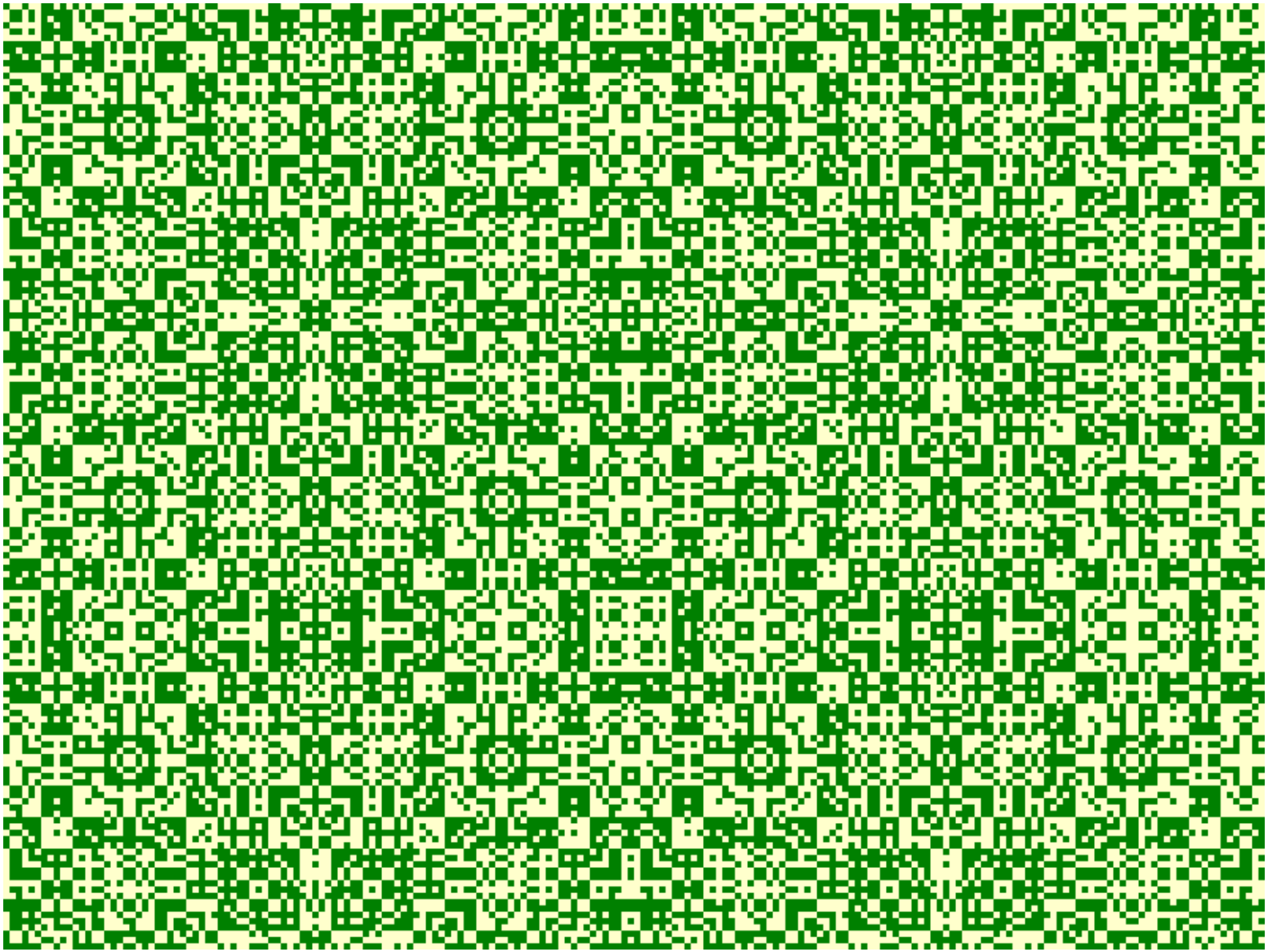


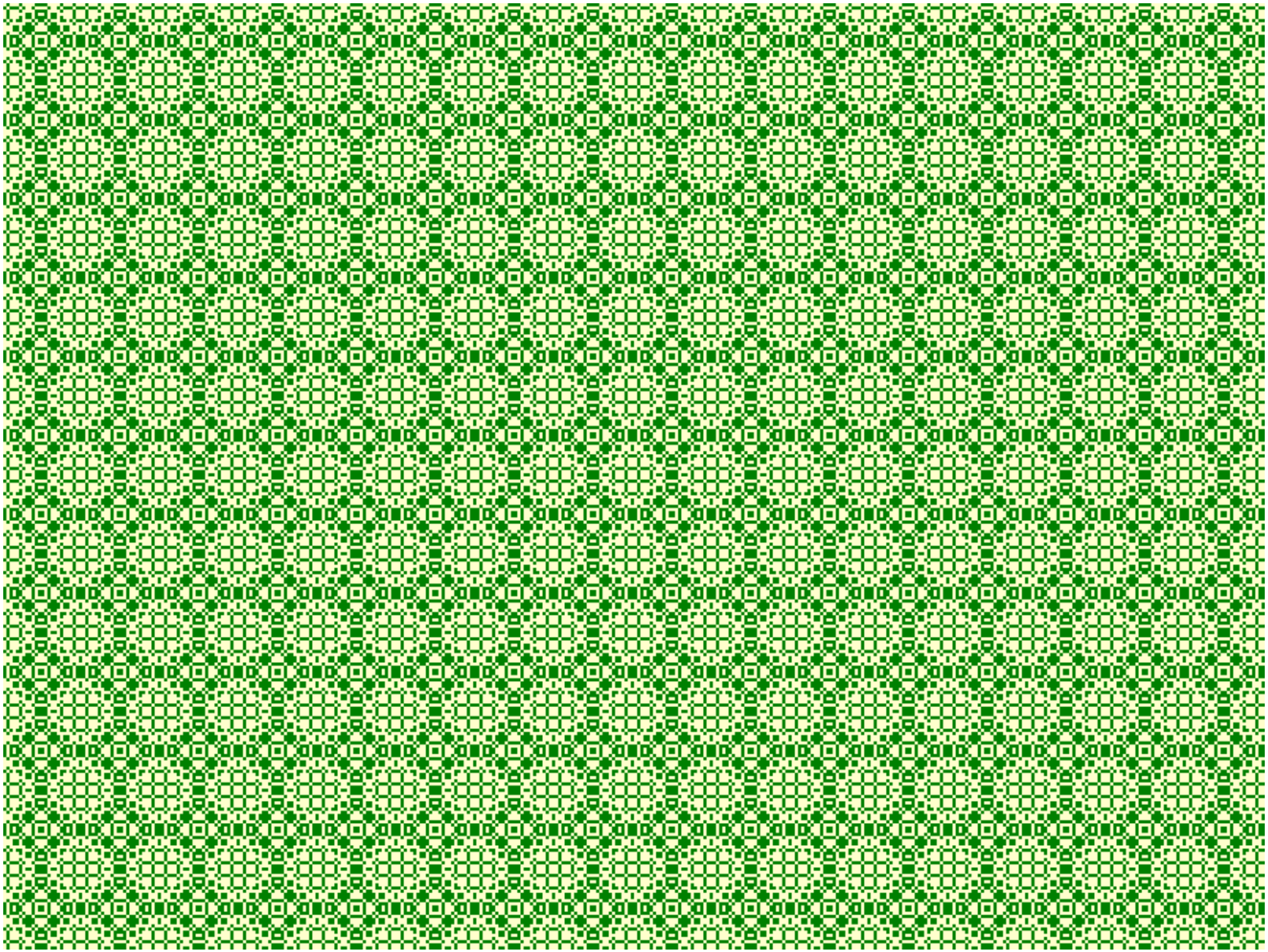


$x^2 + y^2$ 1.8 1.8 2

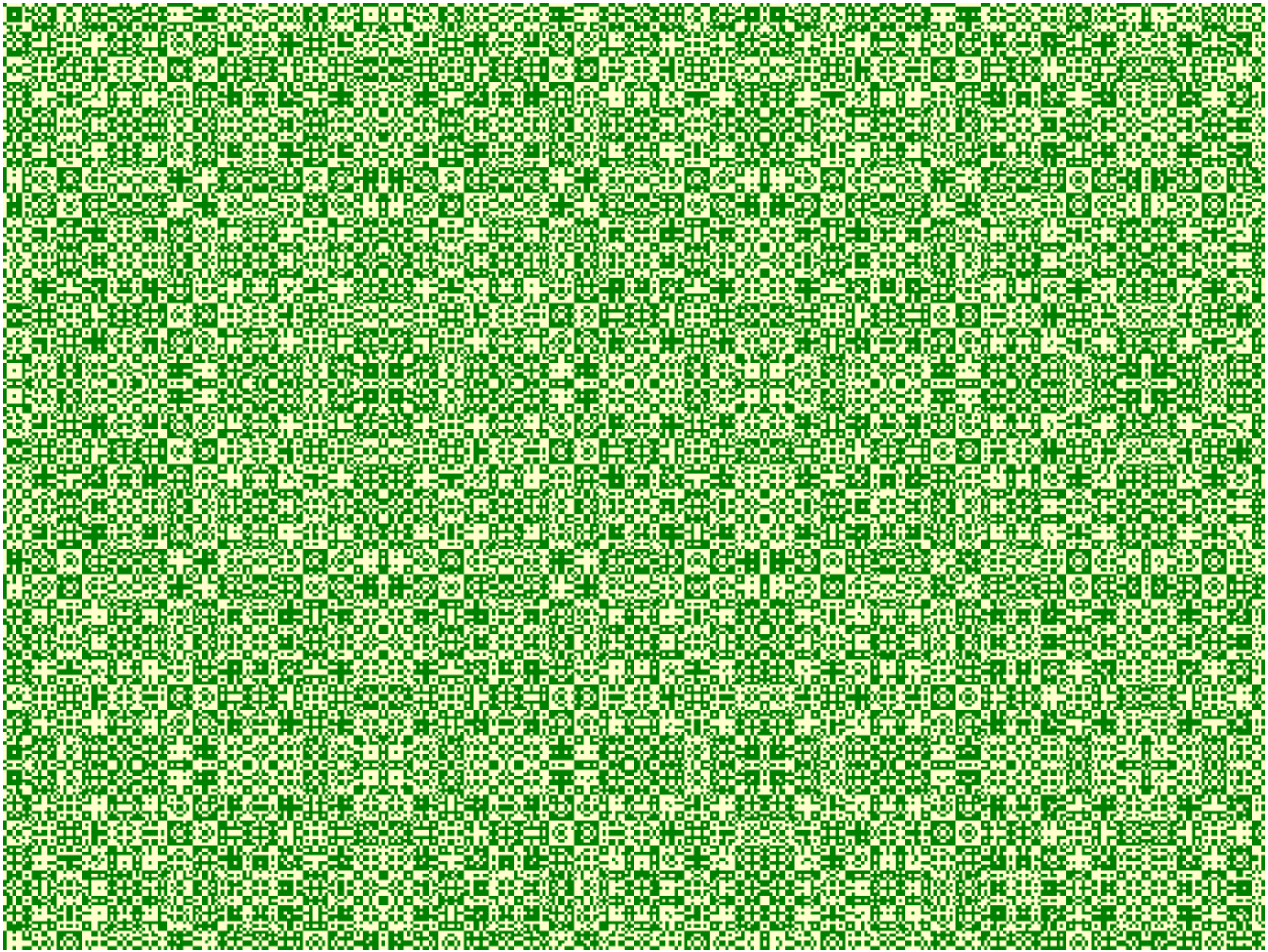




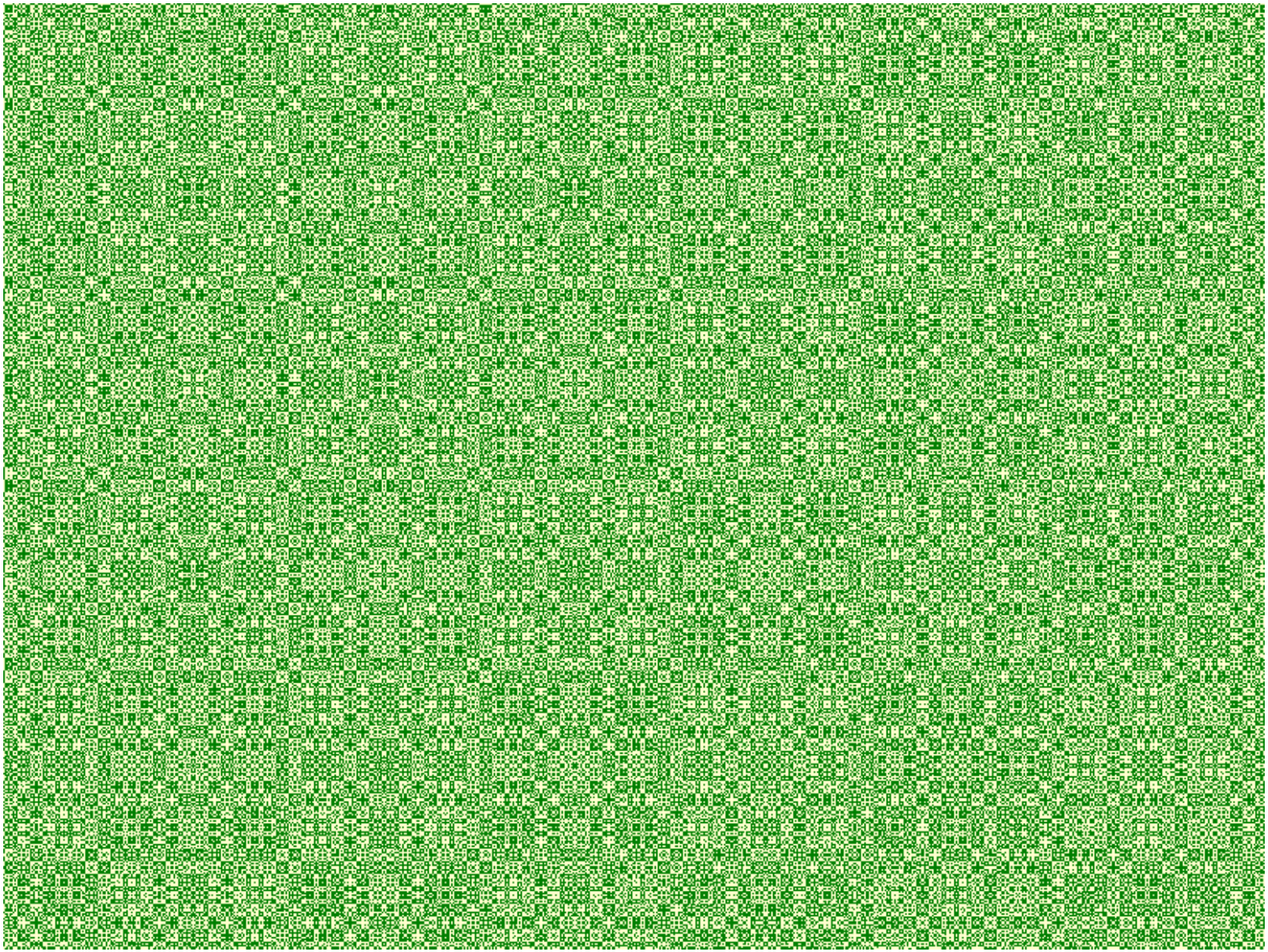




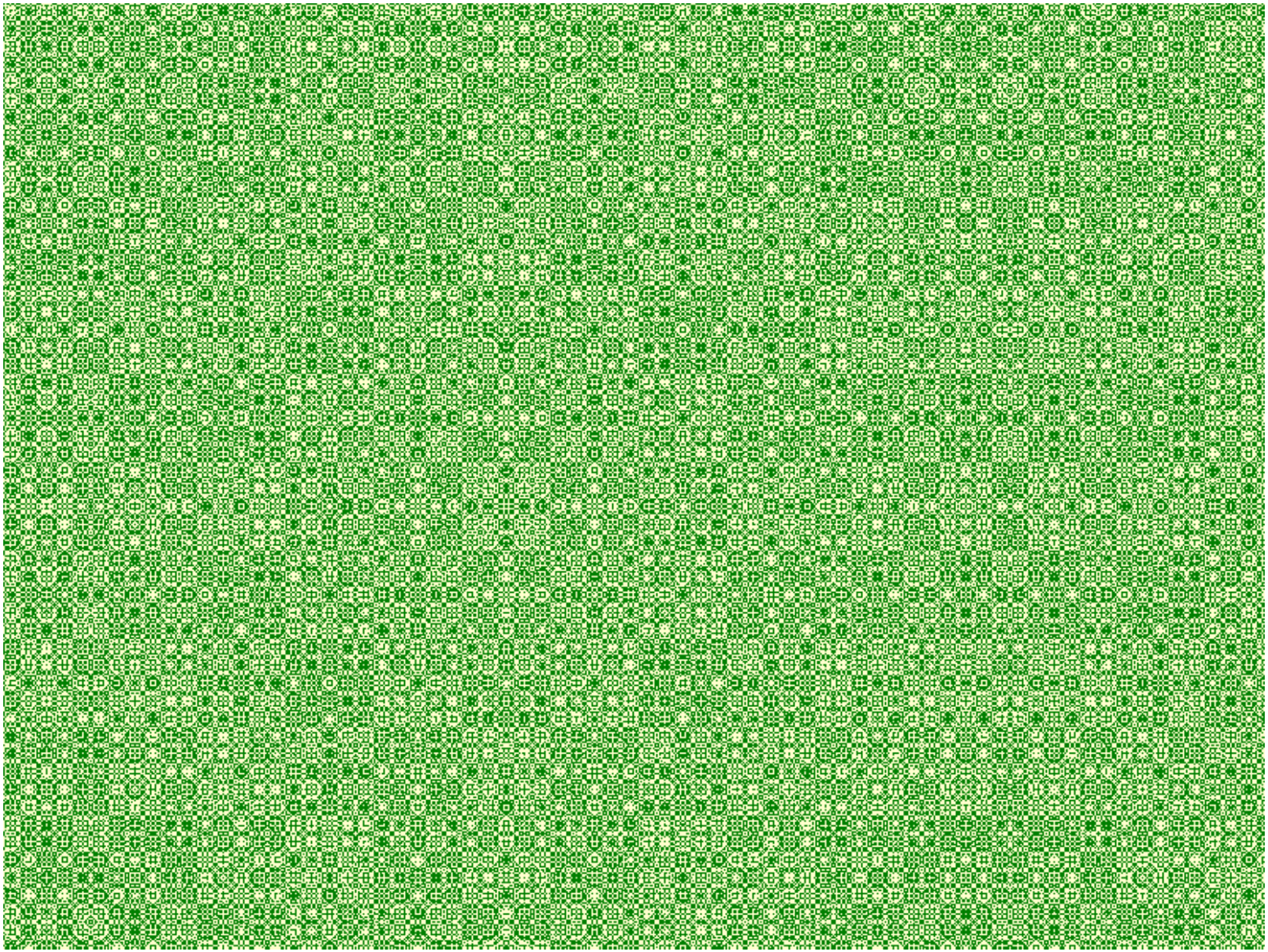
$x^2 + y^2$ 3.41 3.41 2



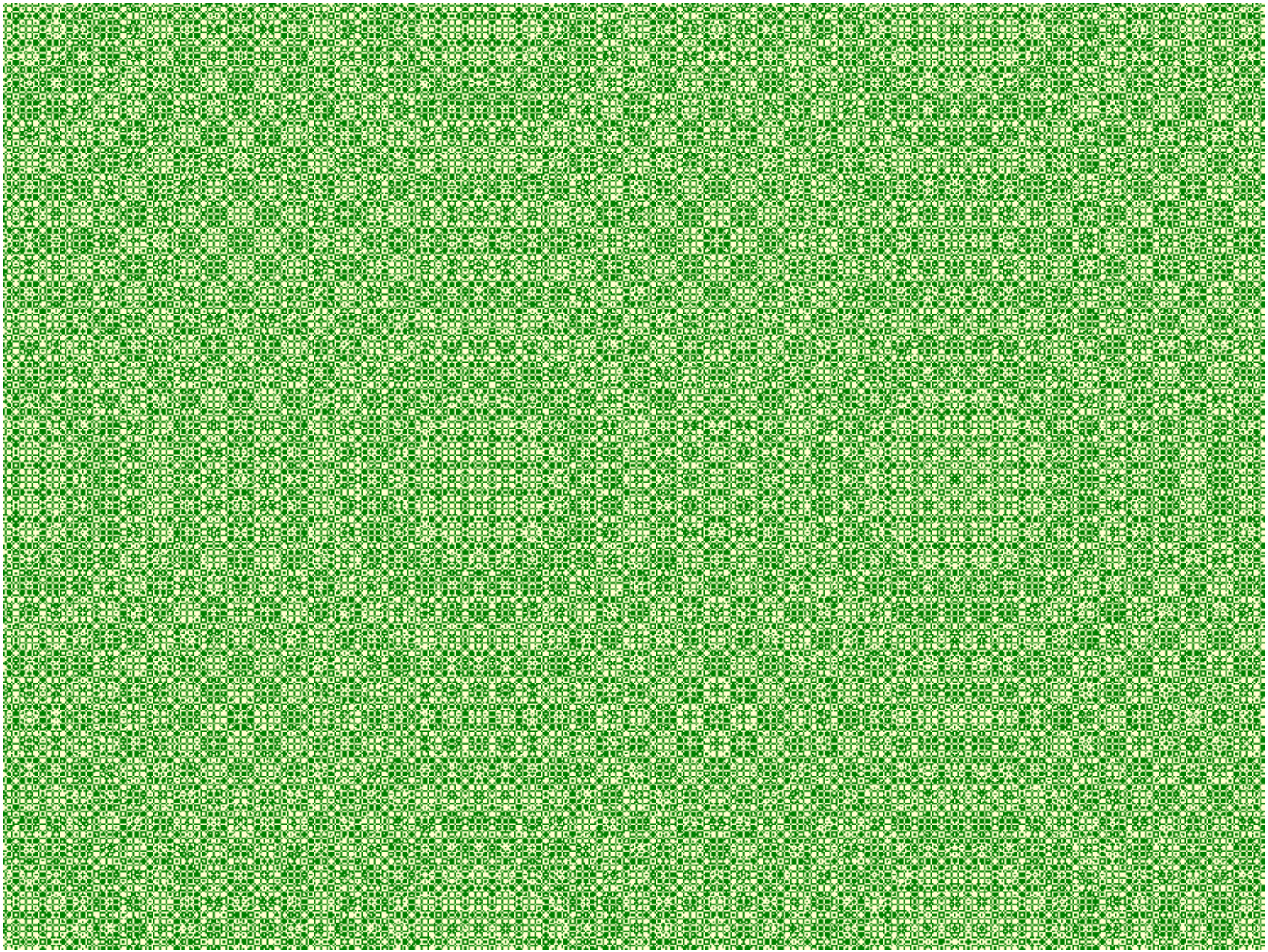
$x^2 + y^2$ 3.41 3.41 1



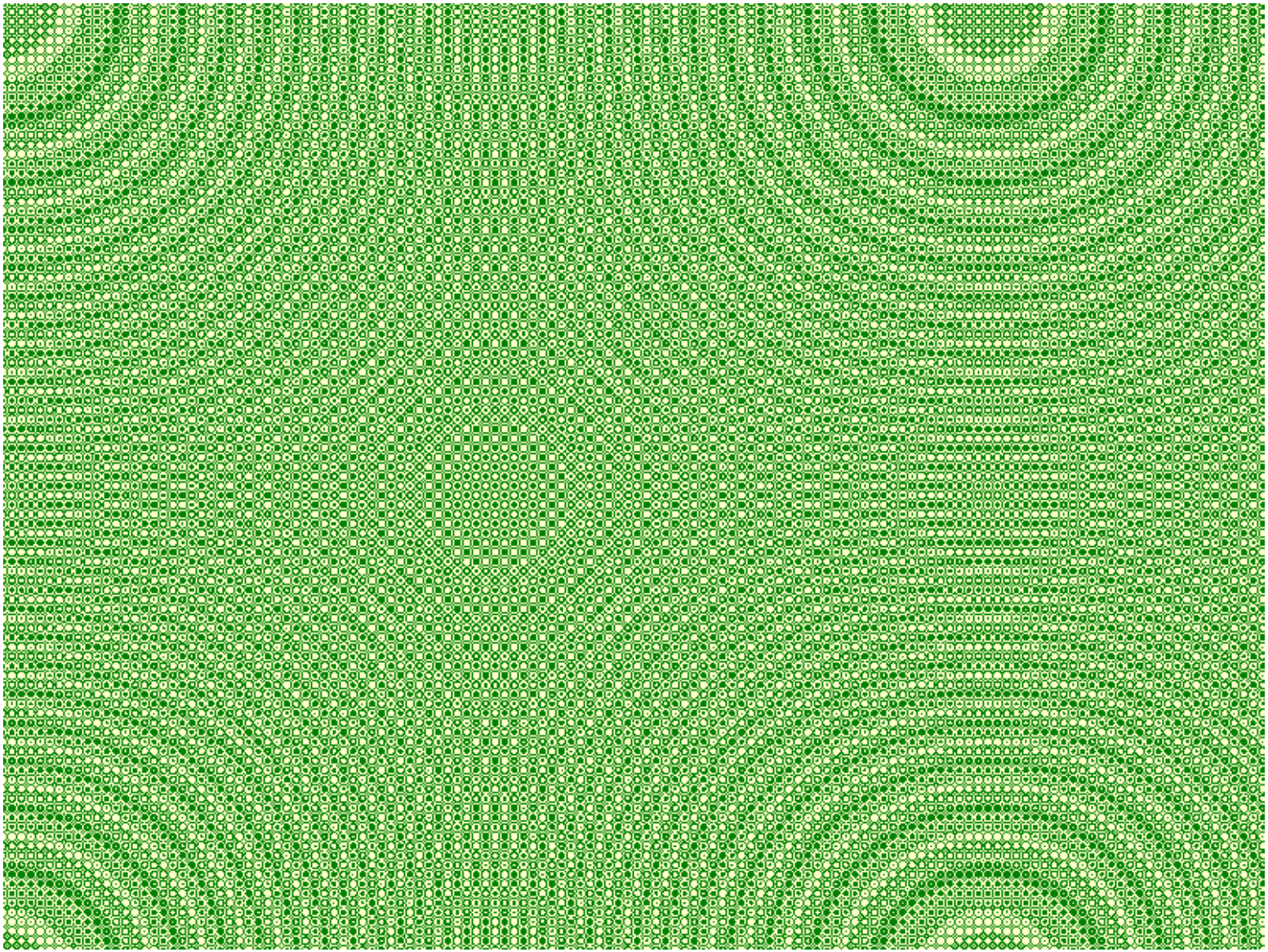
$x^2 + y^2$ 3.42 3.42 1



$x^2 + y^2$ 3.43 3.43 1



$x^2 + y^2$ 3.44 3.44 1



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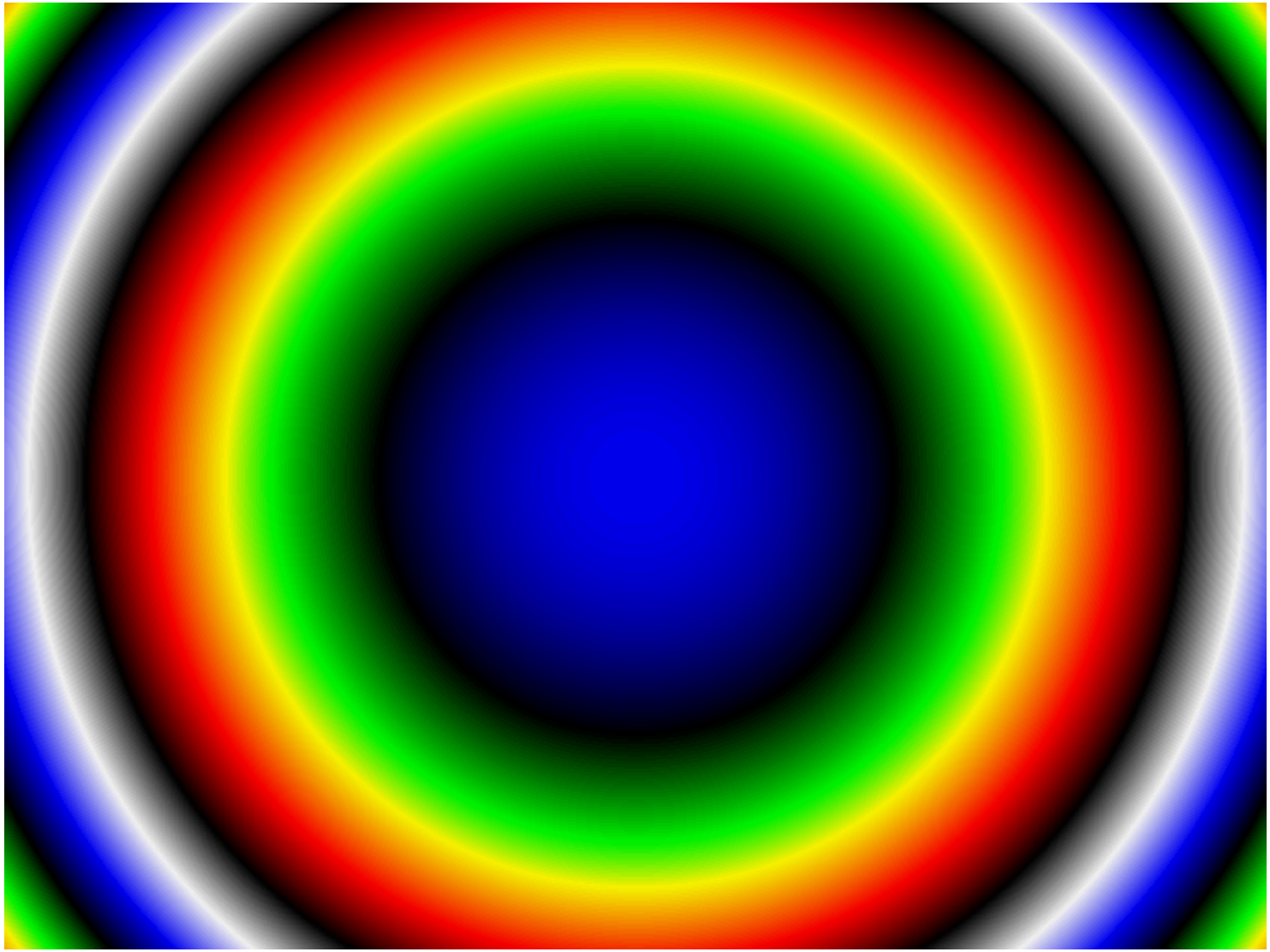

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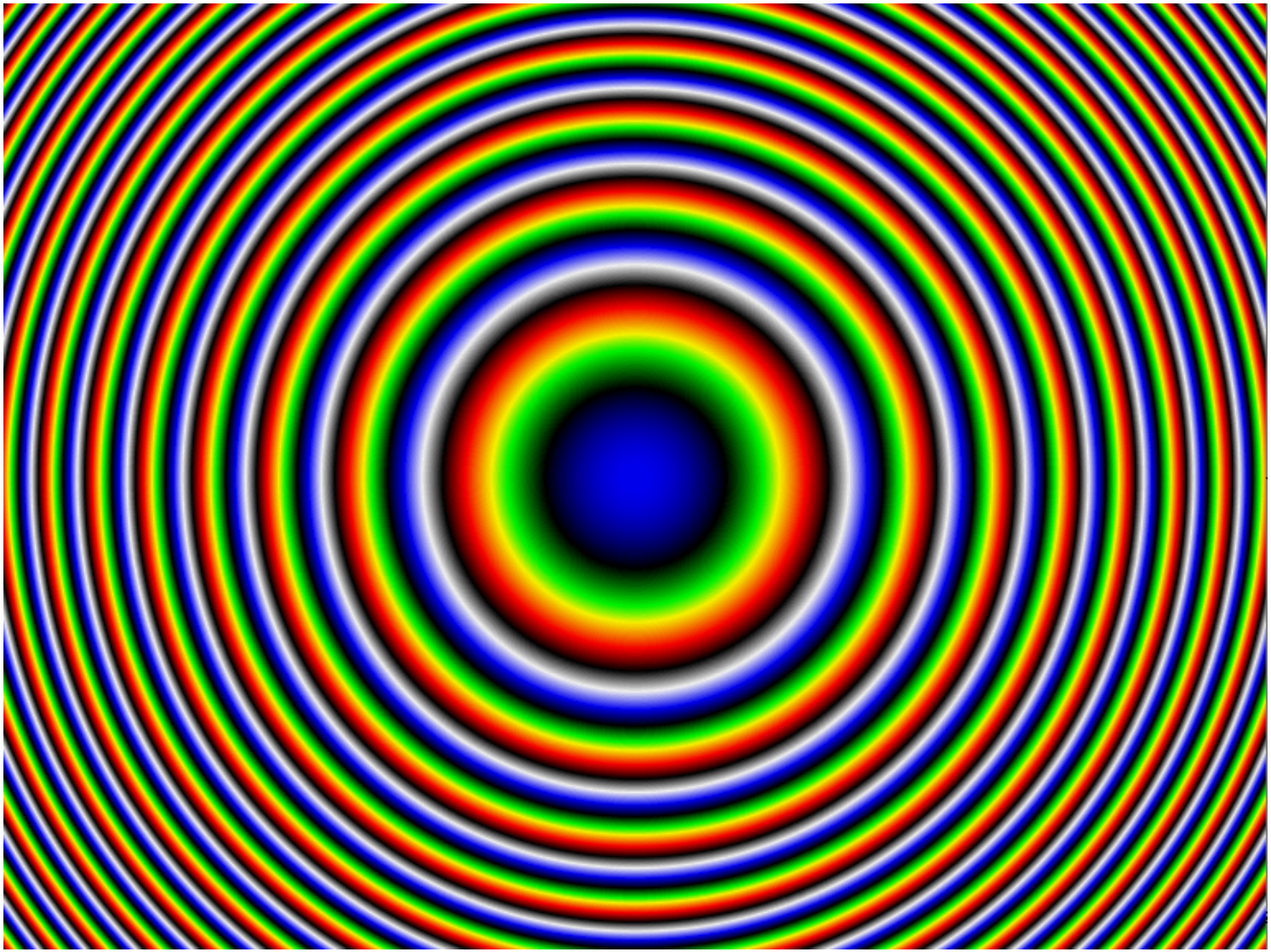

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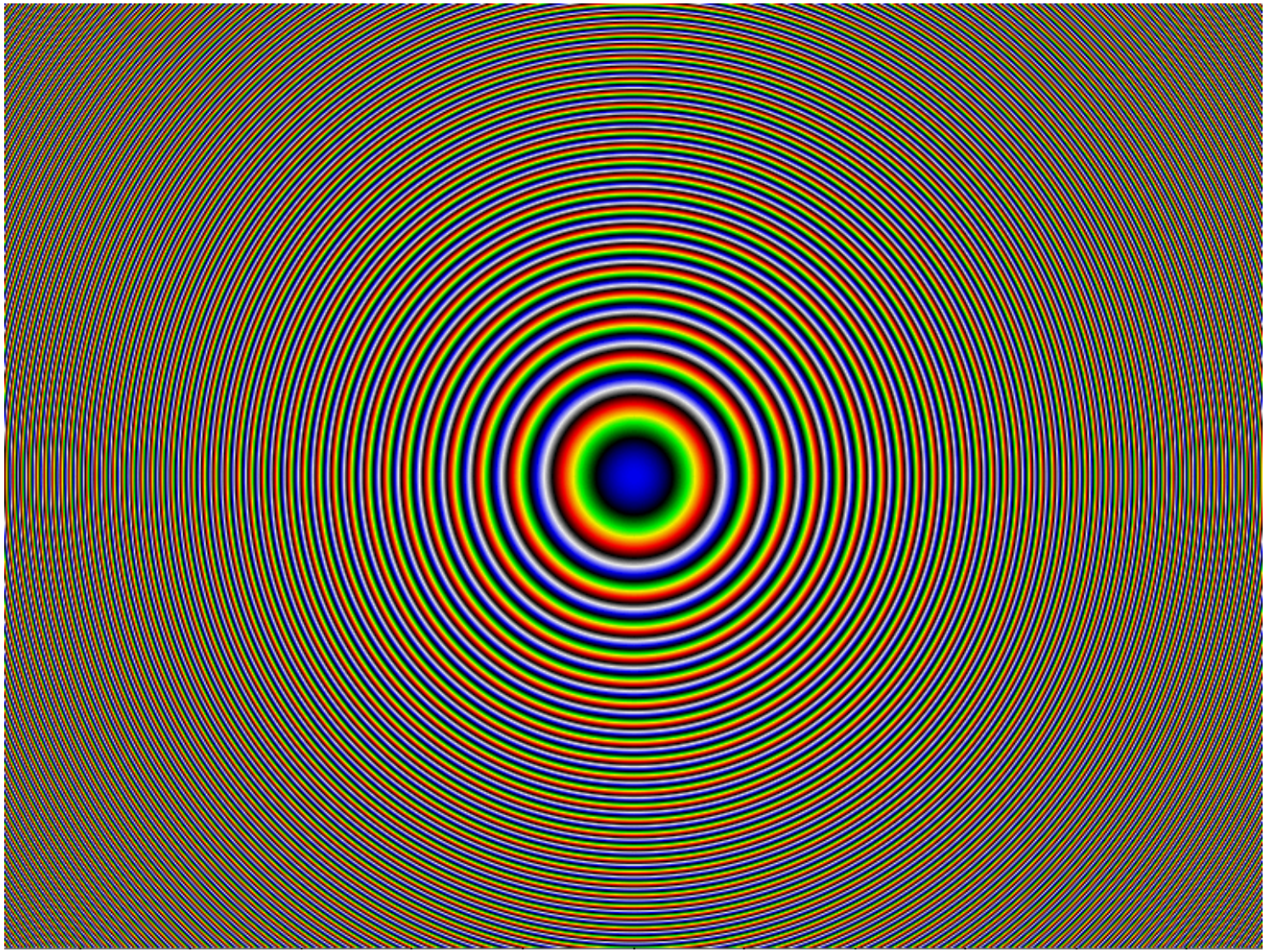

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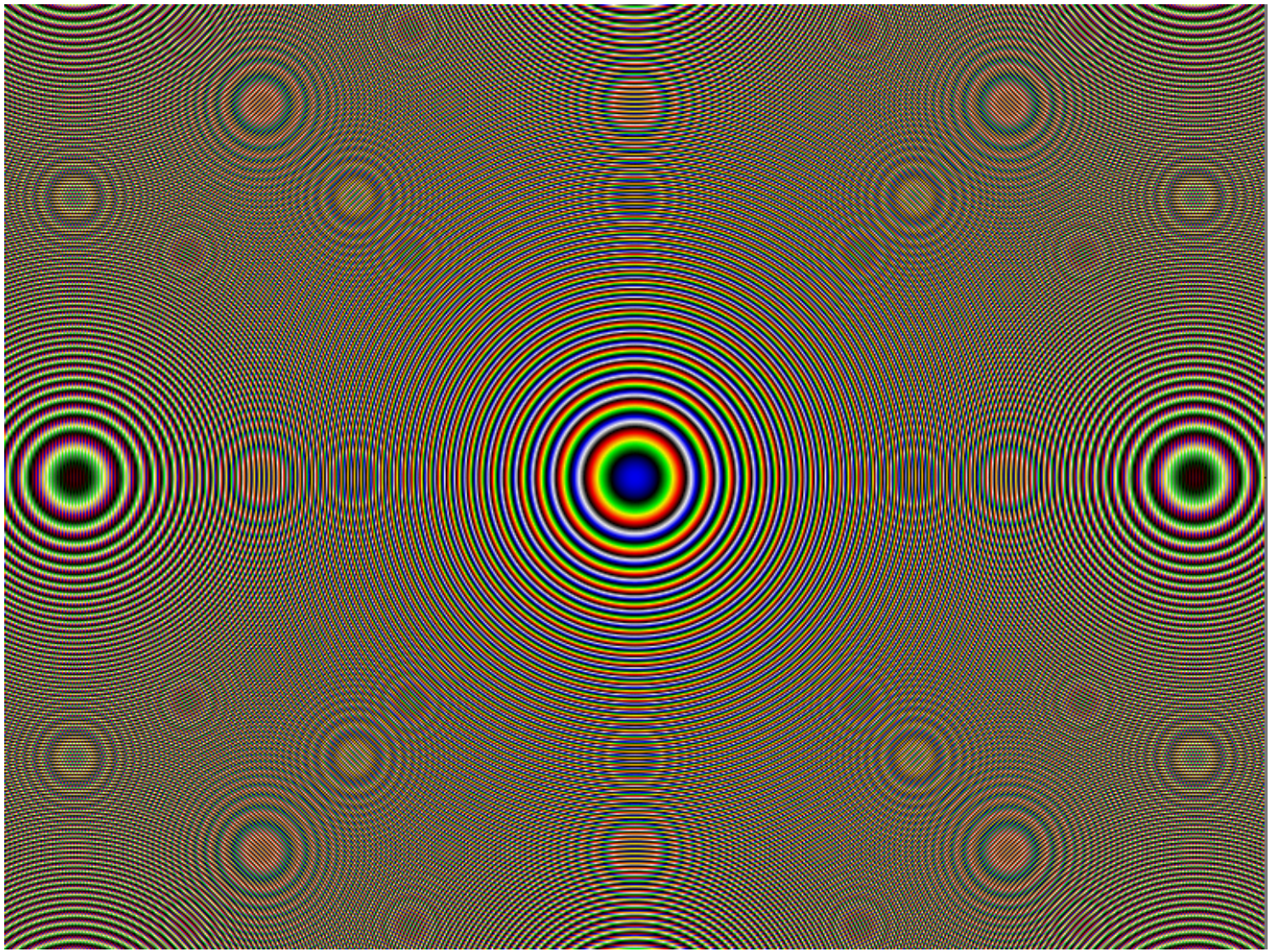

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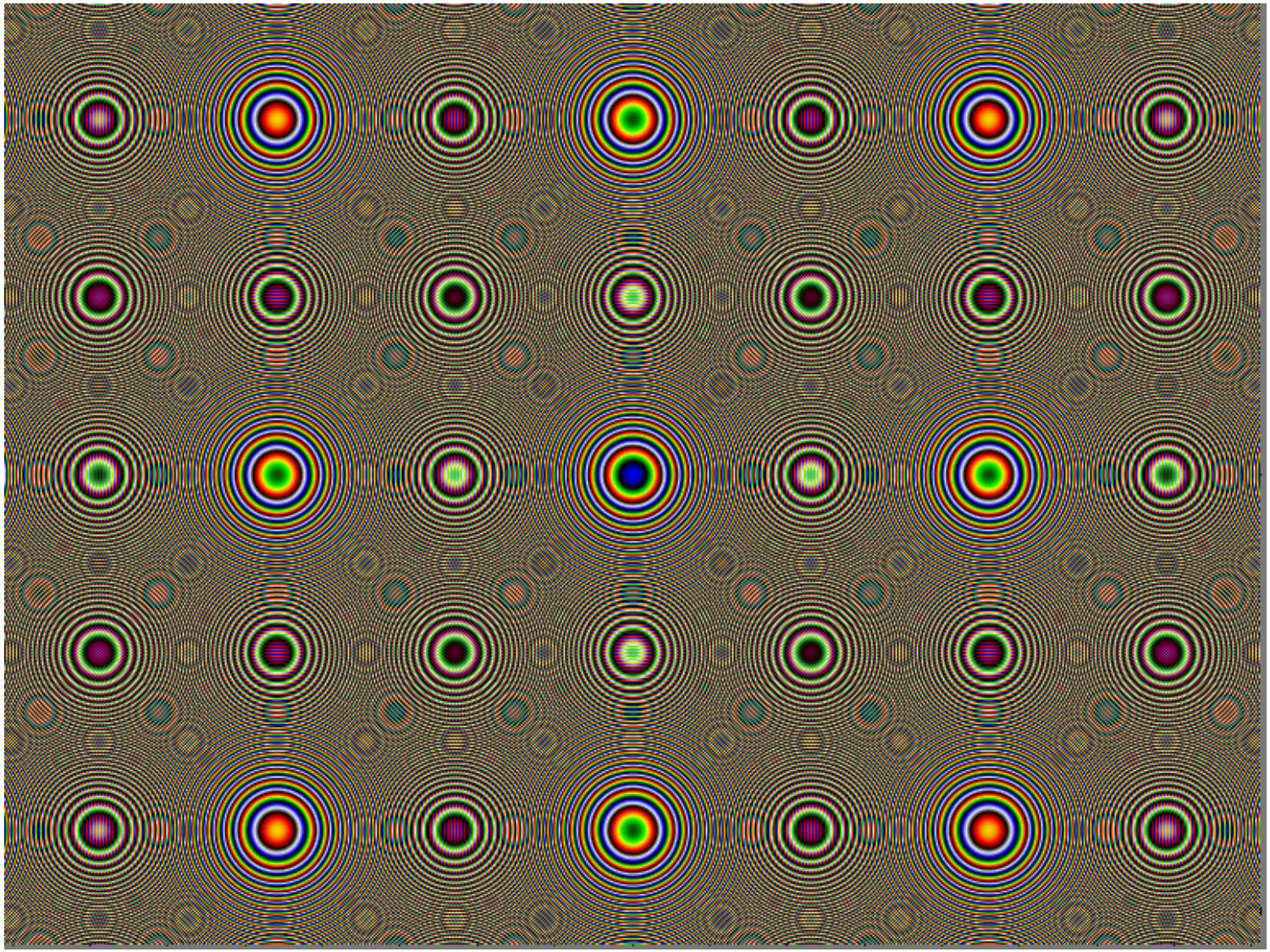

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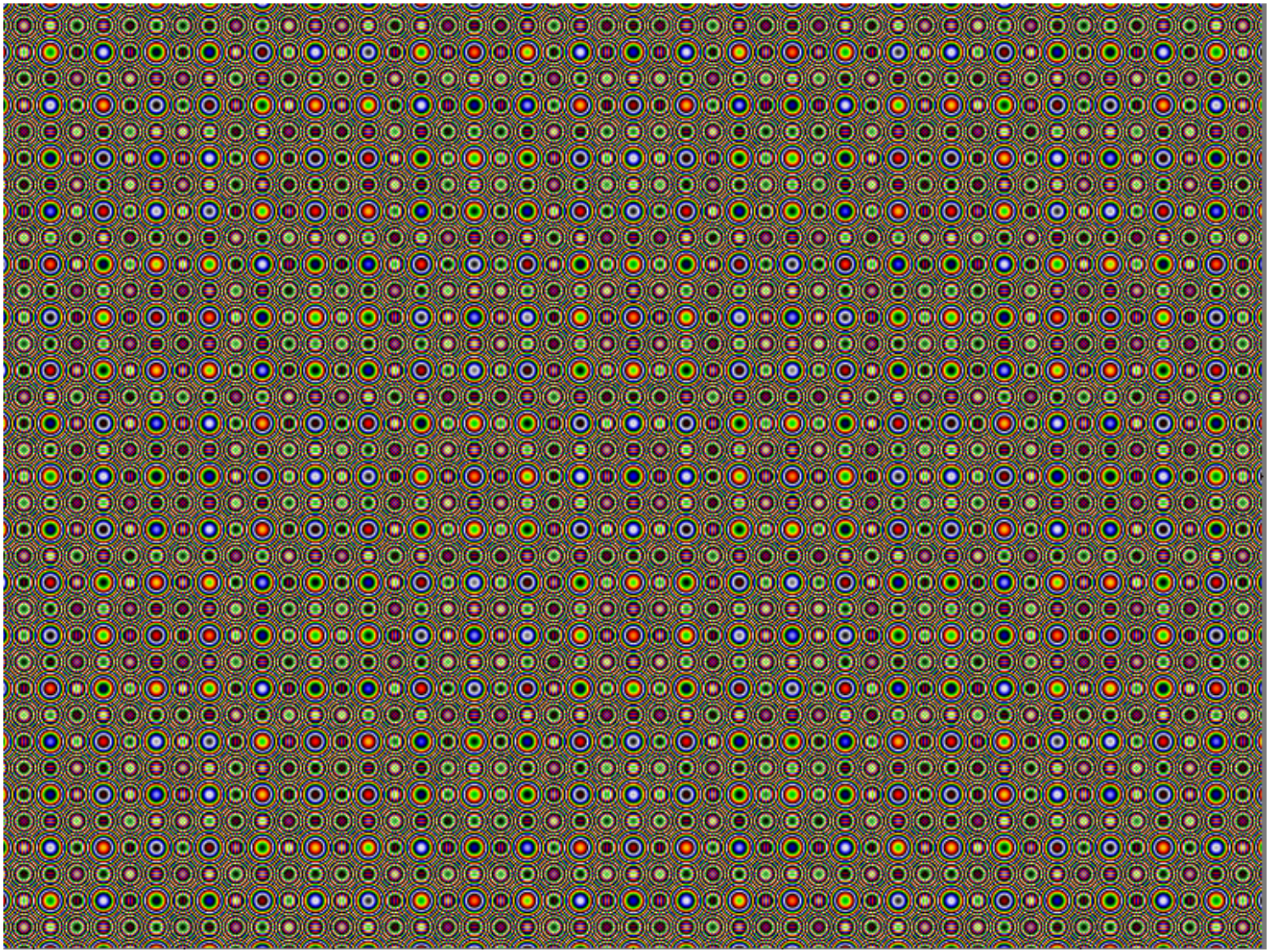

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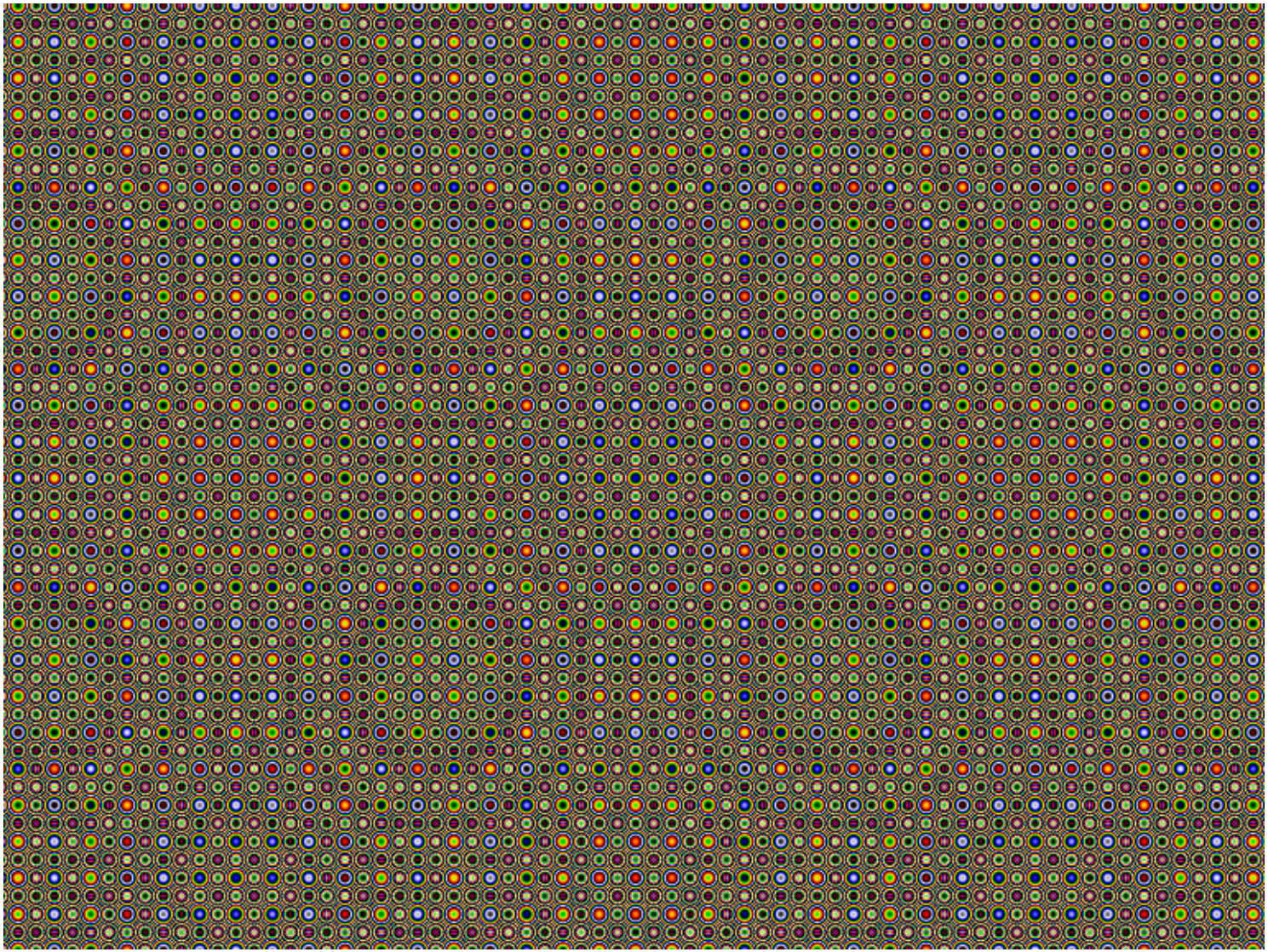

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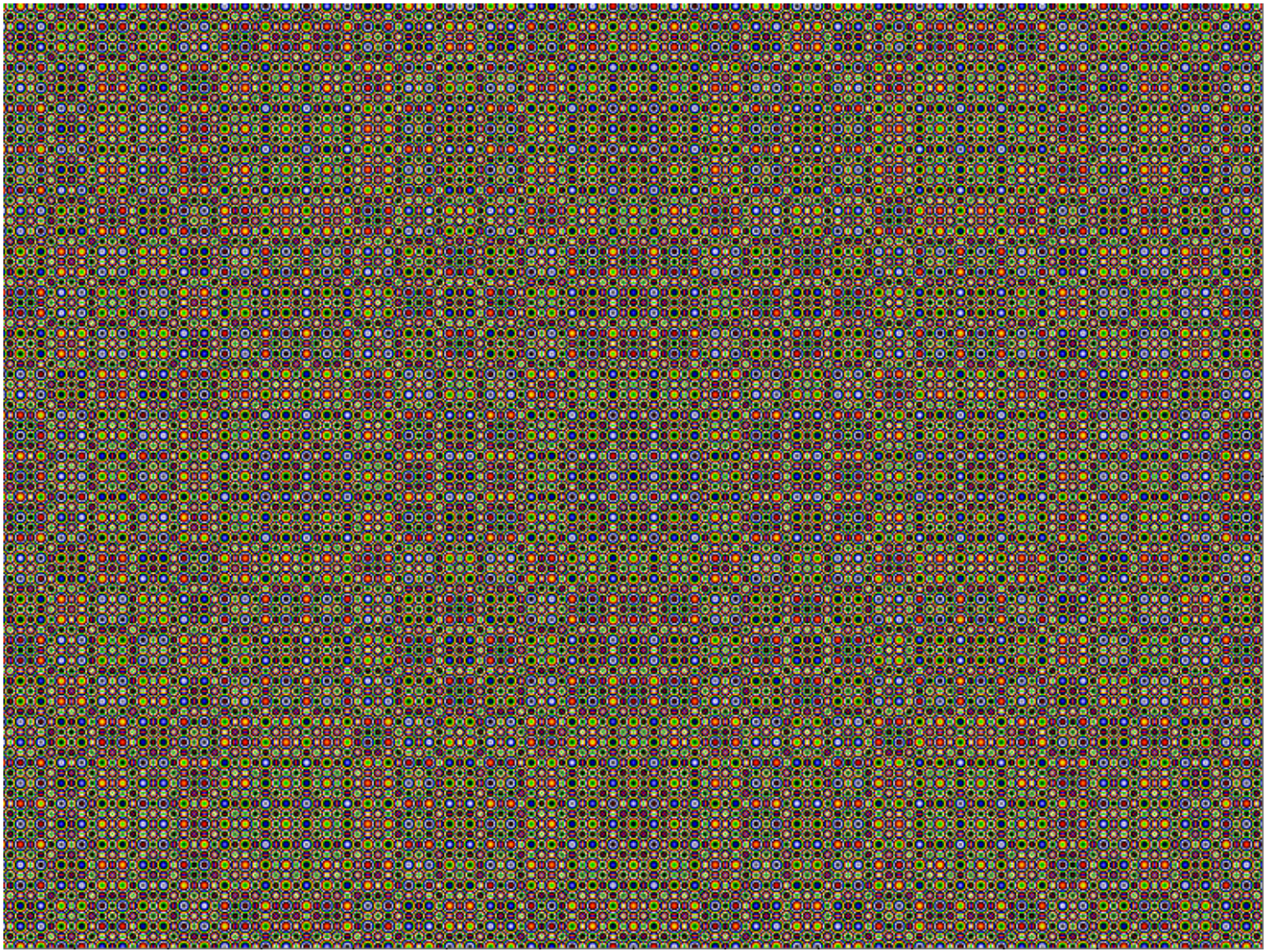
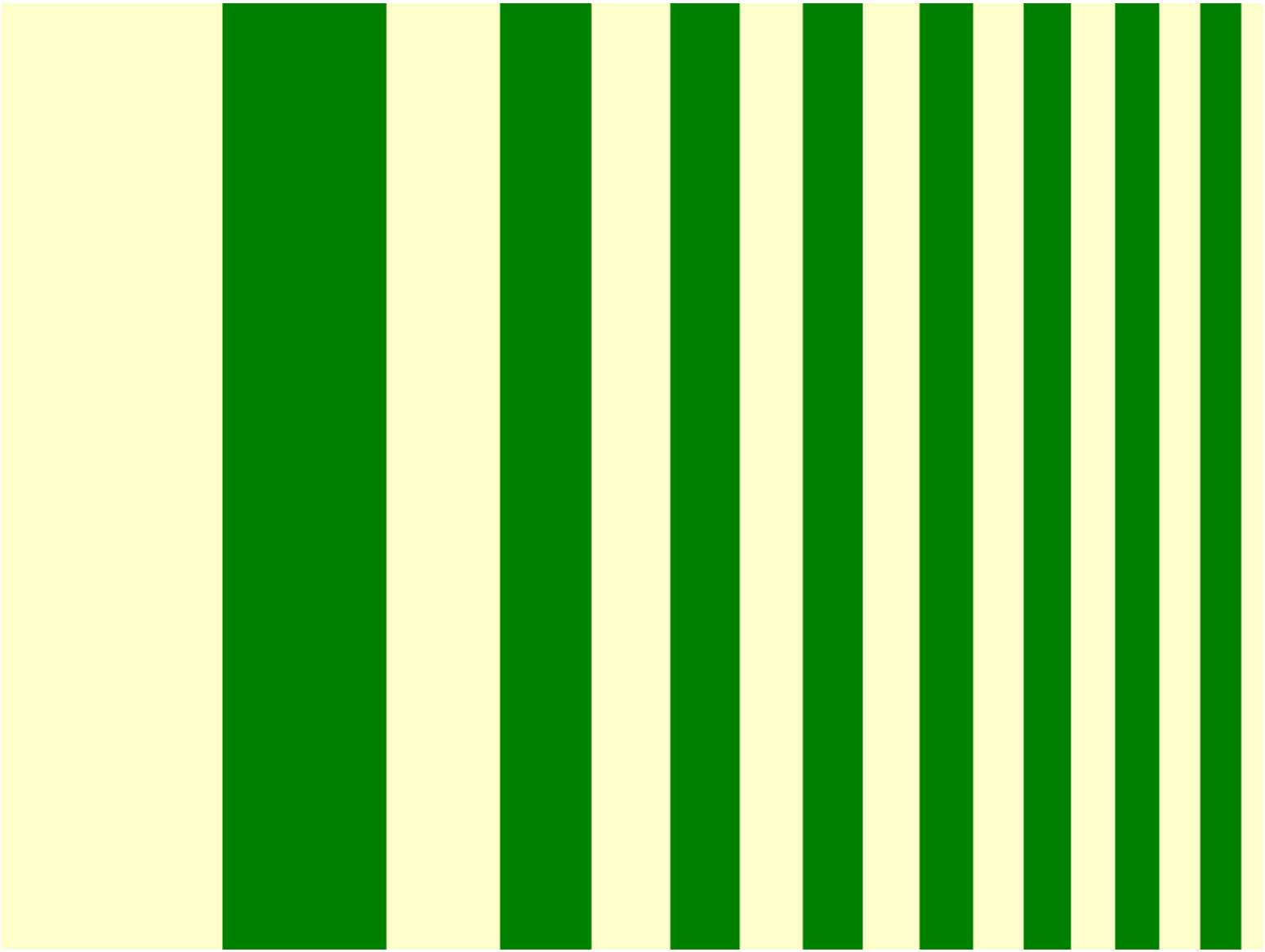
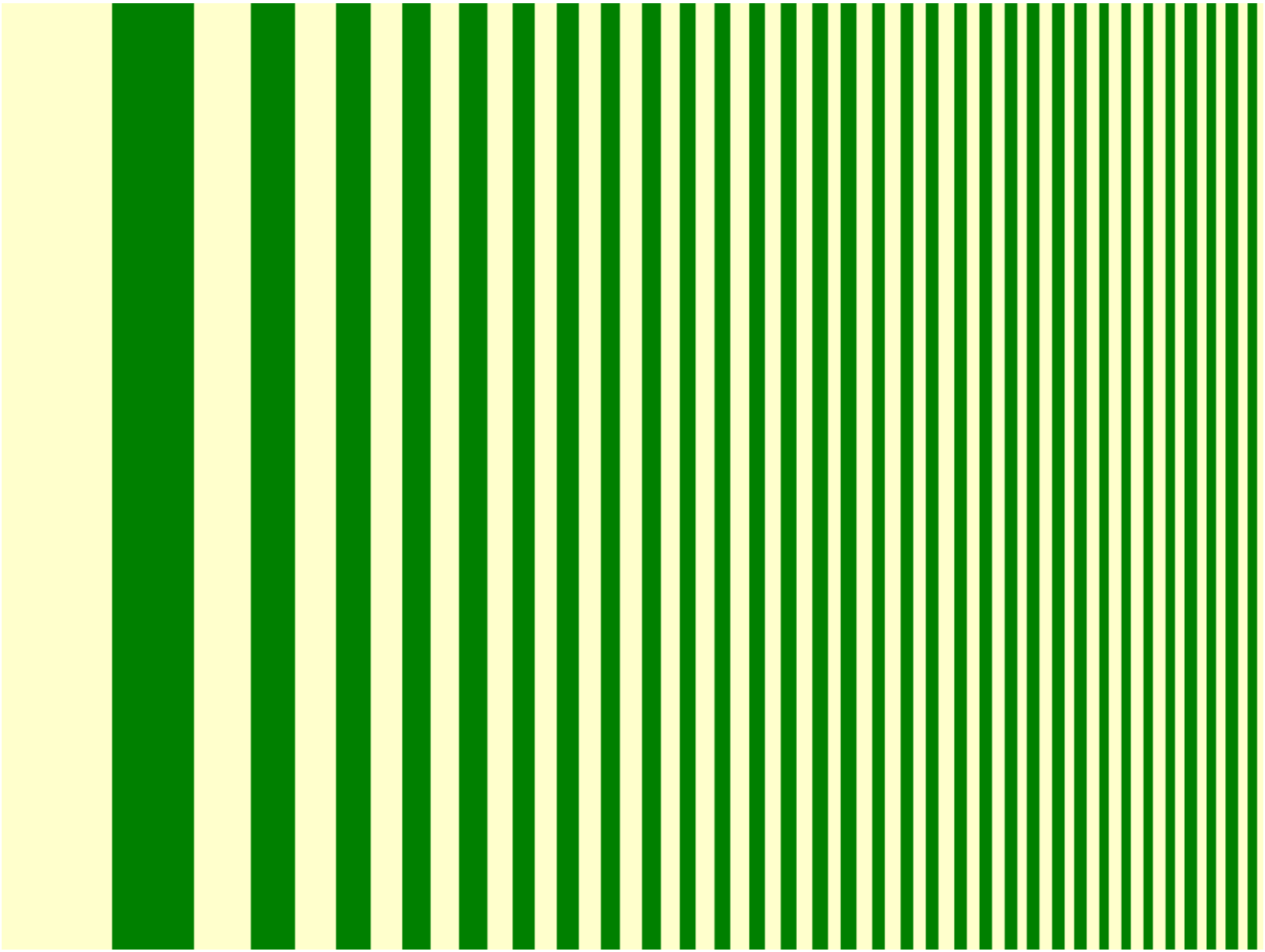
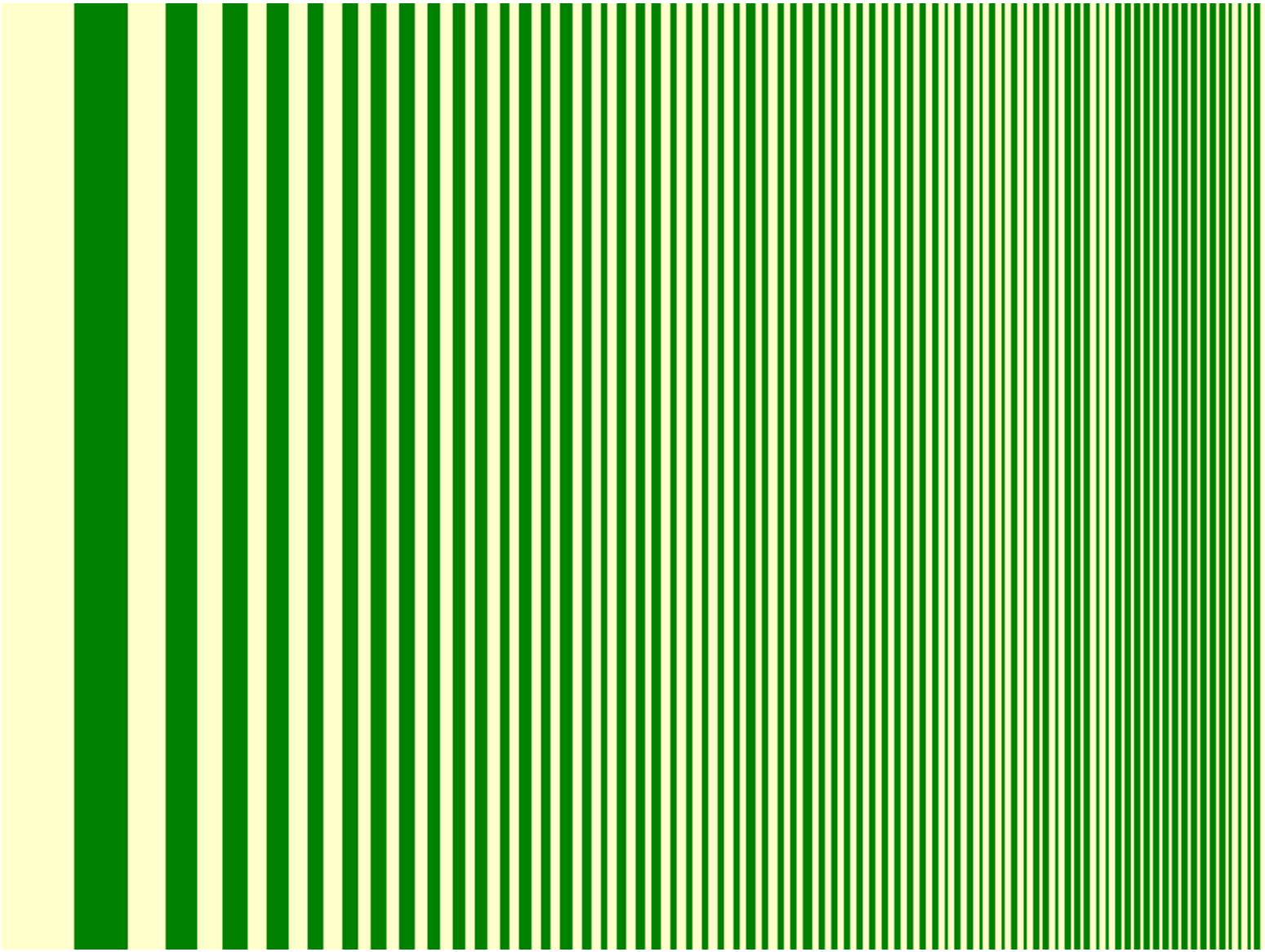
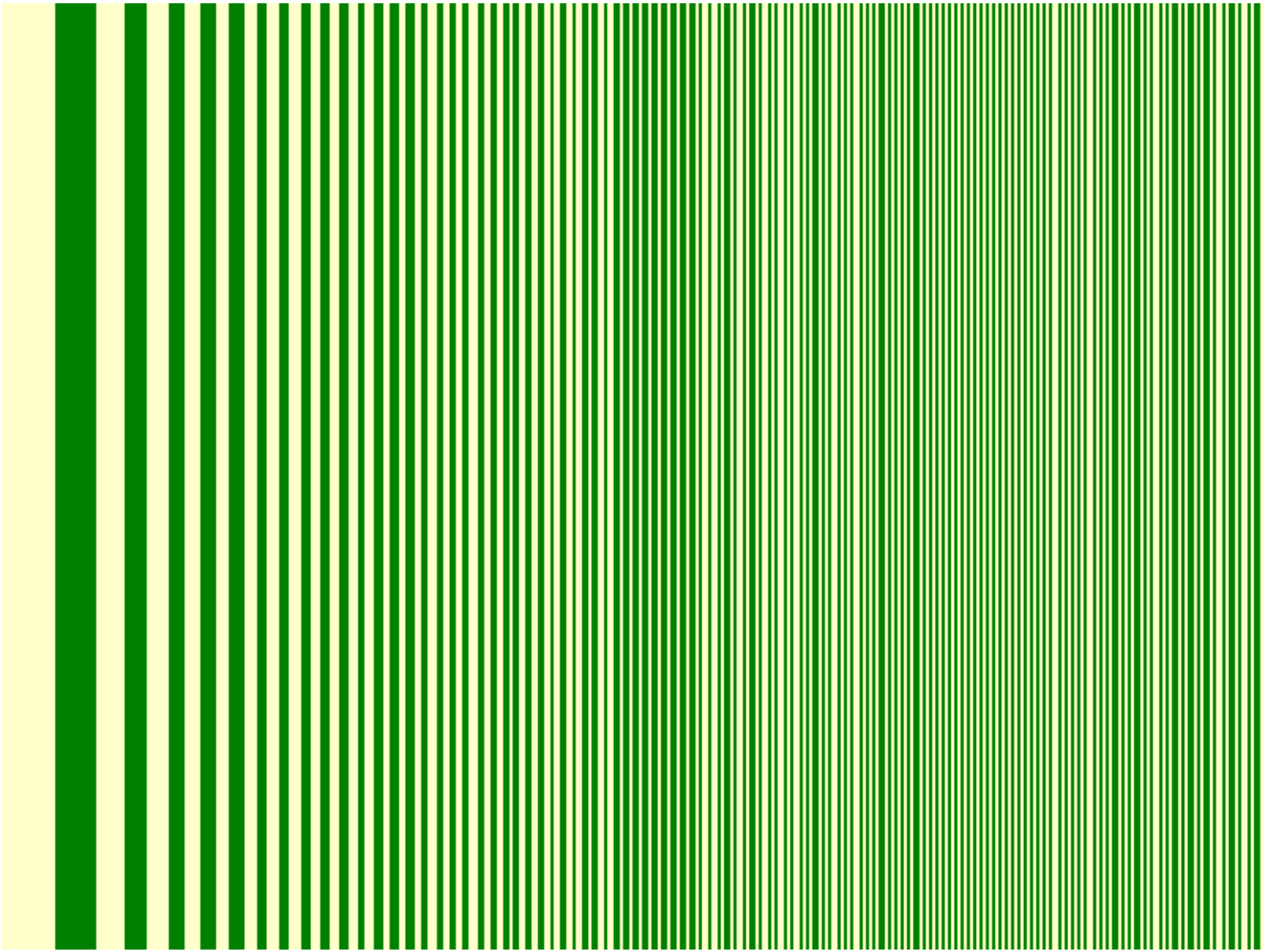


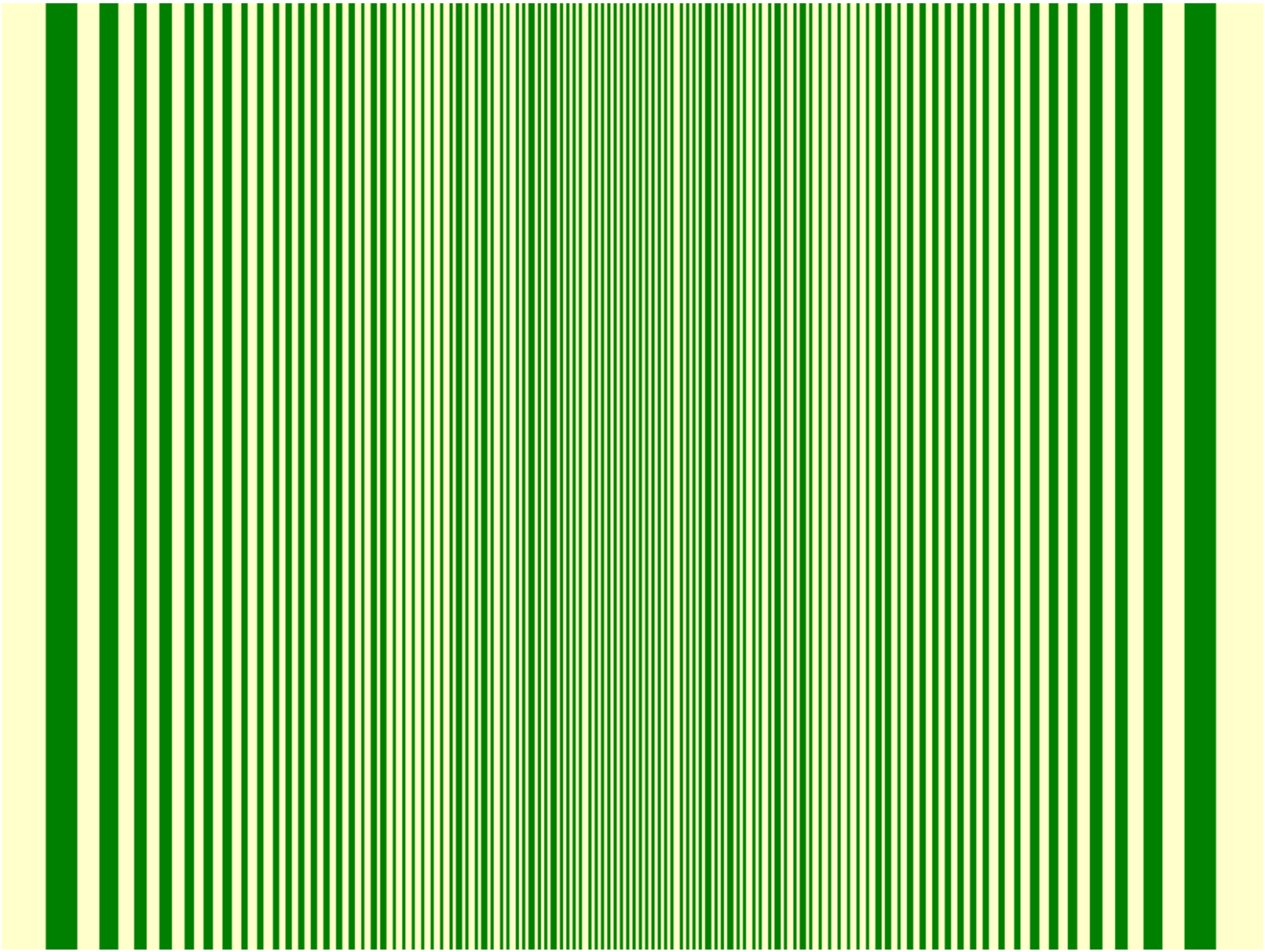


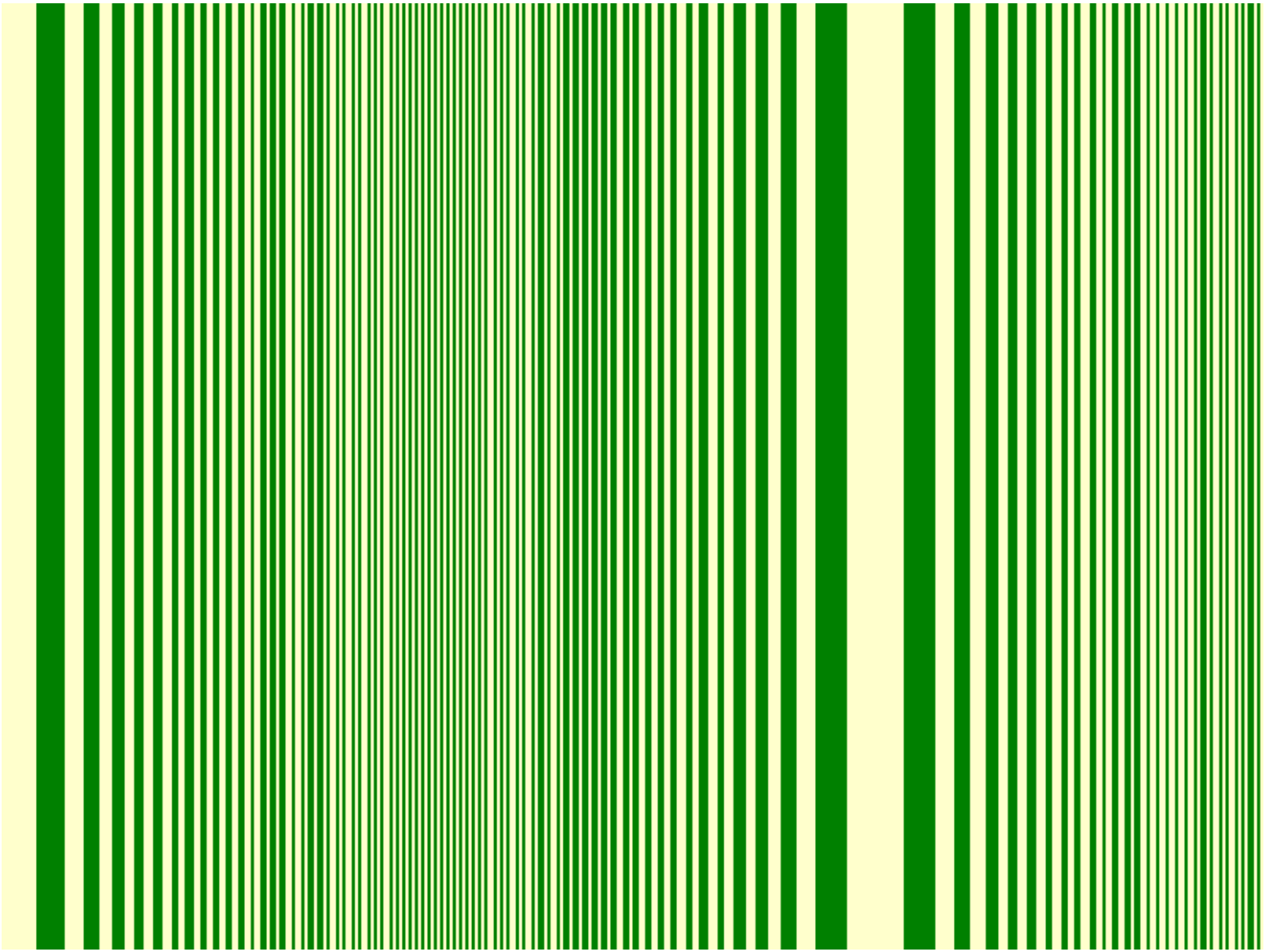


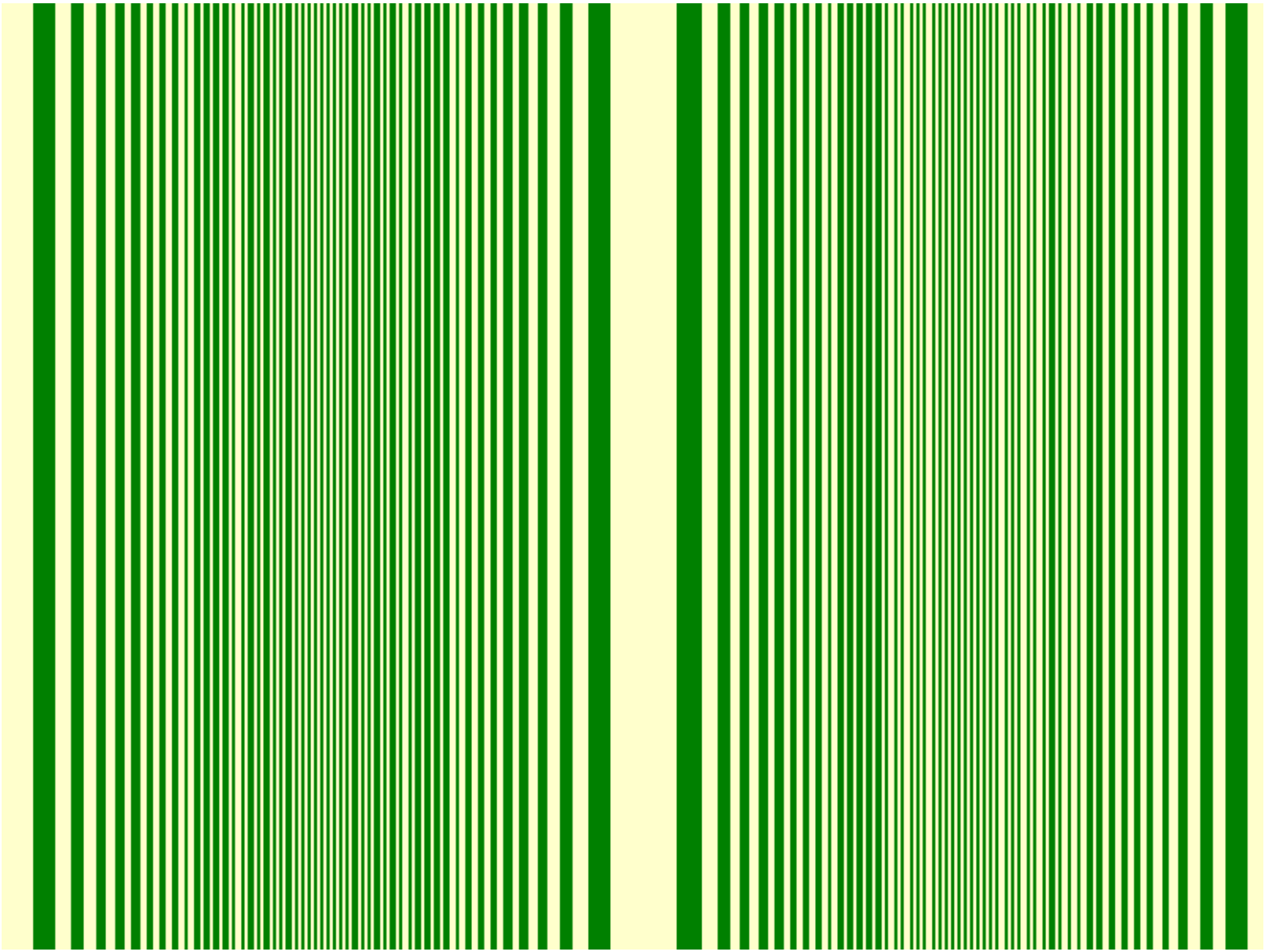


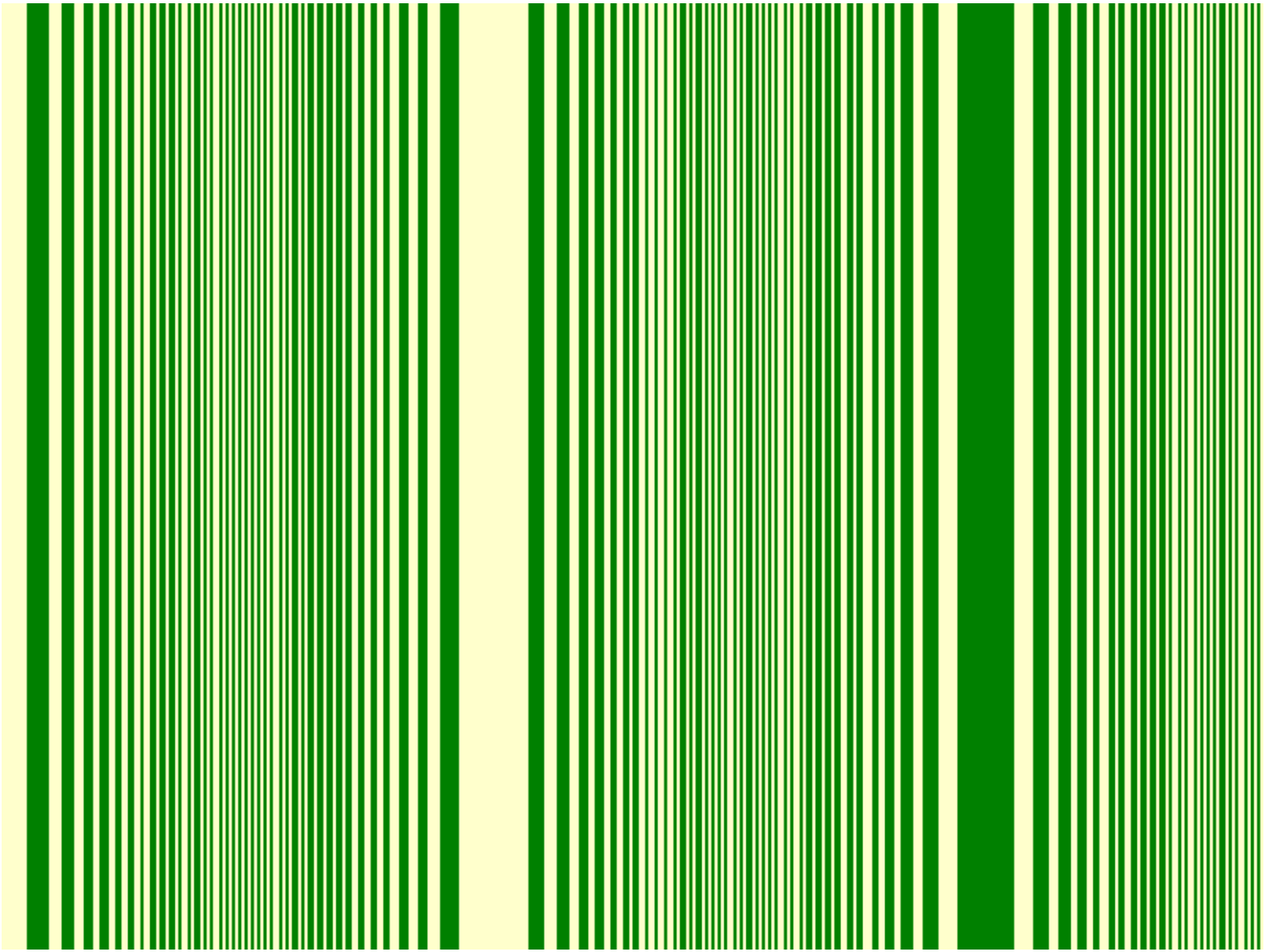


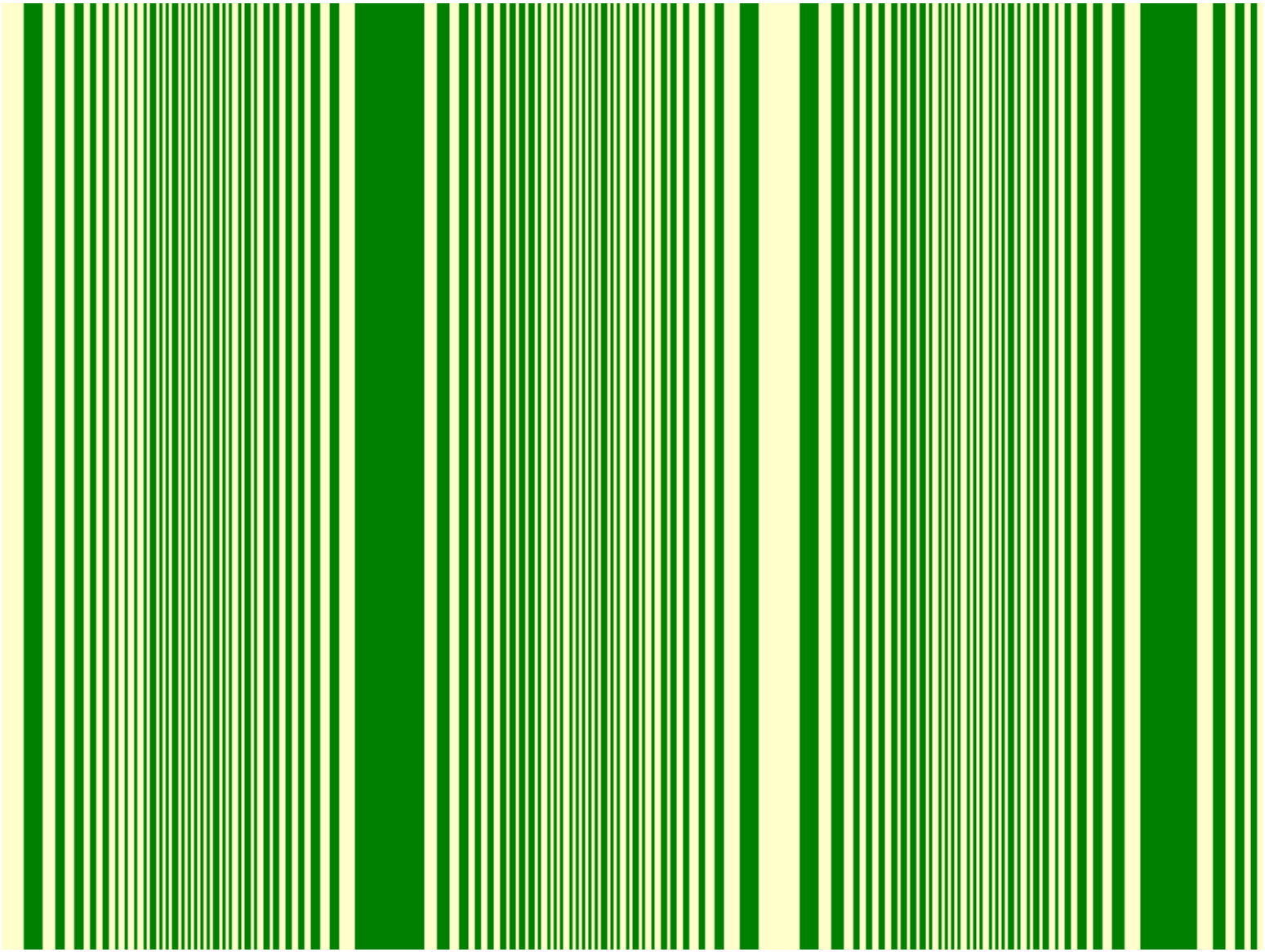
x^2 0.05 0.0 2

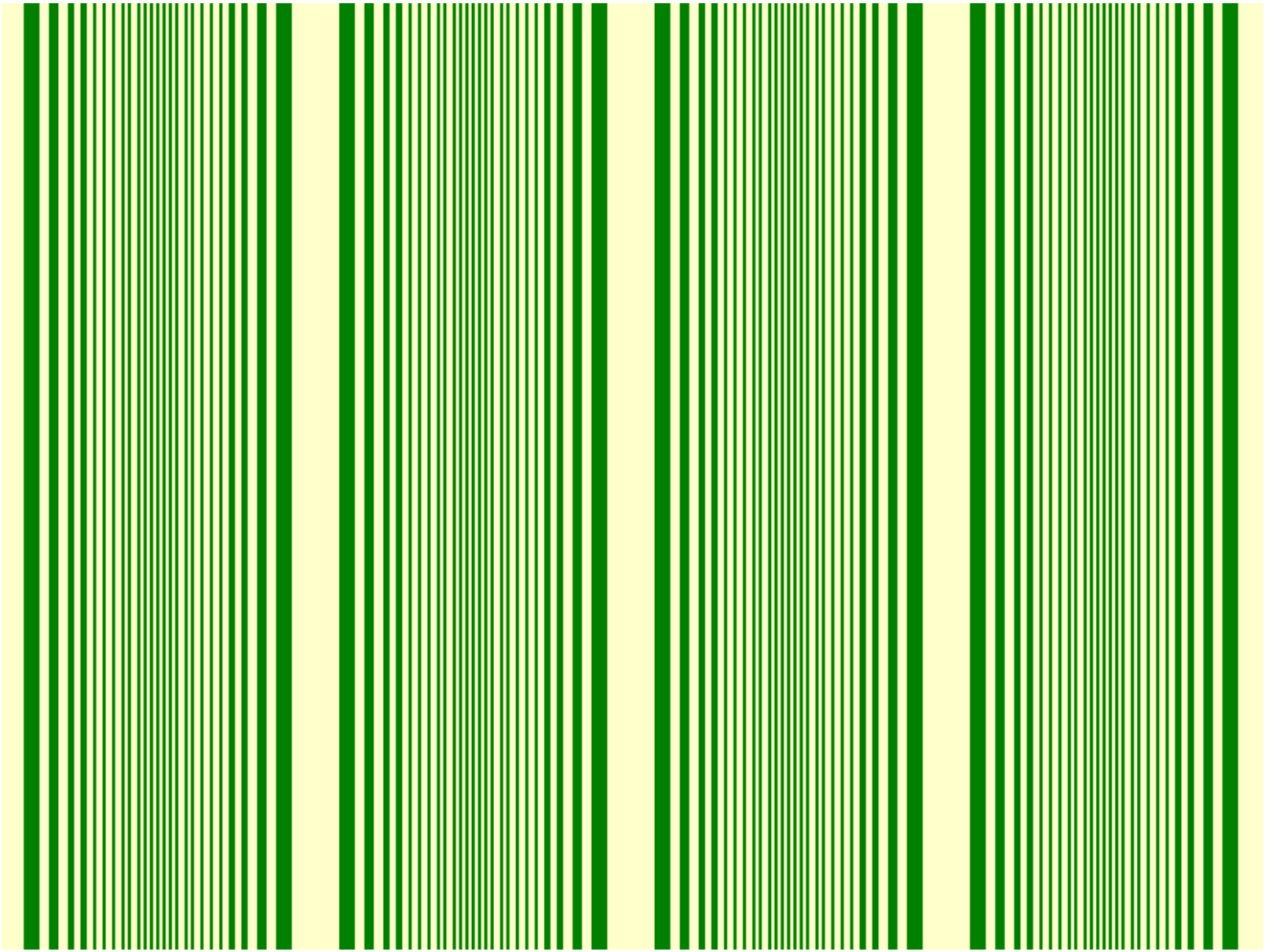


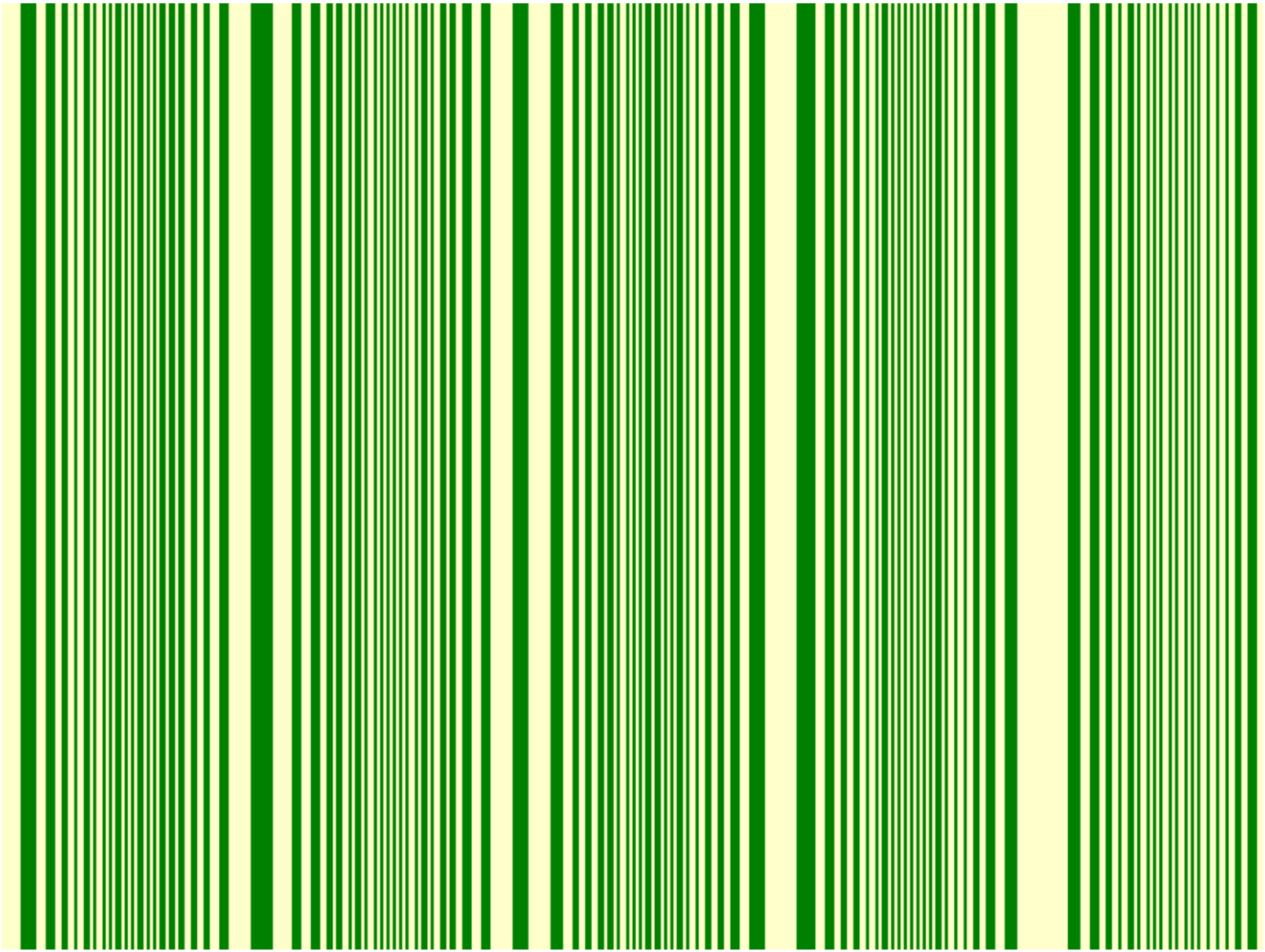


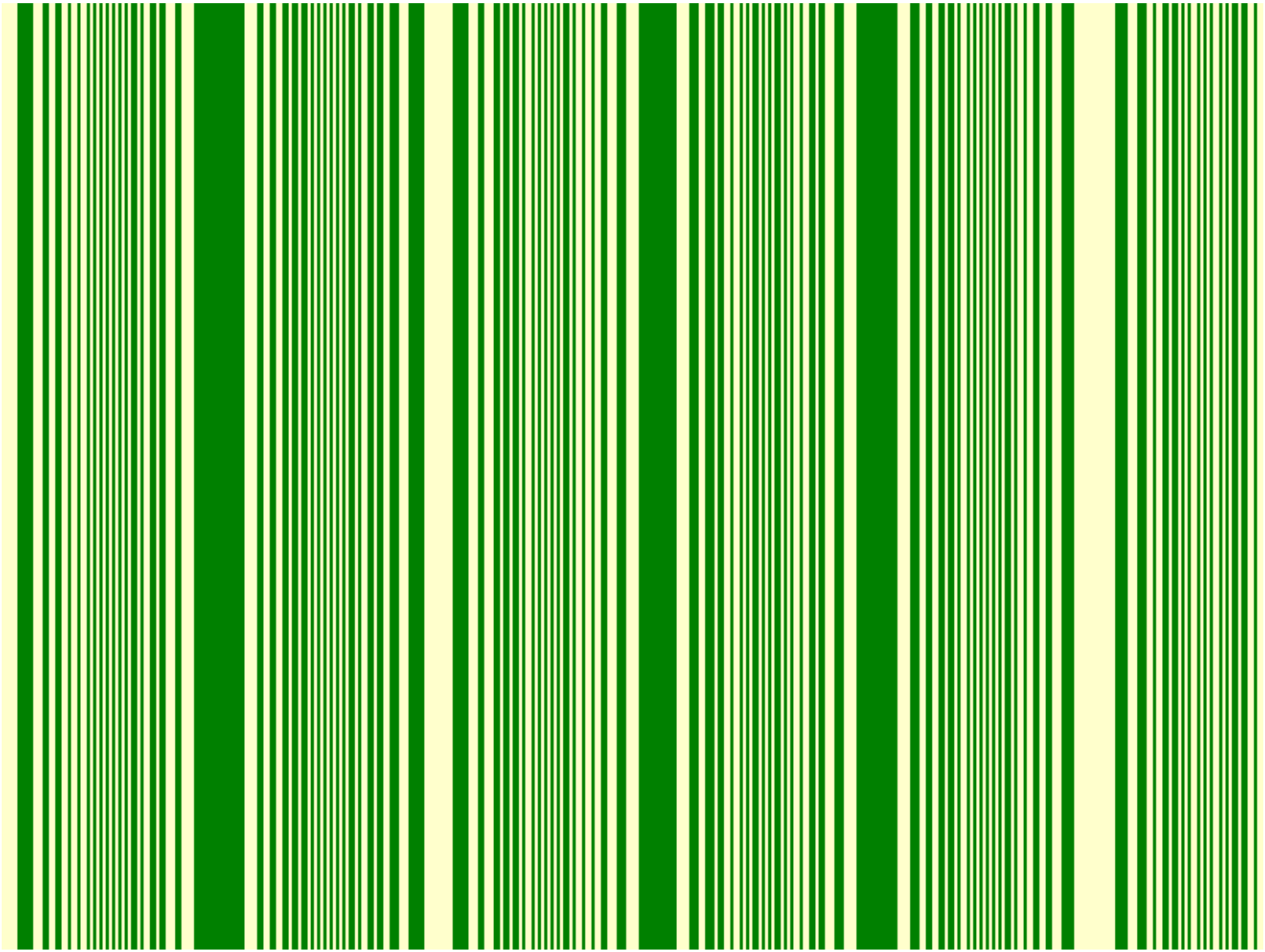


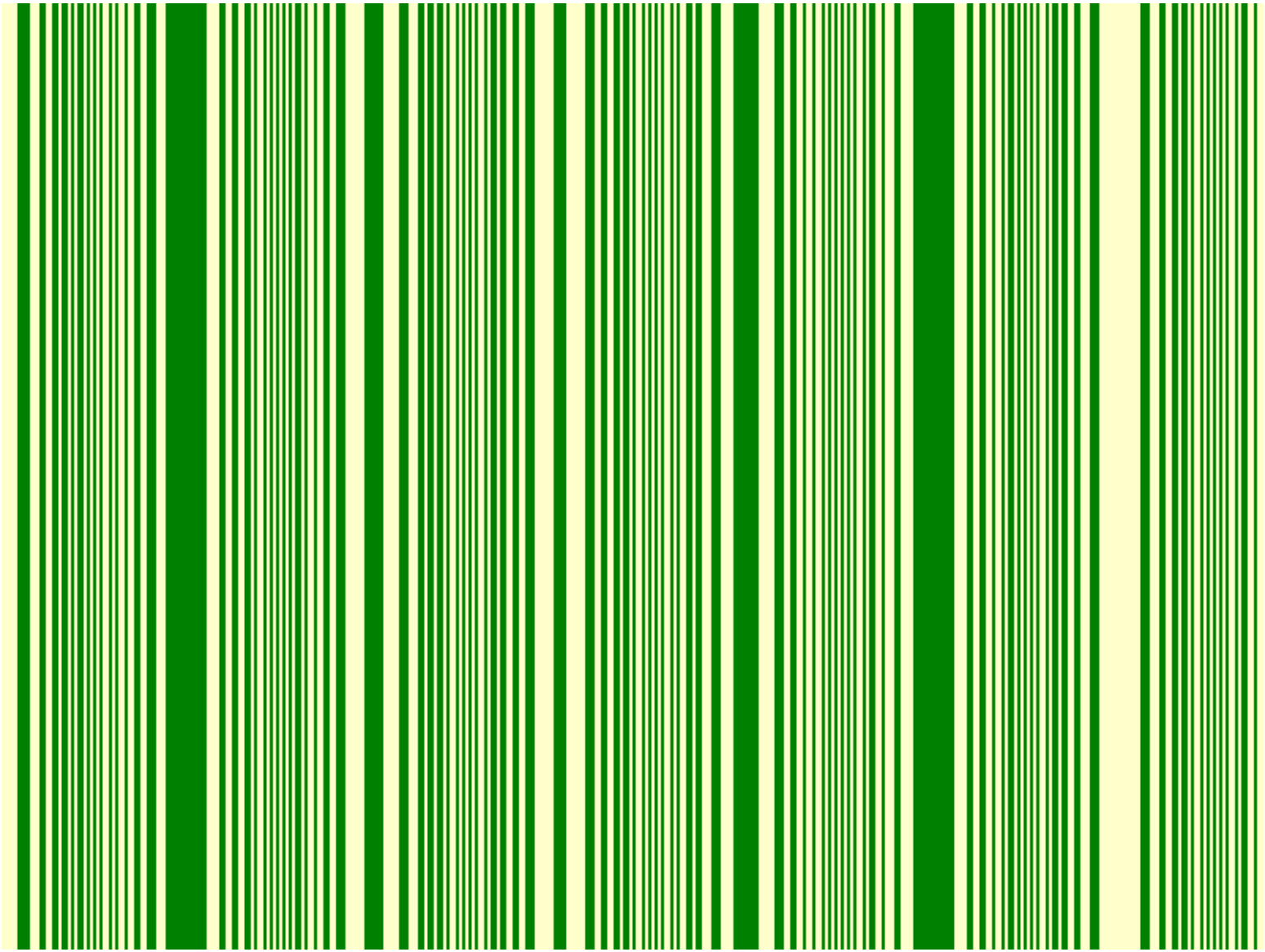


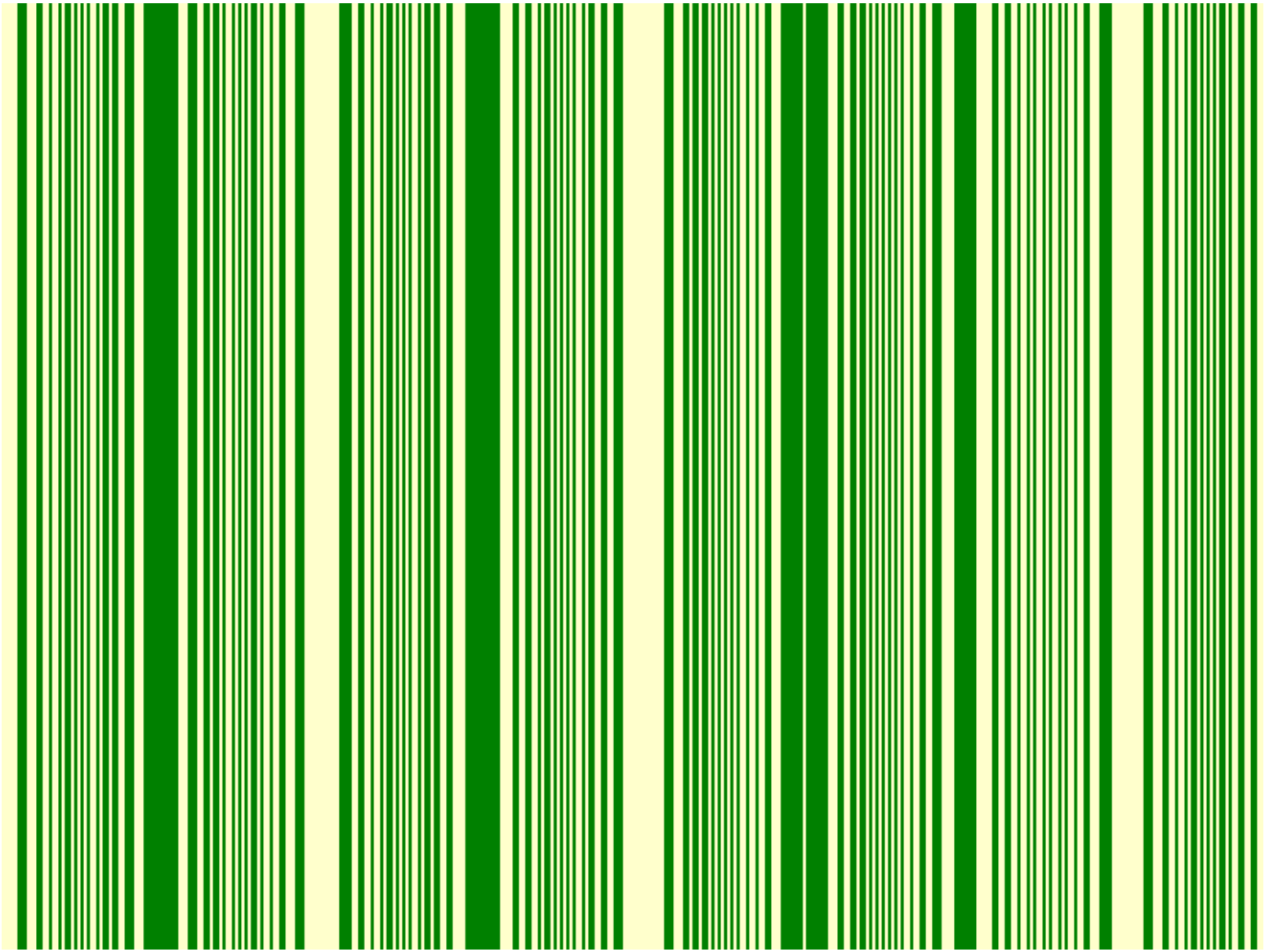


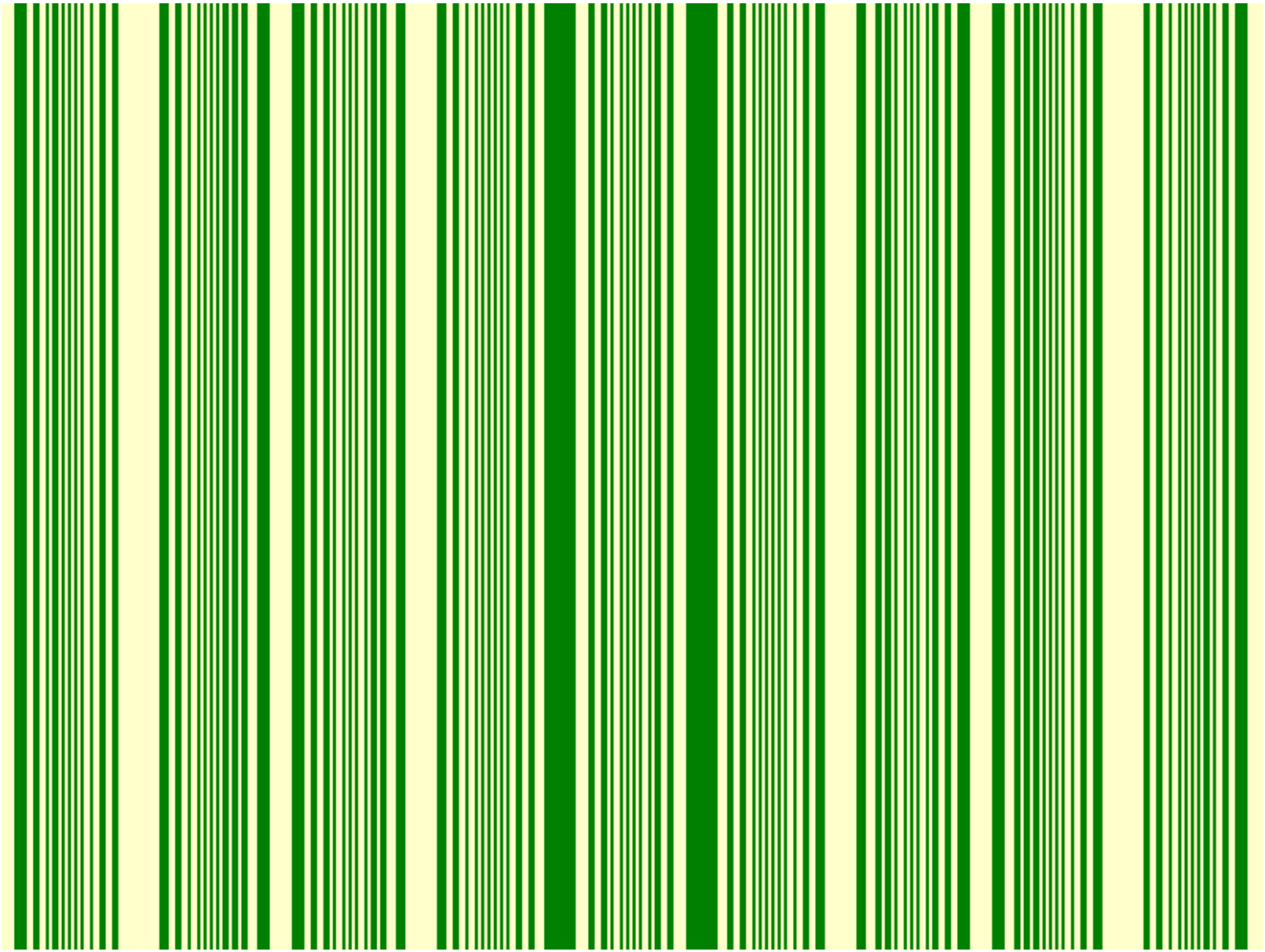


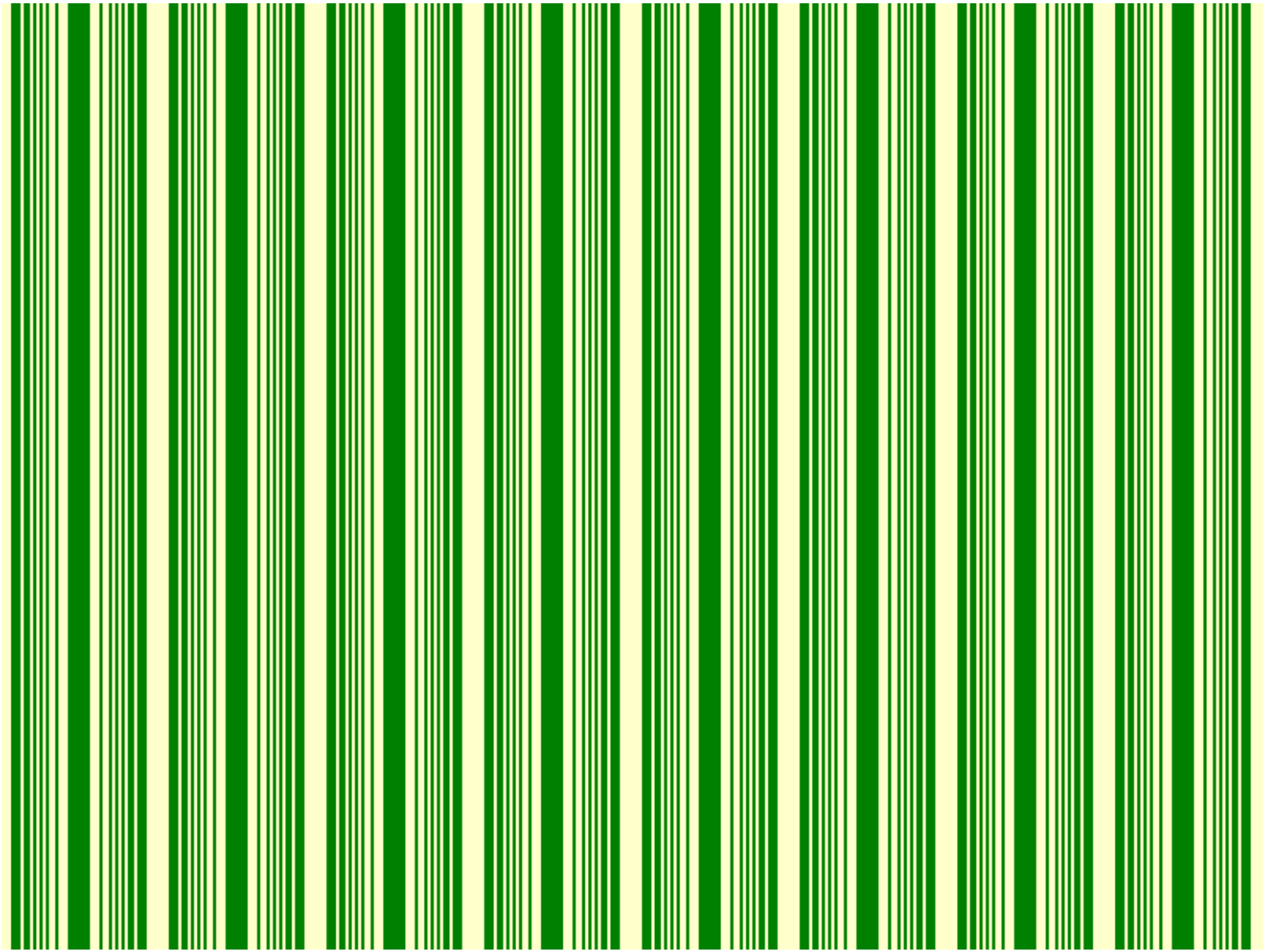


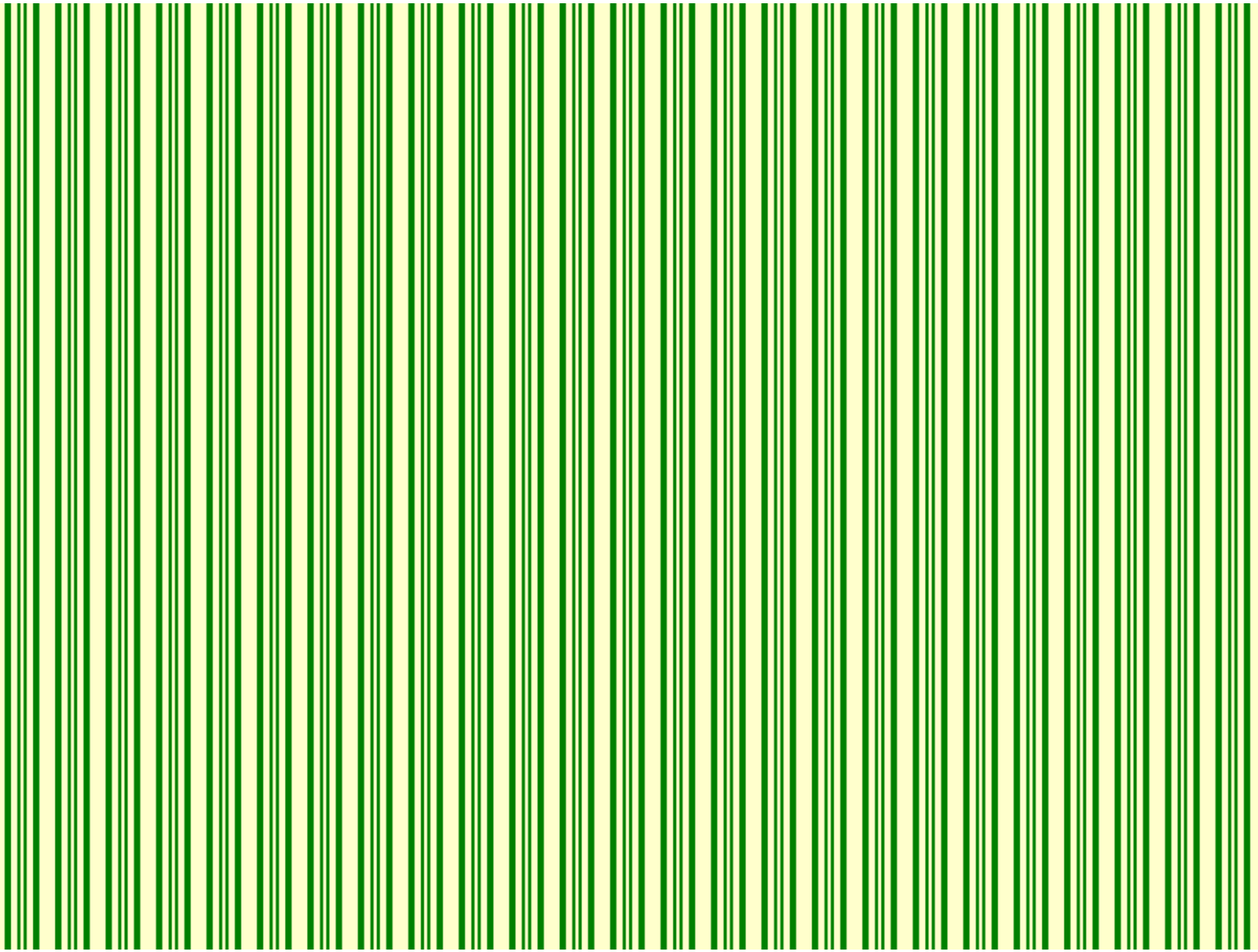


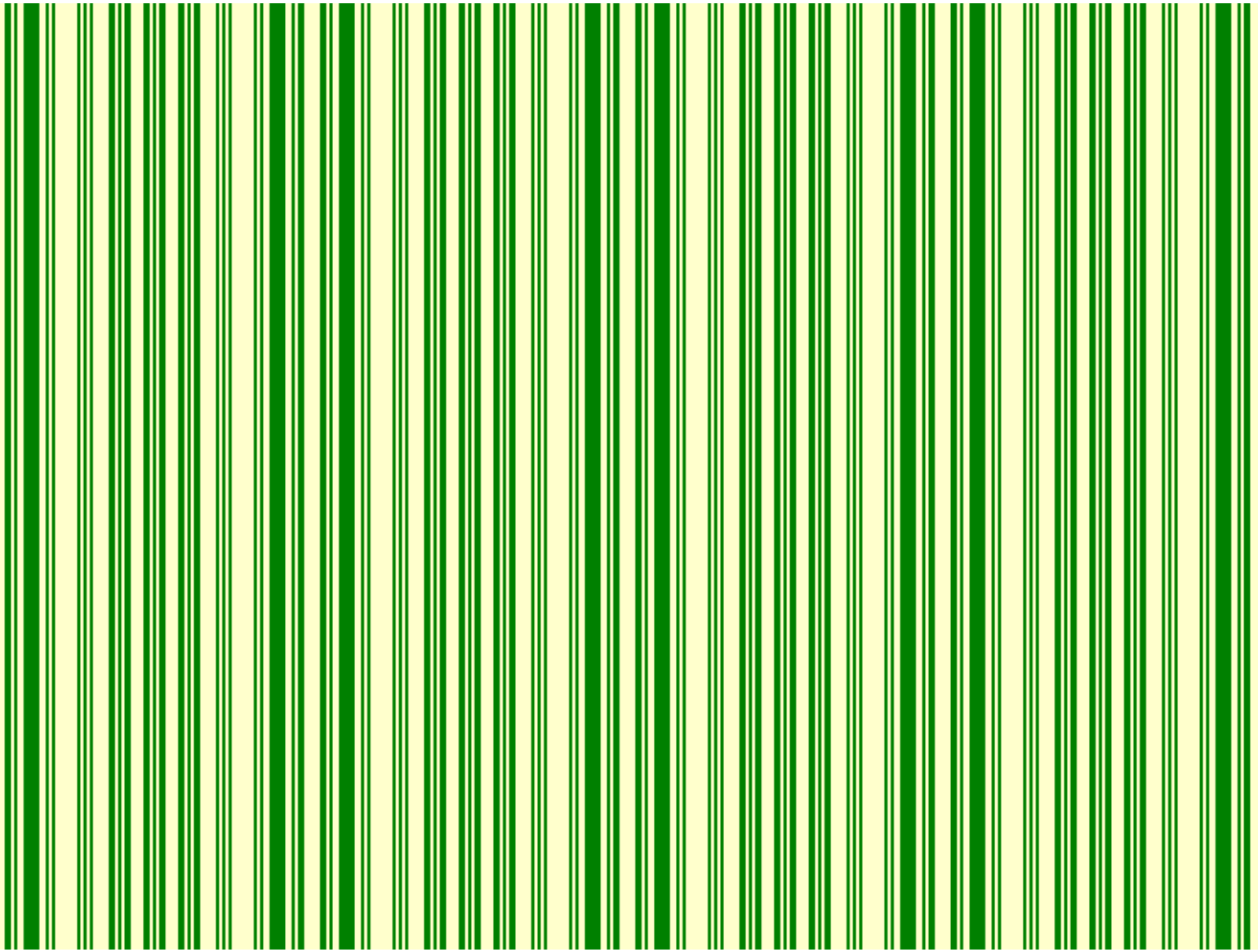


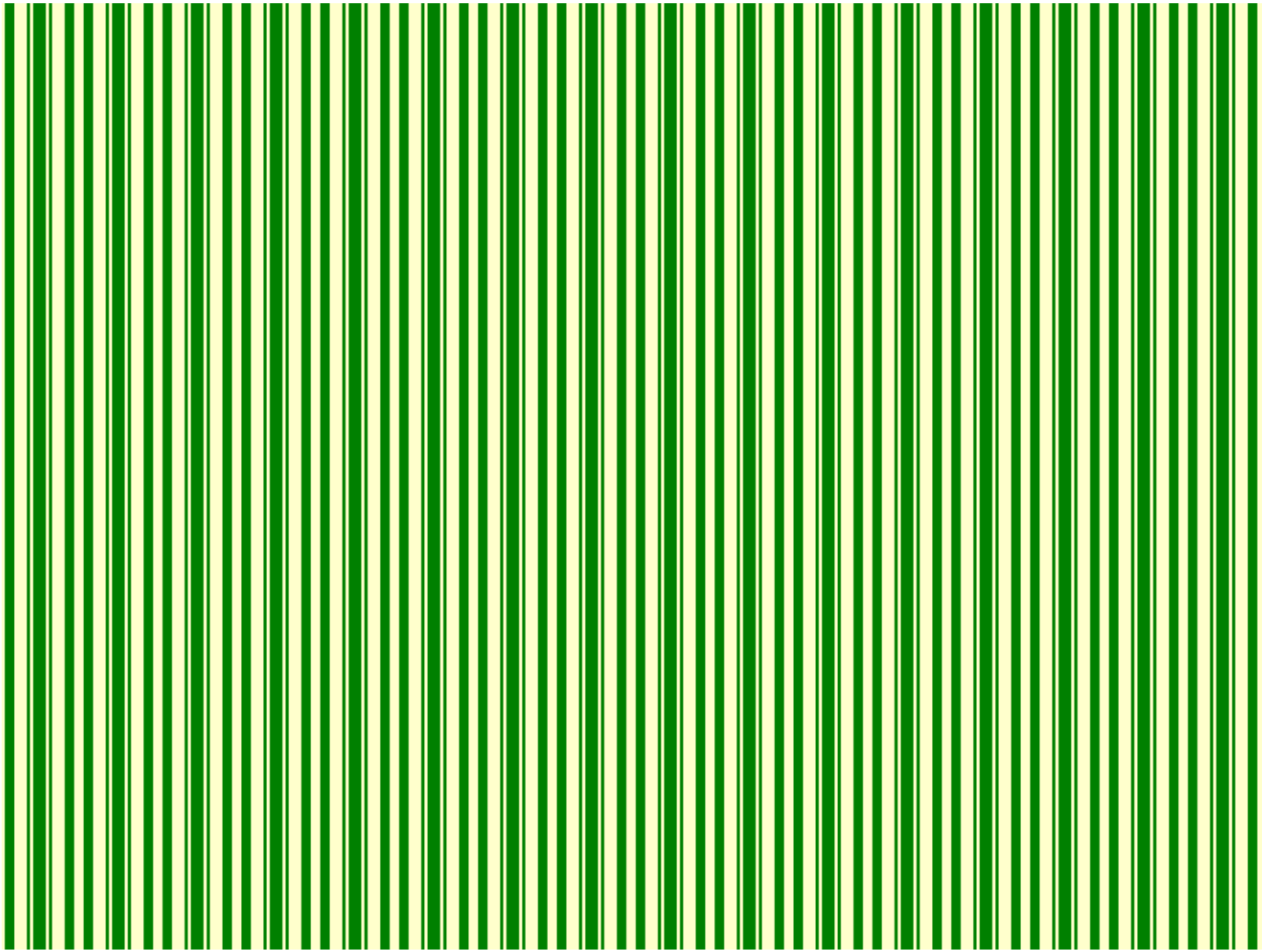


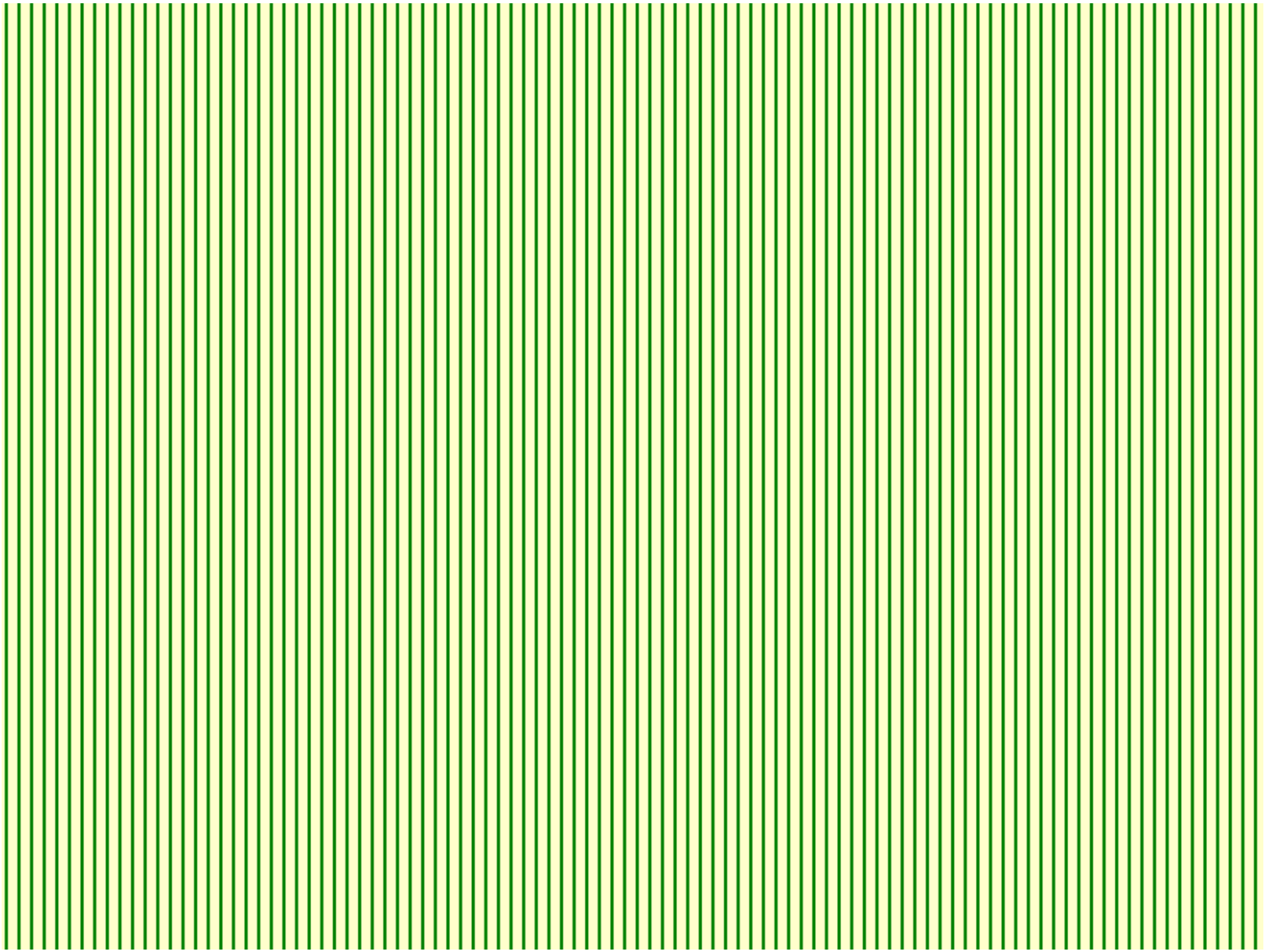


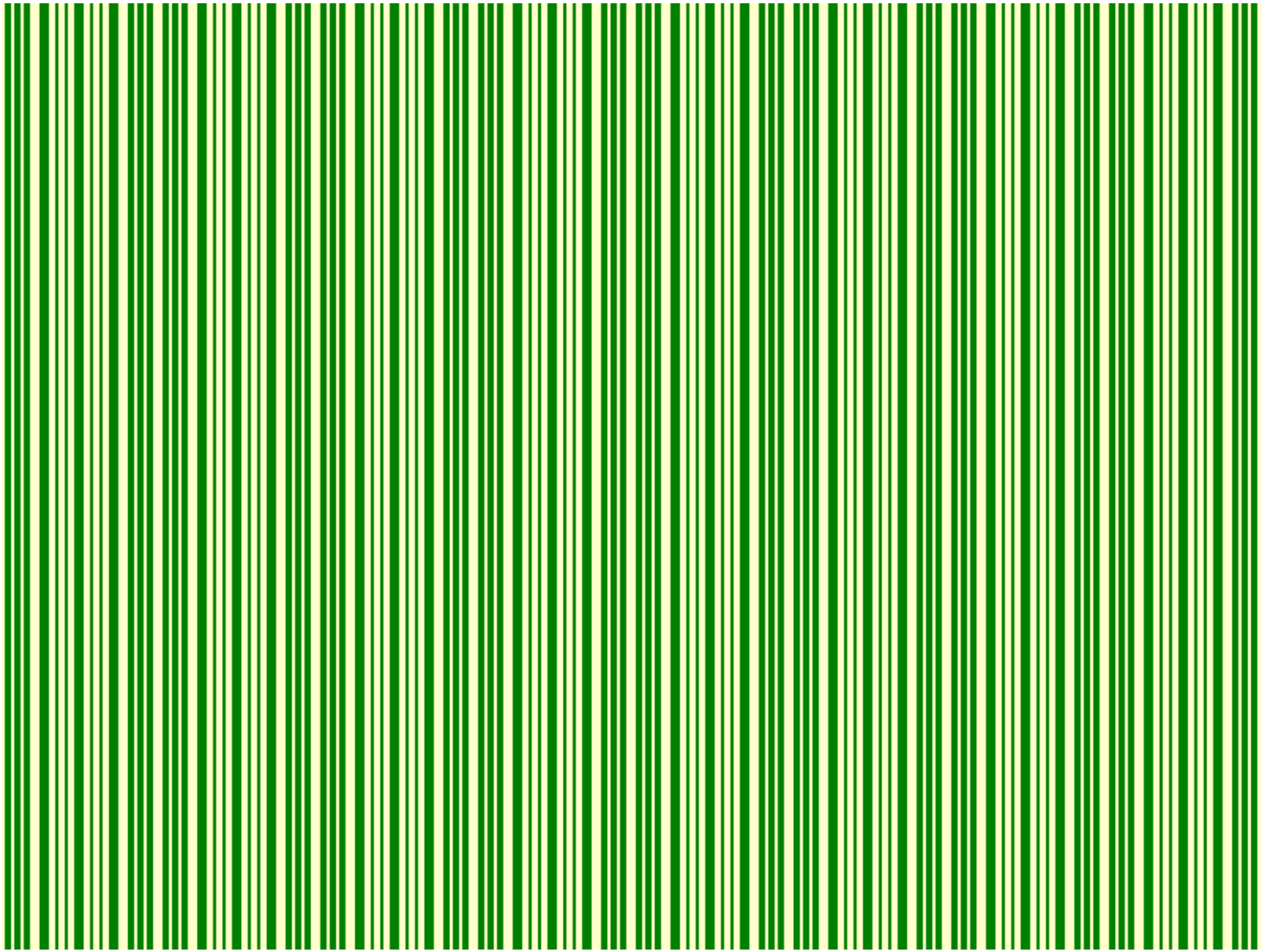


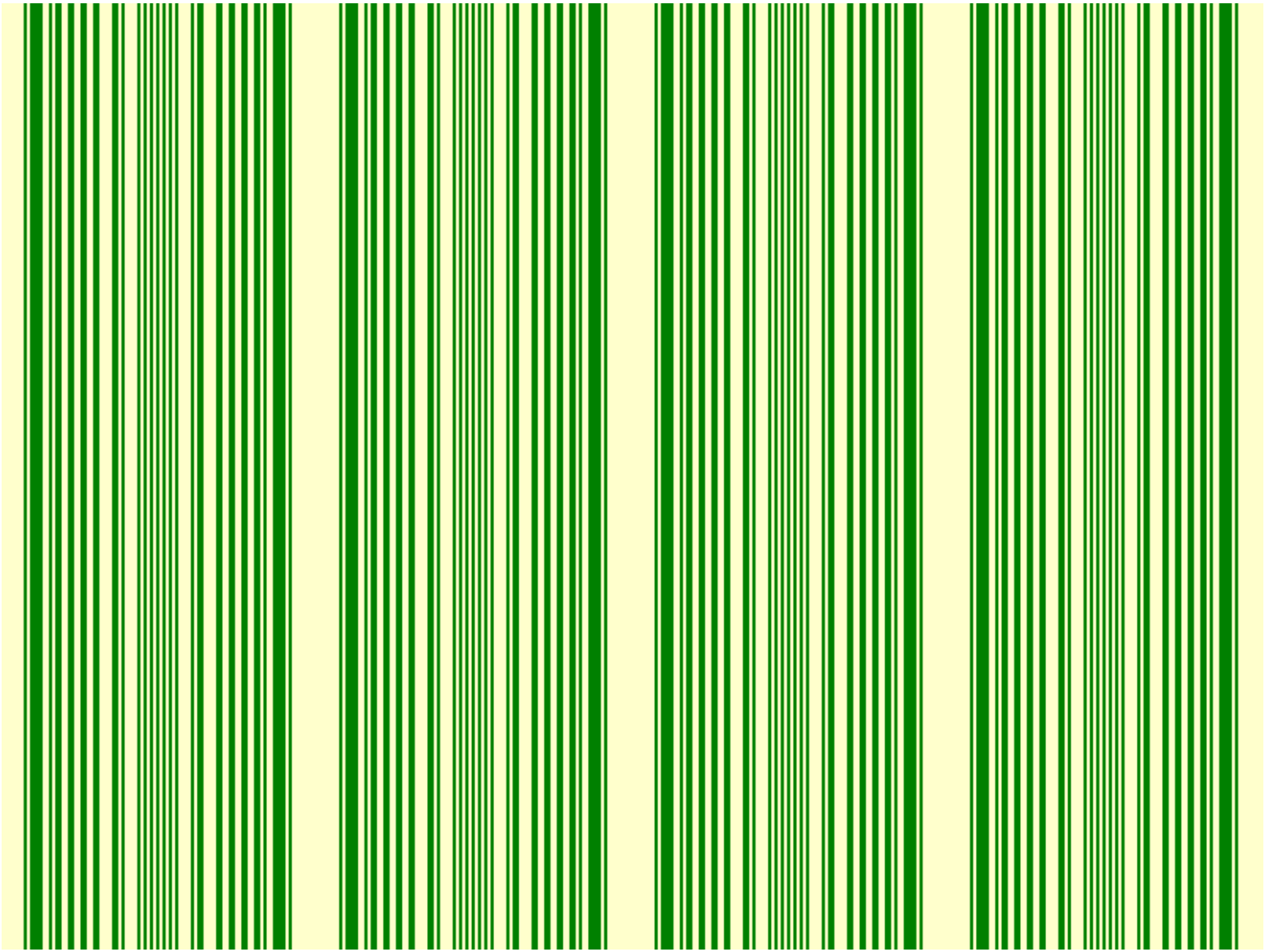


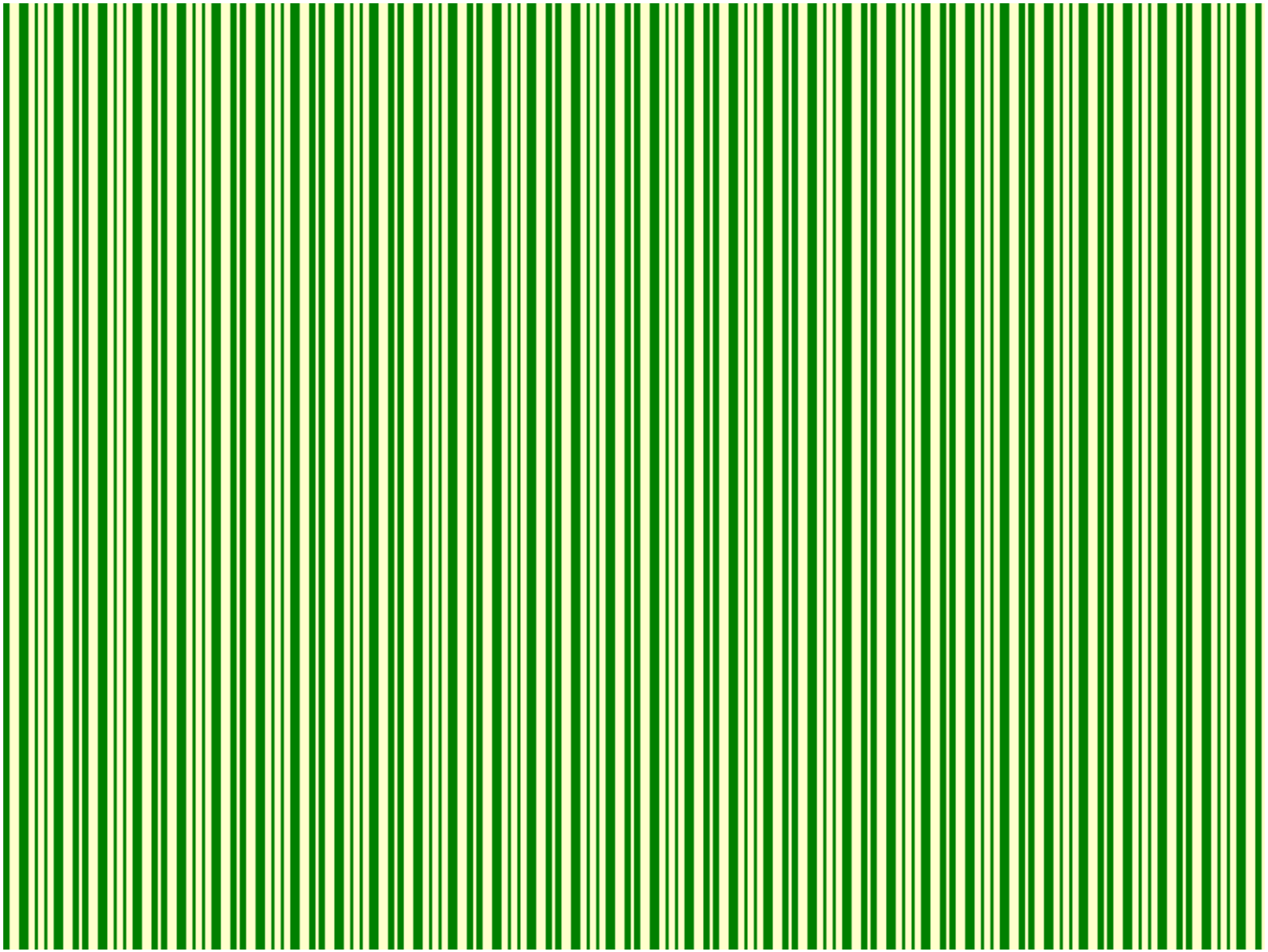


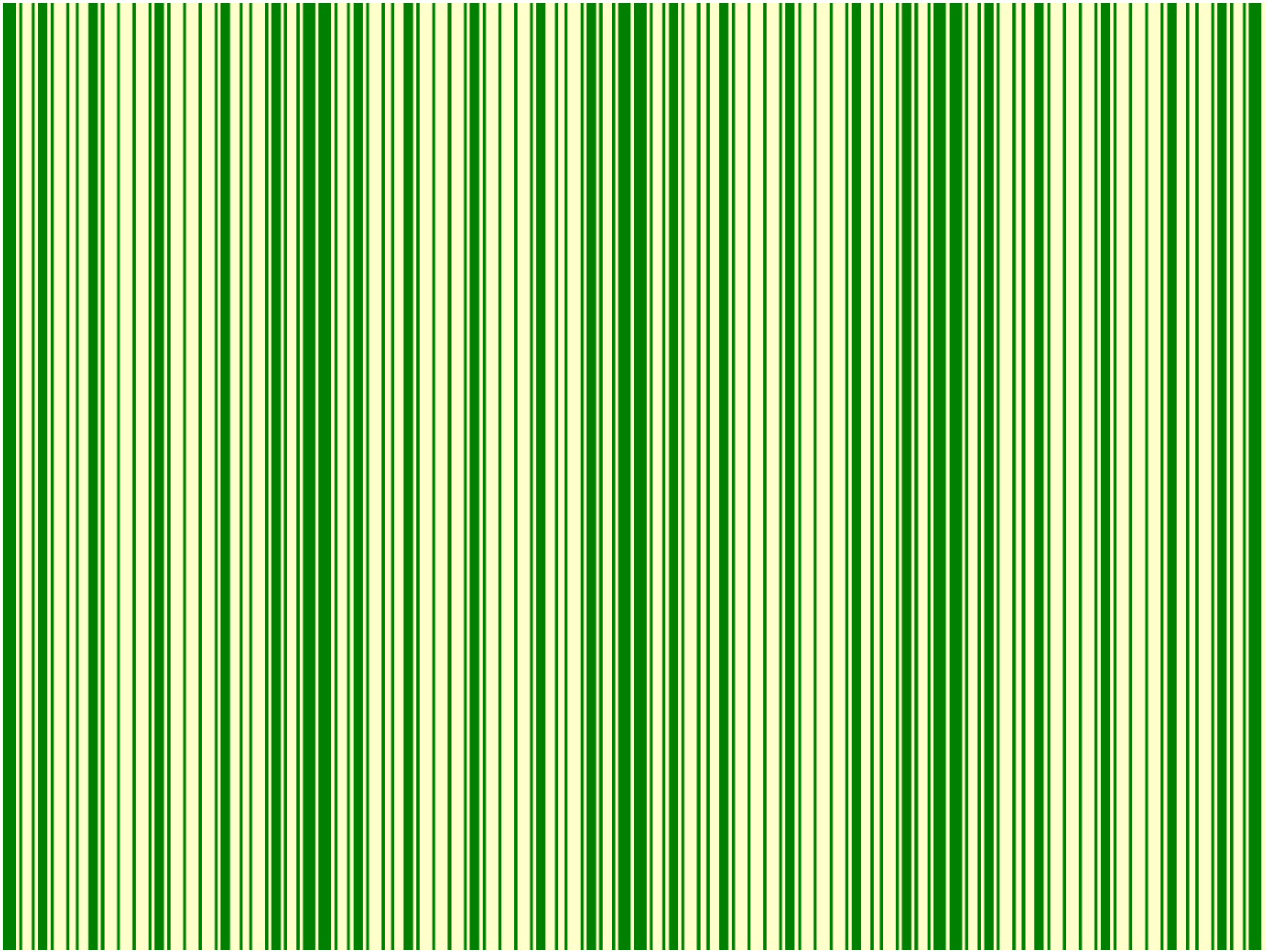


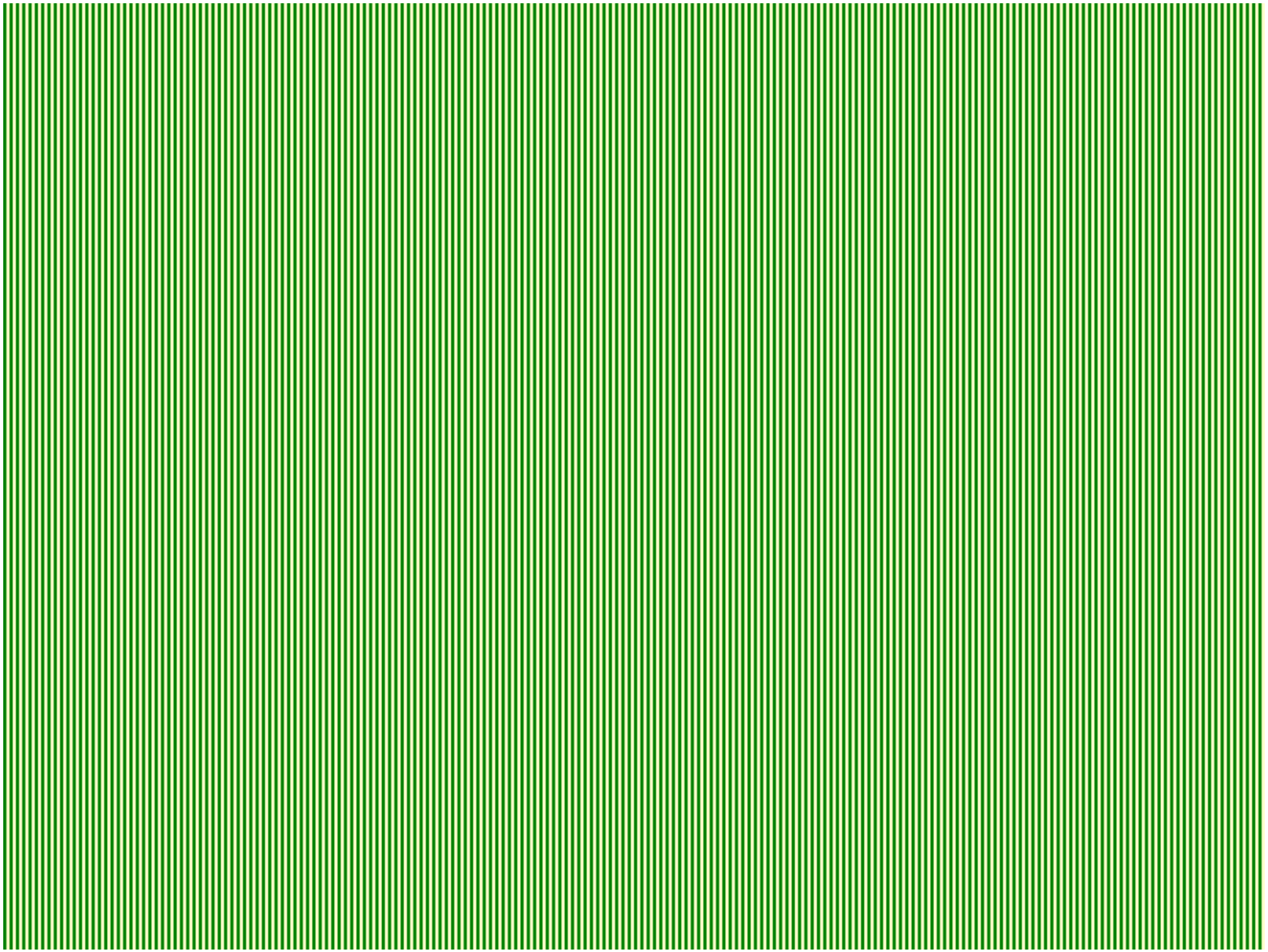


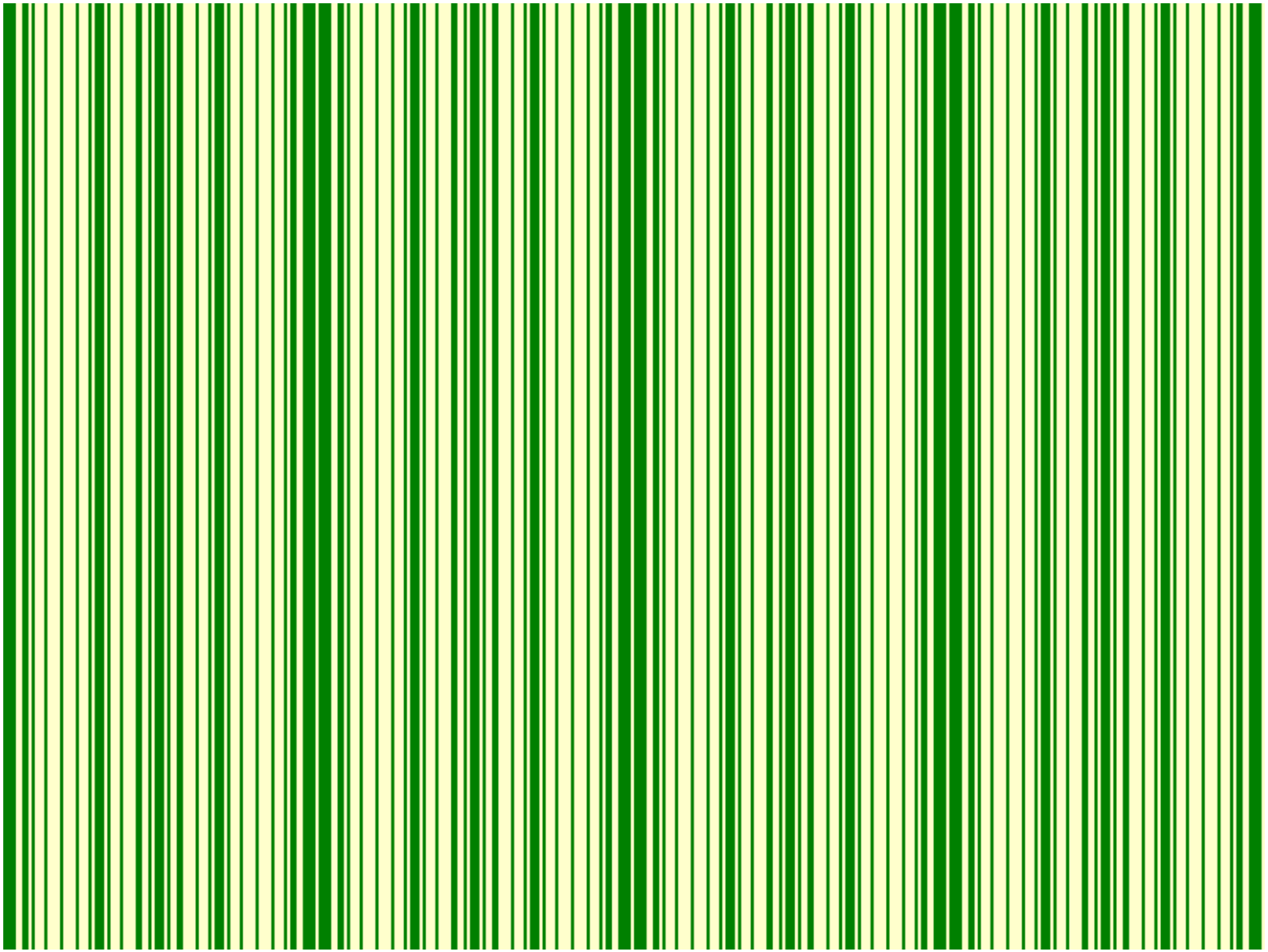










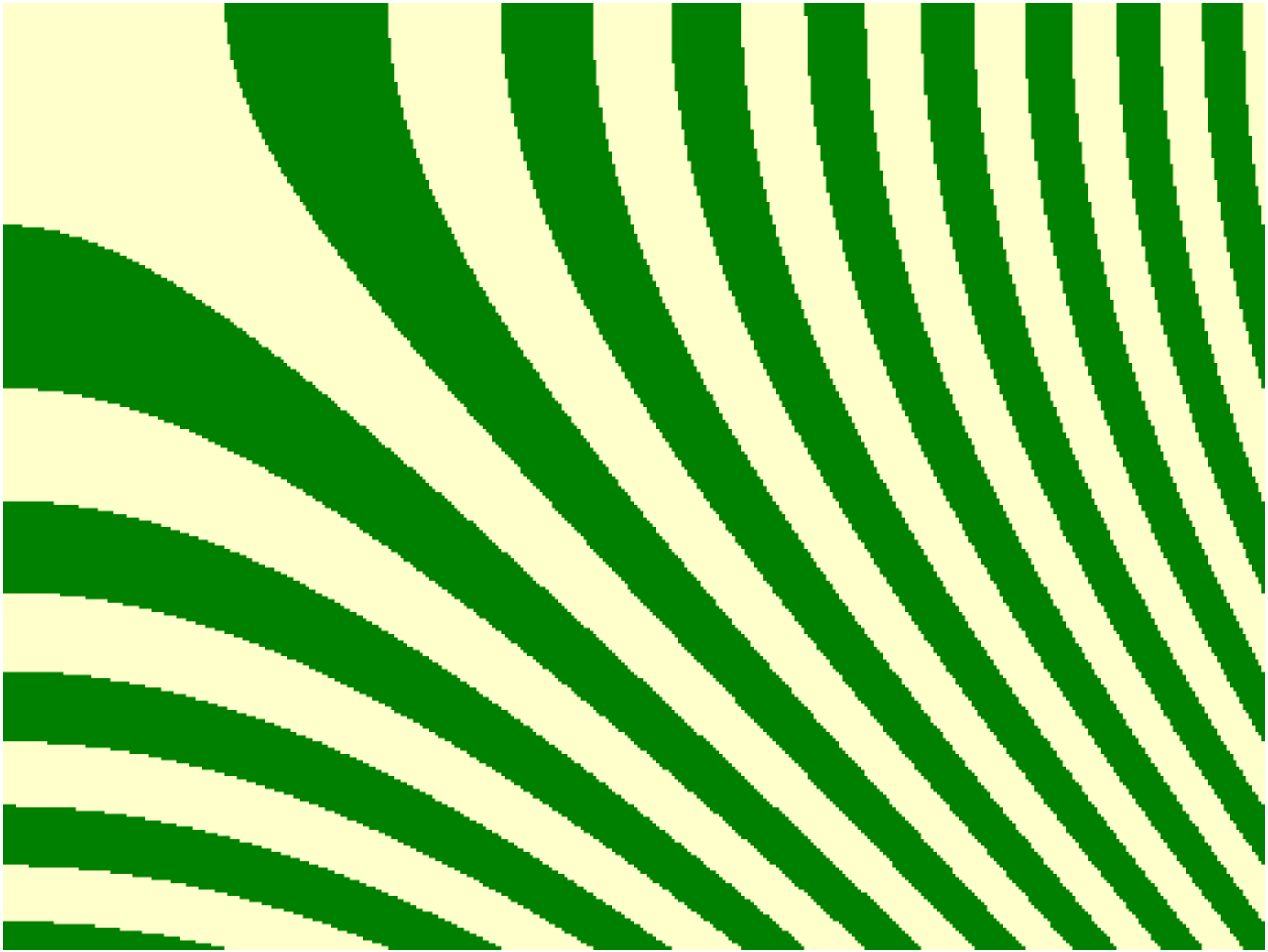


Periodic

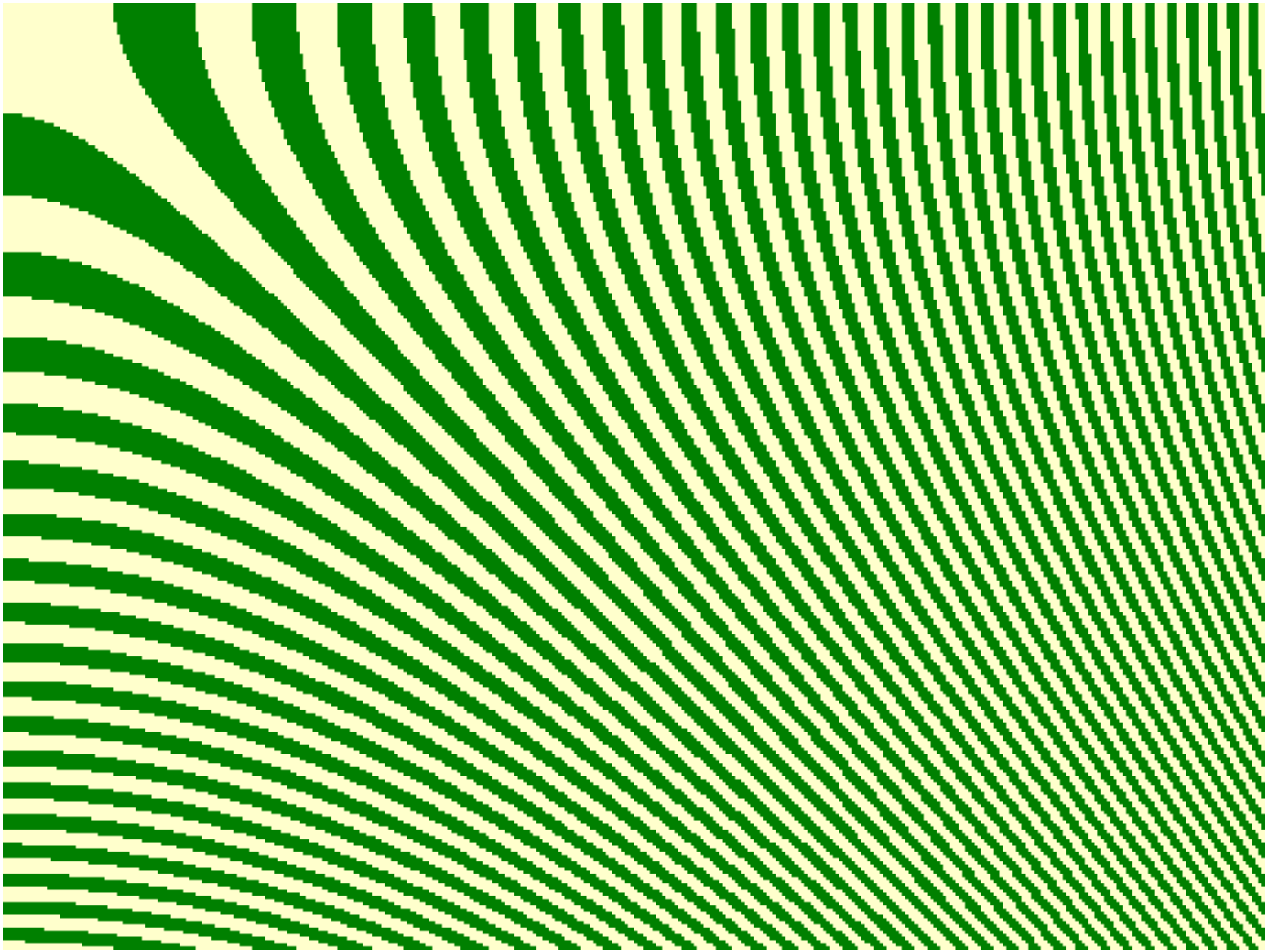
Let increment be p/q in lowest terms
Period: $\alpha = q^2$ if one of p, q even, and $\alpha = 2q^2$ otherwise

$$\left(k\frac{p}{q} + \alpha\frac{p}{q}\right)^2 - \left(k\frac{p}{q}\right)^2 = 2k\frac{p^2}{q^2}\alpha + \alpha^2\frac{p^2}{q^2}$$

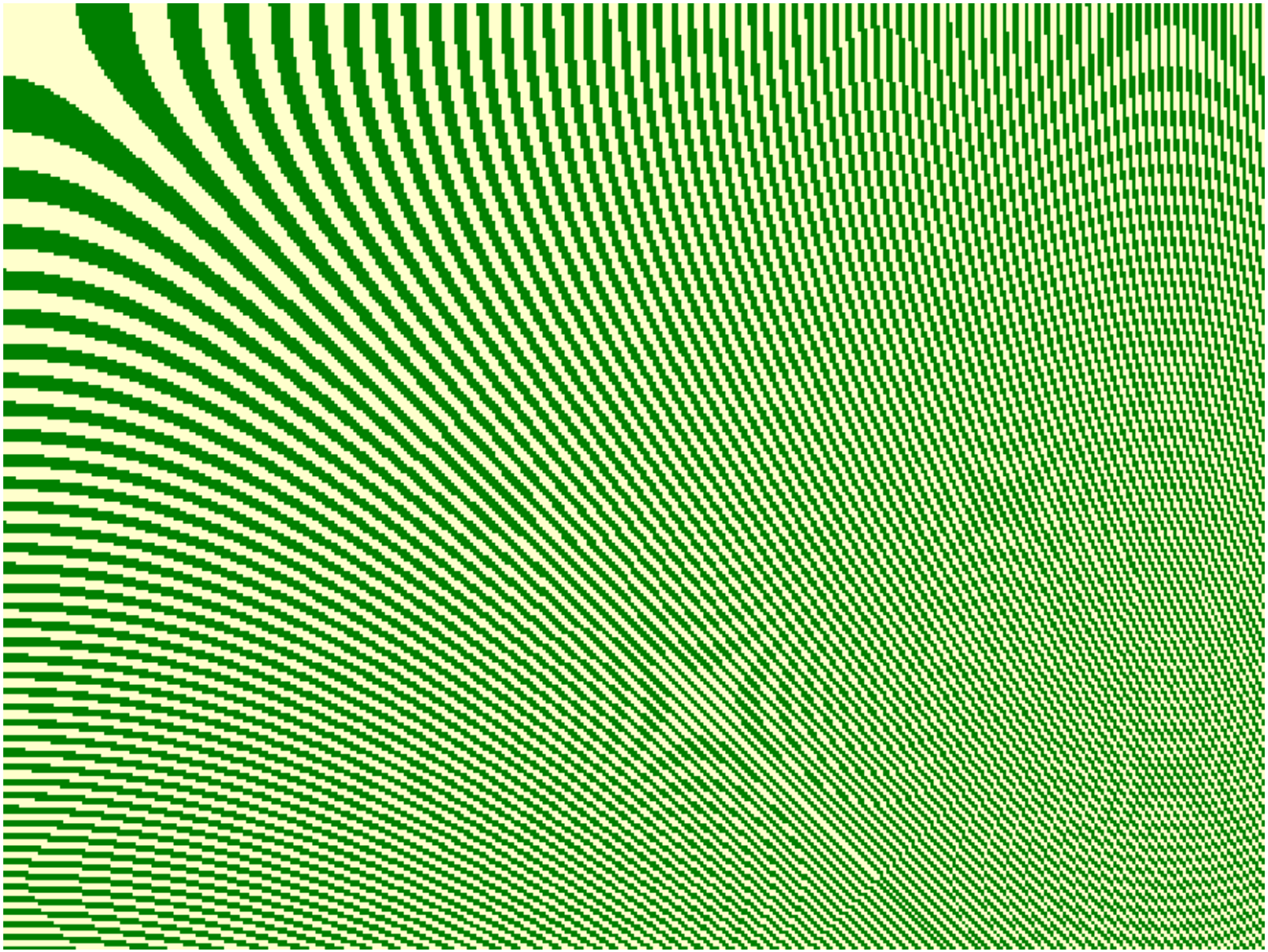
Want this to be an integer divisible by 2
This means $q^2|\alpha$, and if p, q are both odd, then $2|\alpha$.



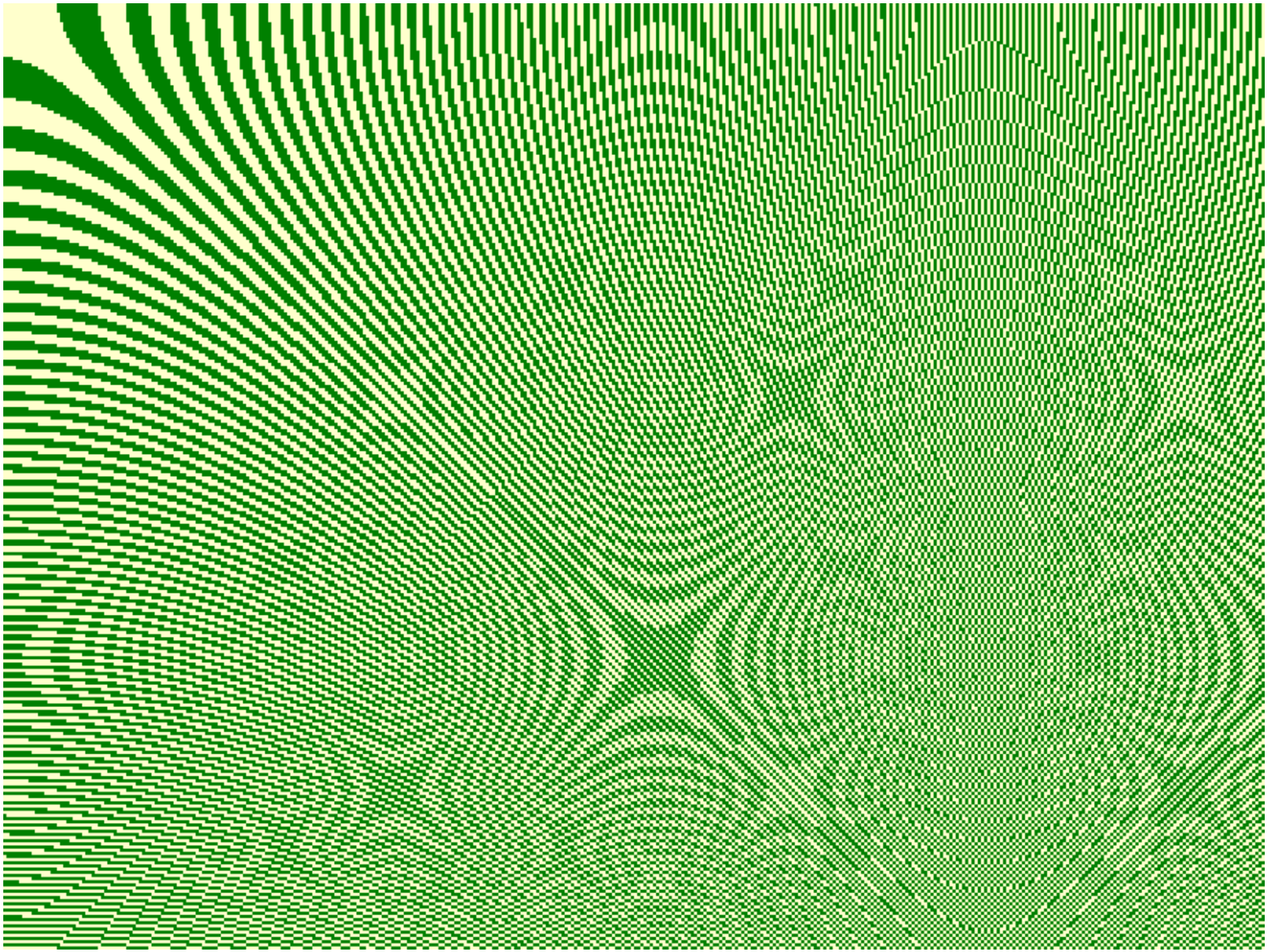
$x^2 - y^2$ 0.02 0.02 2



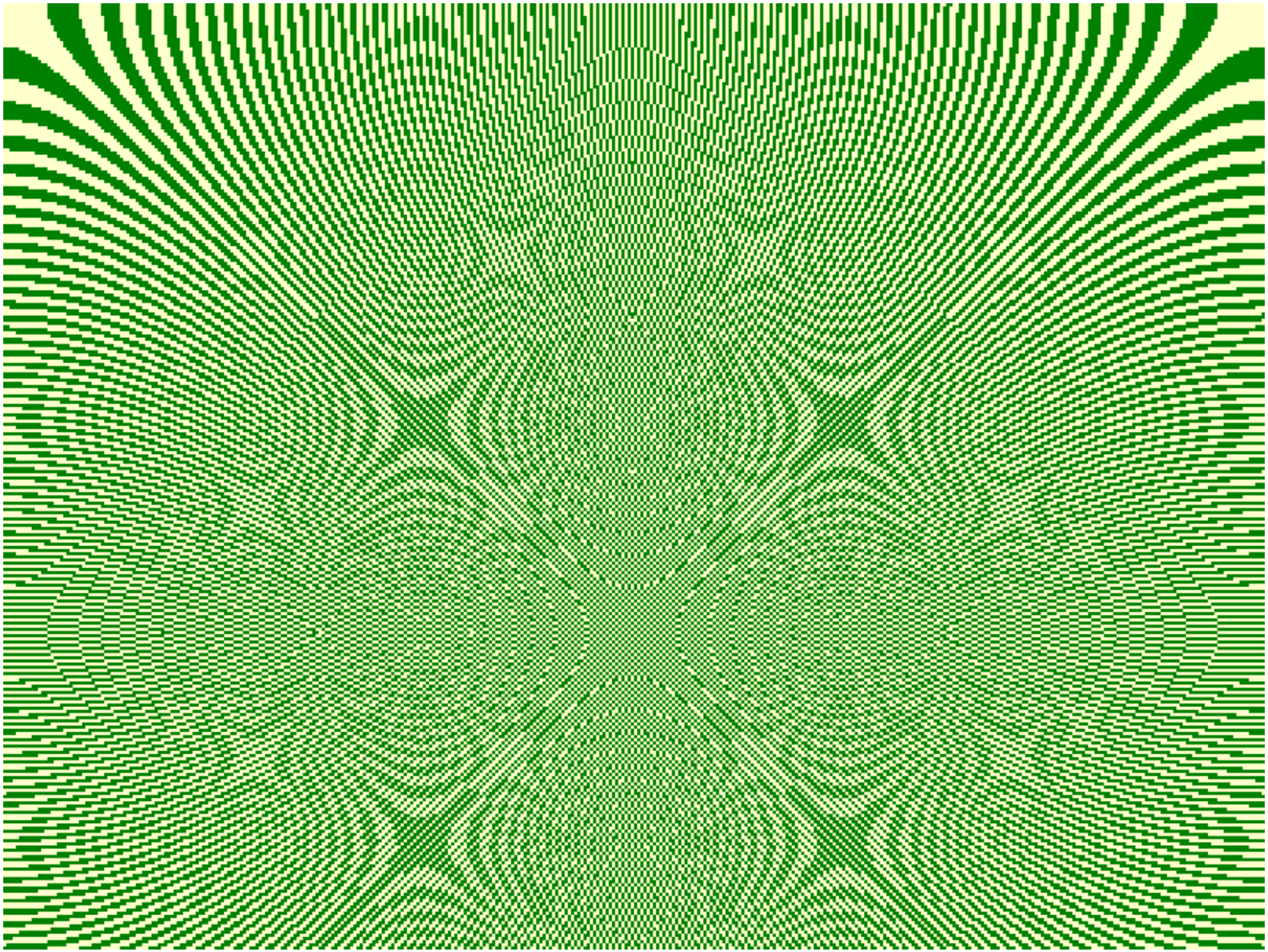
$x^2 - y^2$ 0.03 0.03 2



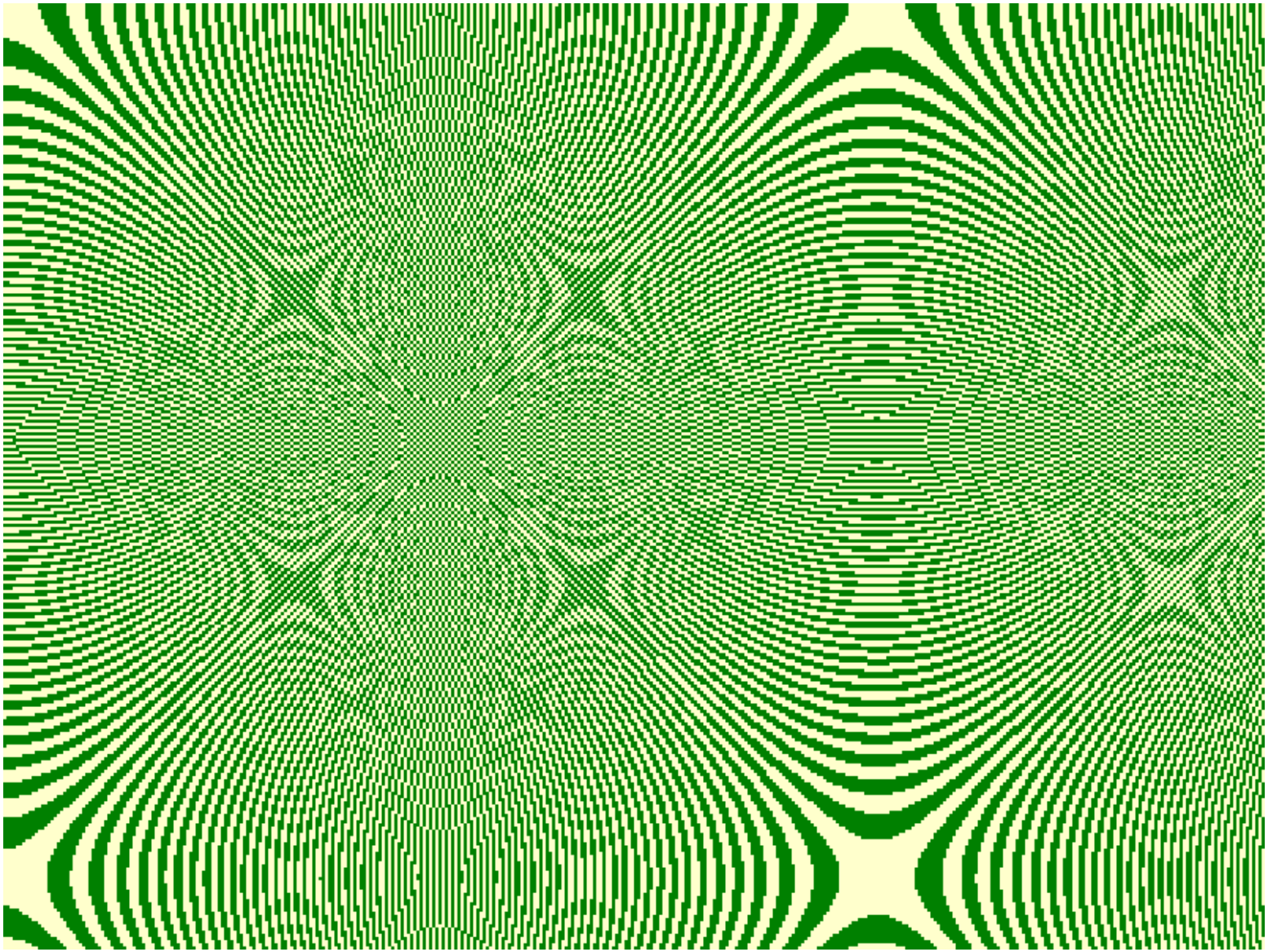
$x^2 - y^2$ 0.04 0.04 2



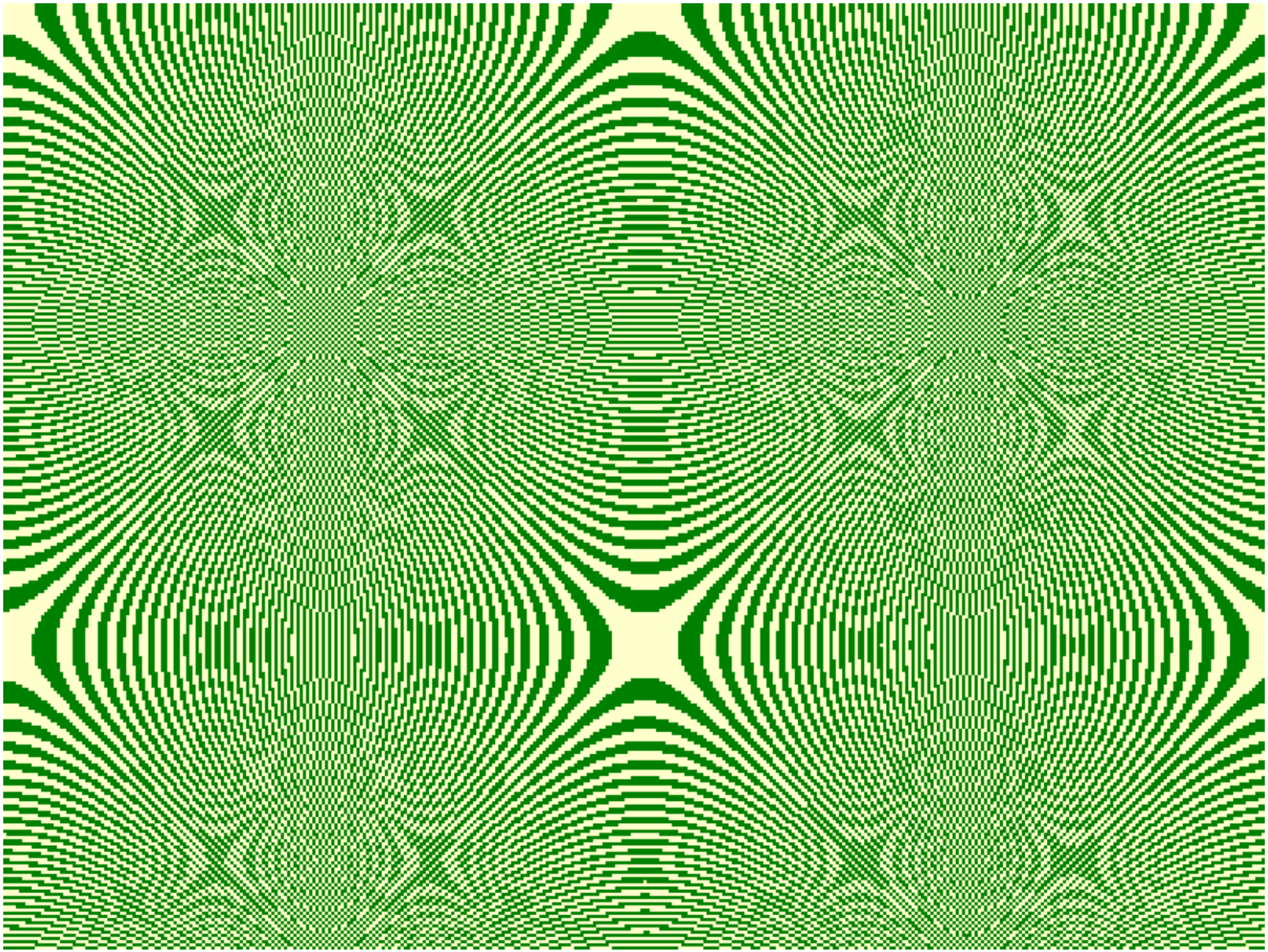
$x^2 - y^2$ 0.05 0.05 2



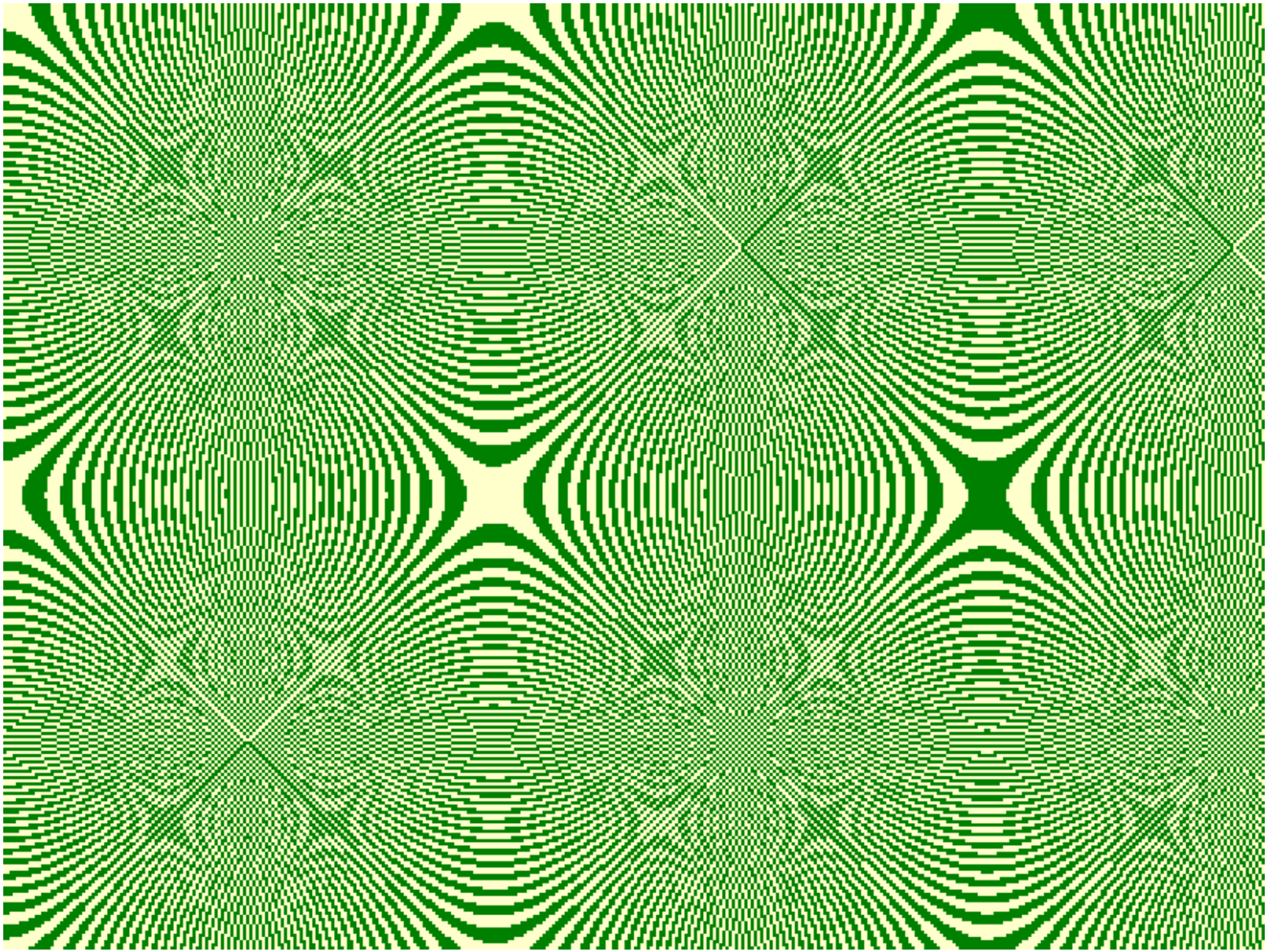
$x^2 - y^2$ 0.06 0.06 2



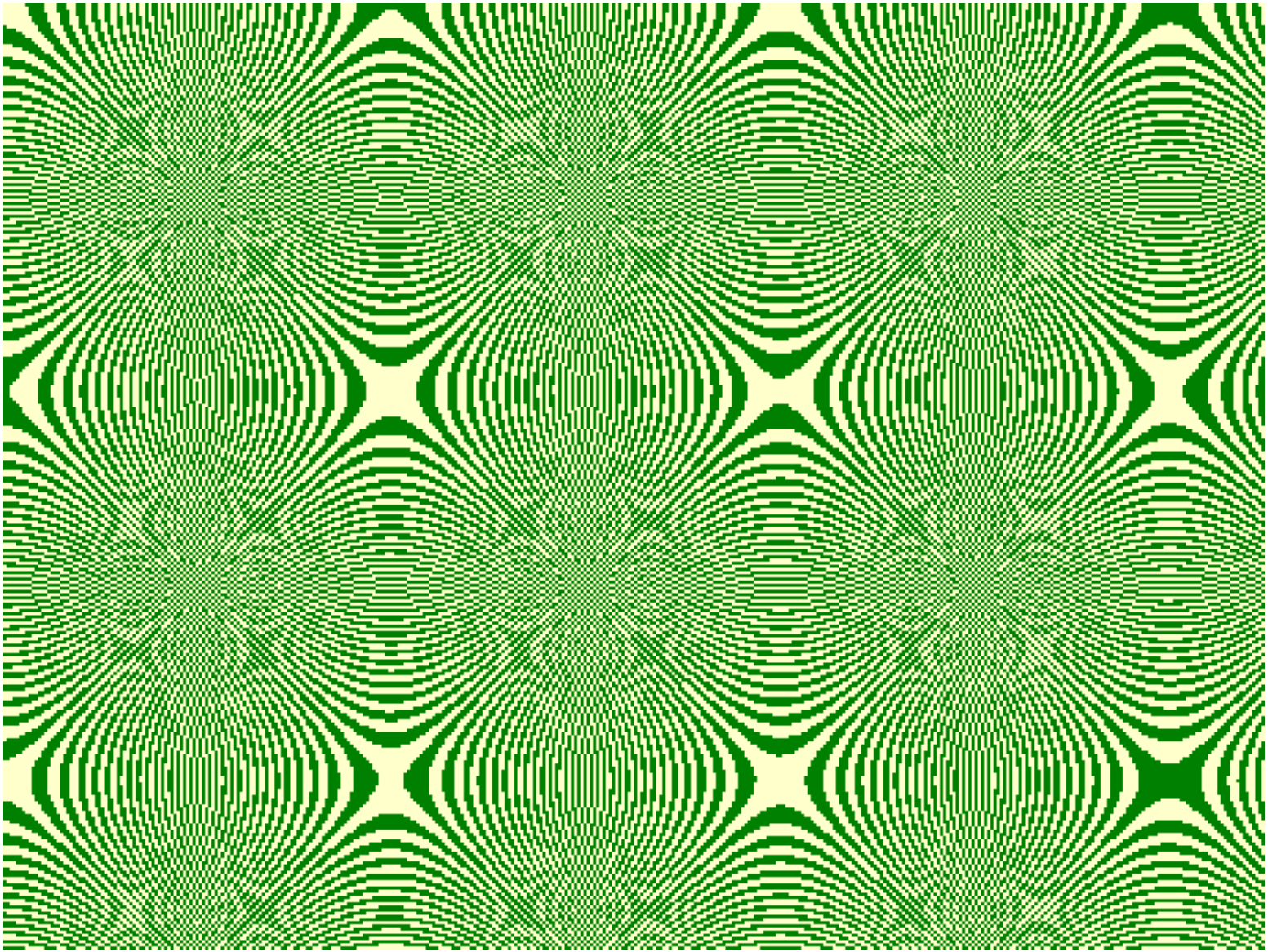
$x^2 - y^2$ 0.07 0.07 2



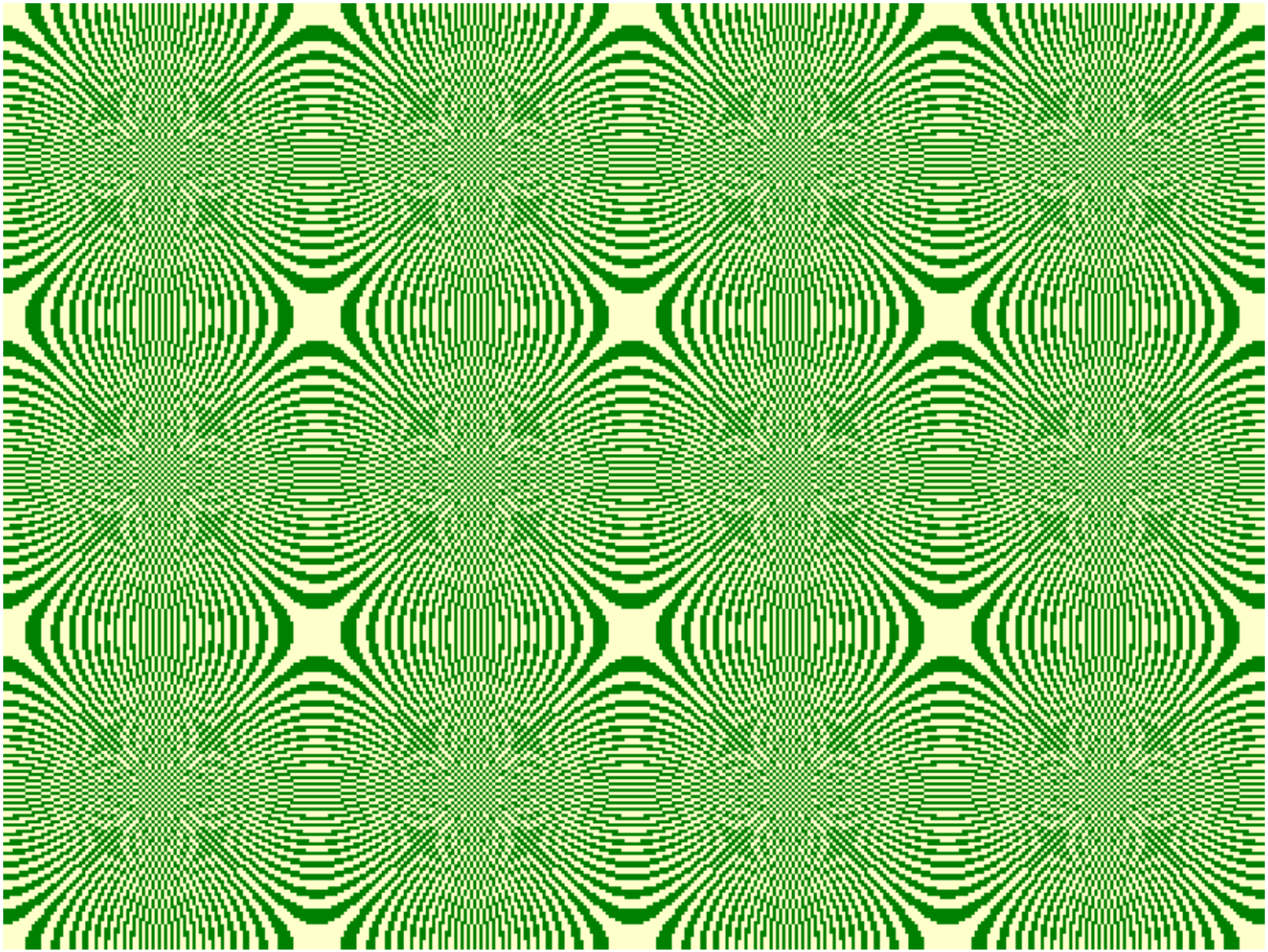
$x^2 - y^2$ 0.08 0.08 2

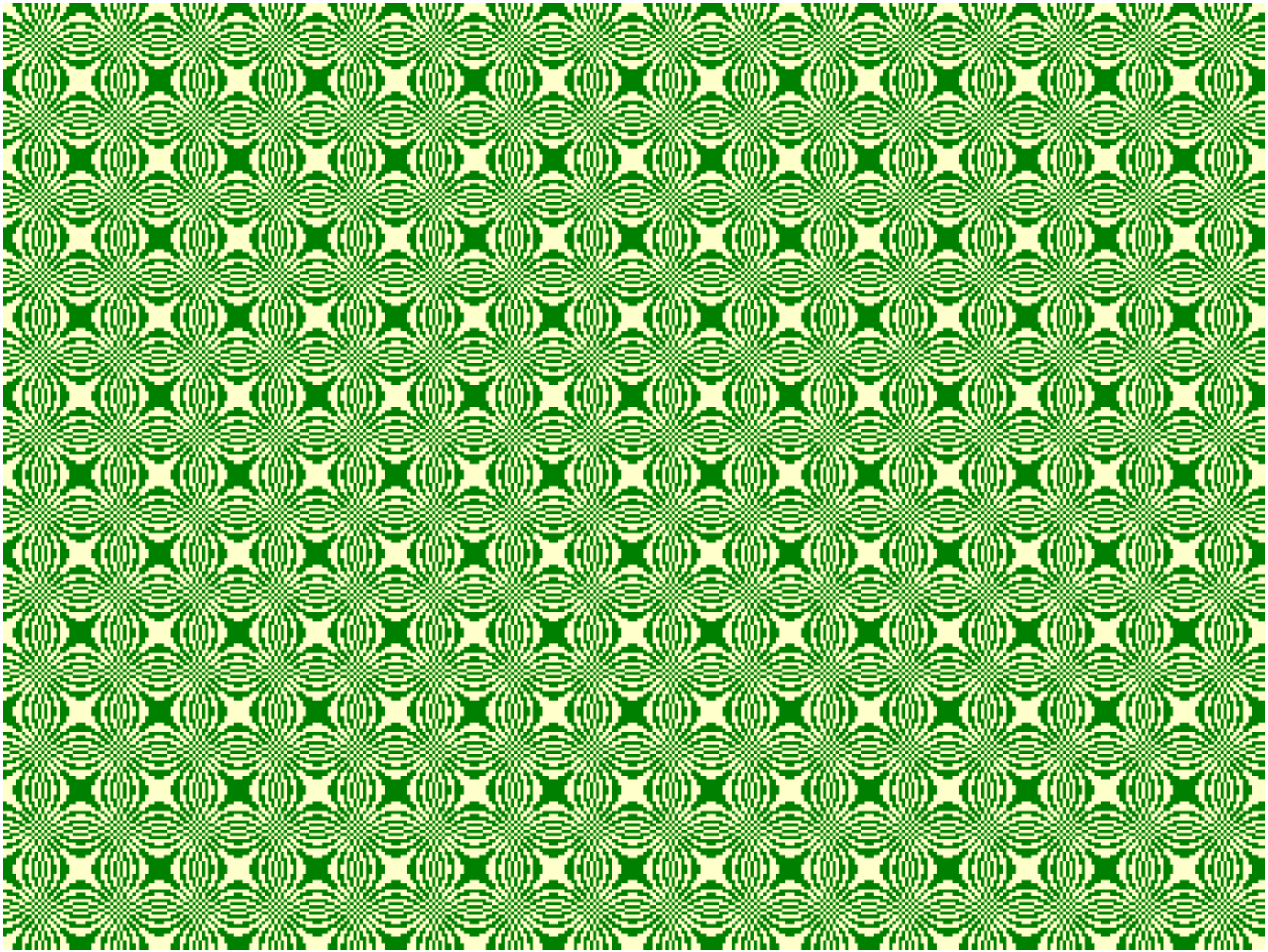


$x^2 - y^2$ 0.09 0.09 2

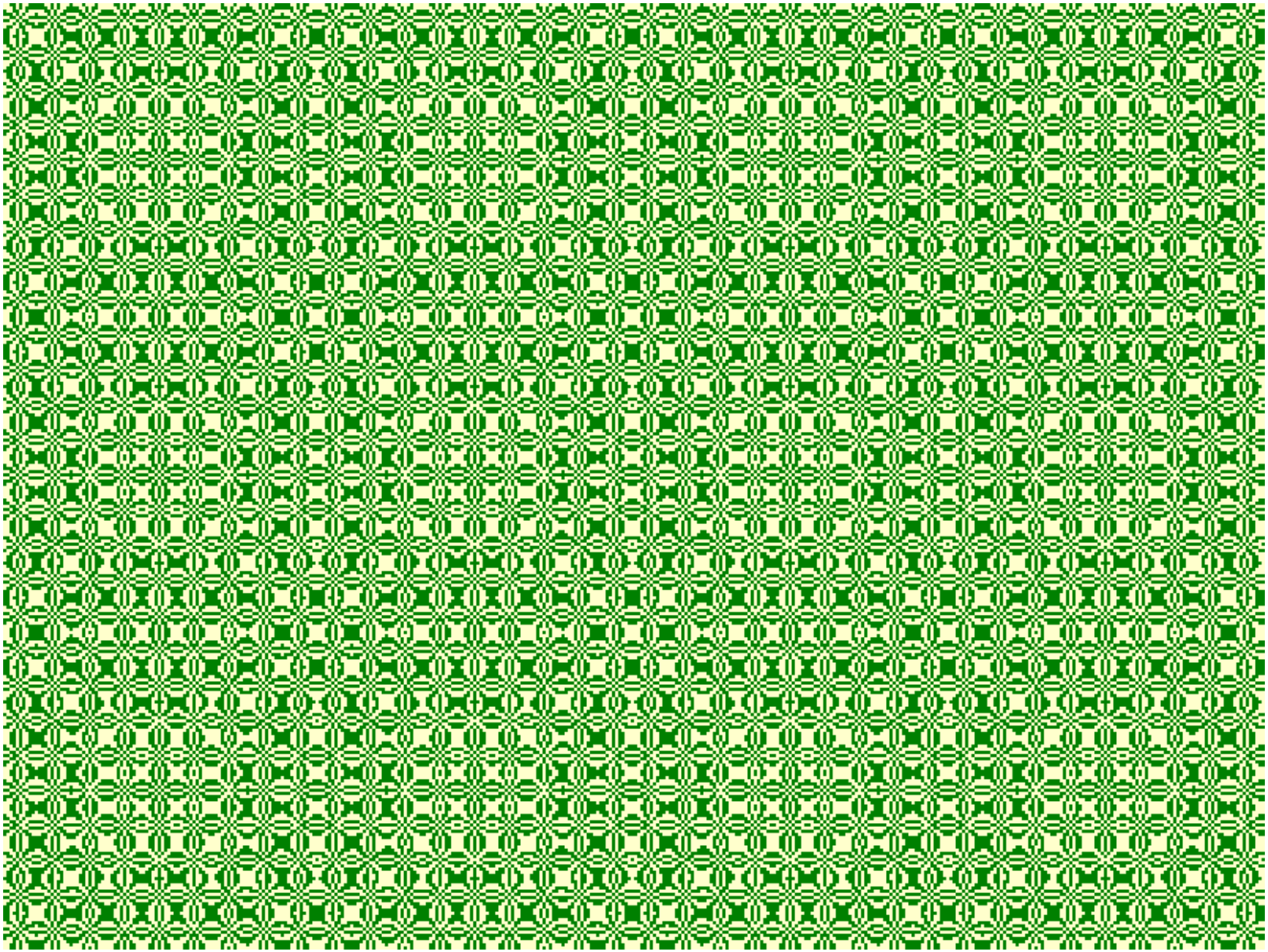


$x^2 - y^2$ 0.1 0.1 2

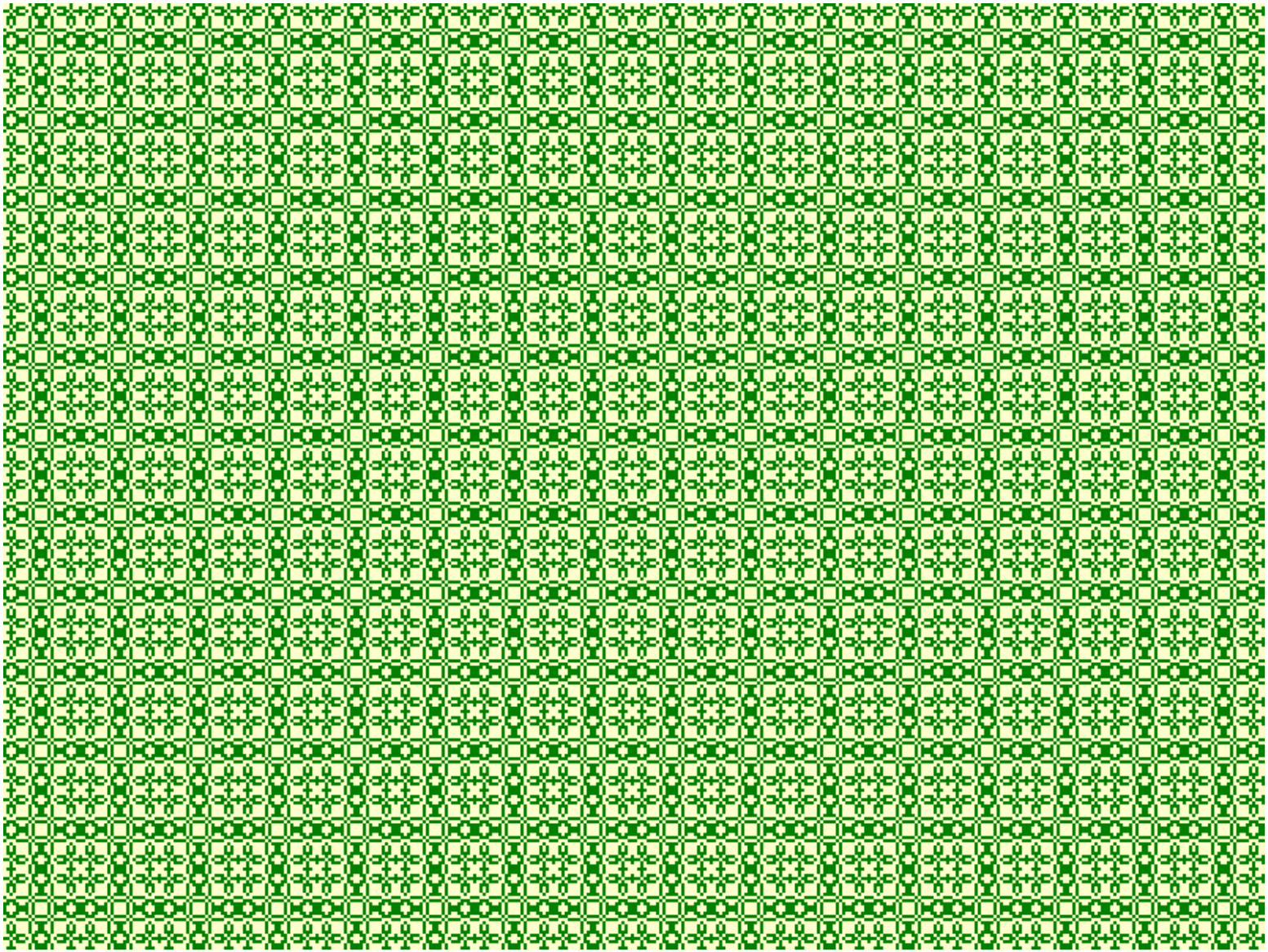




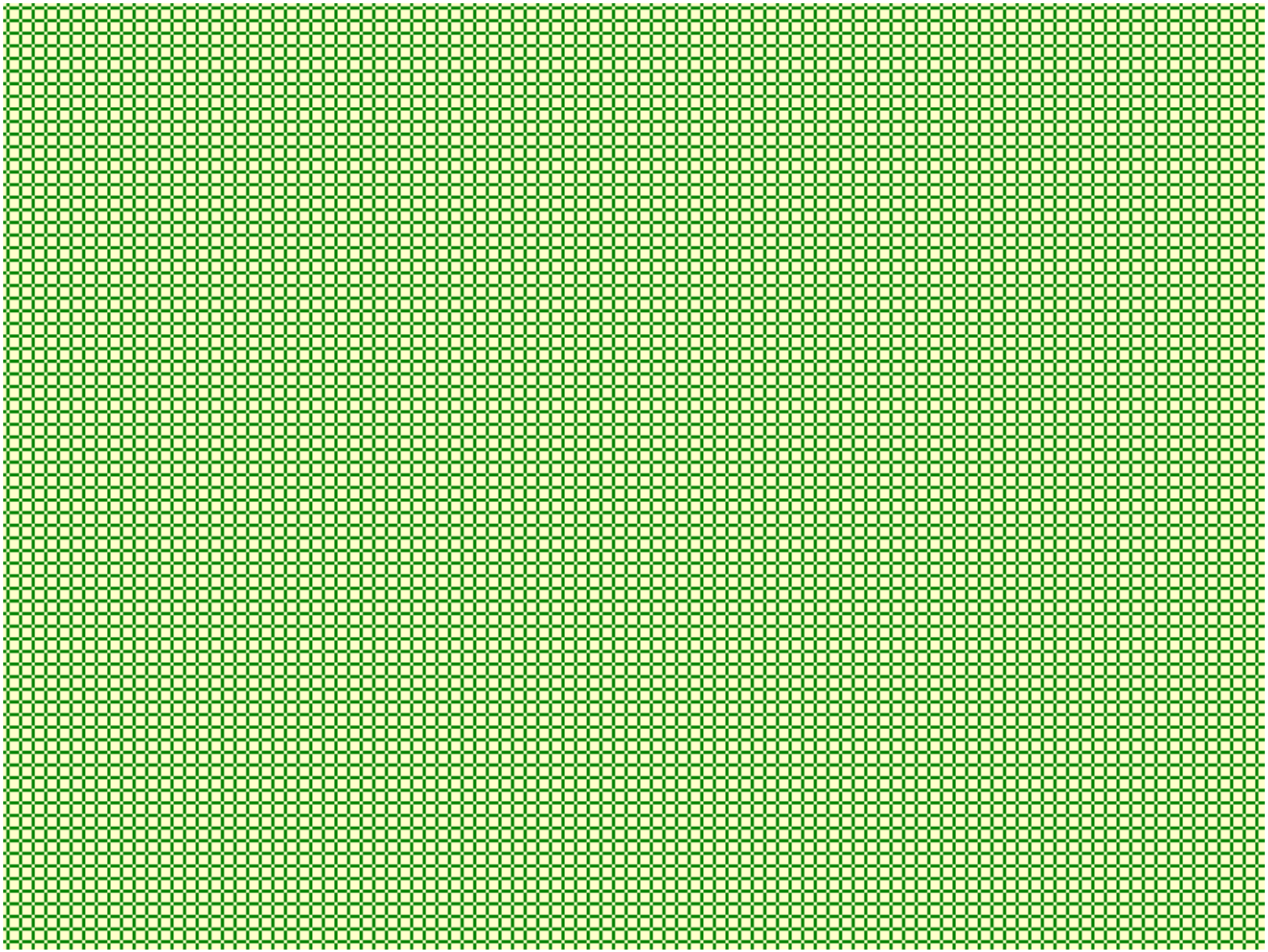
$x^2 - y^2$ 0.3 0.3 2



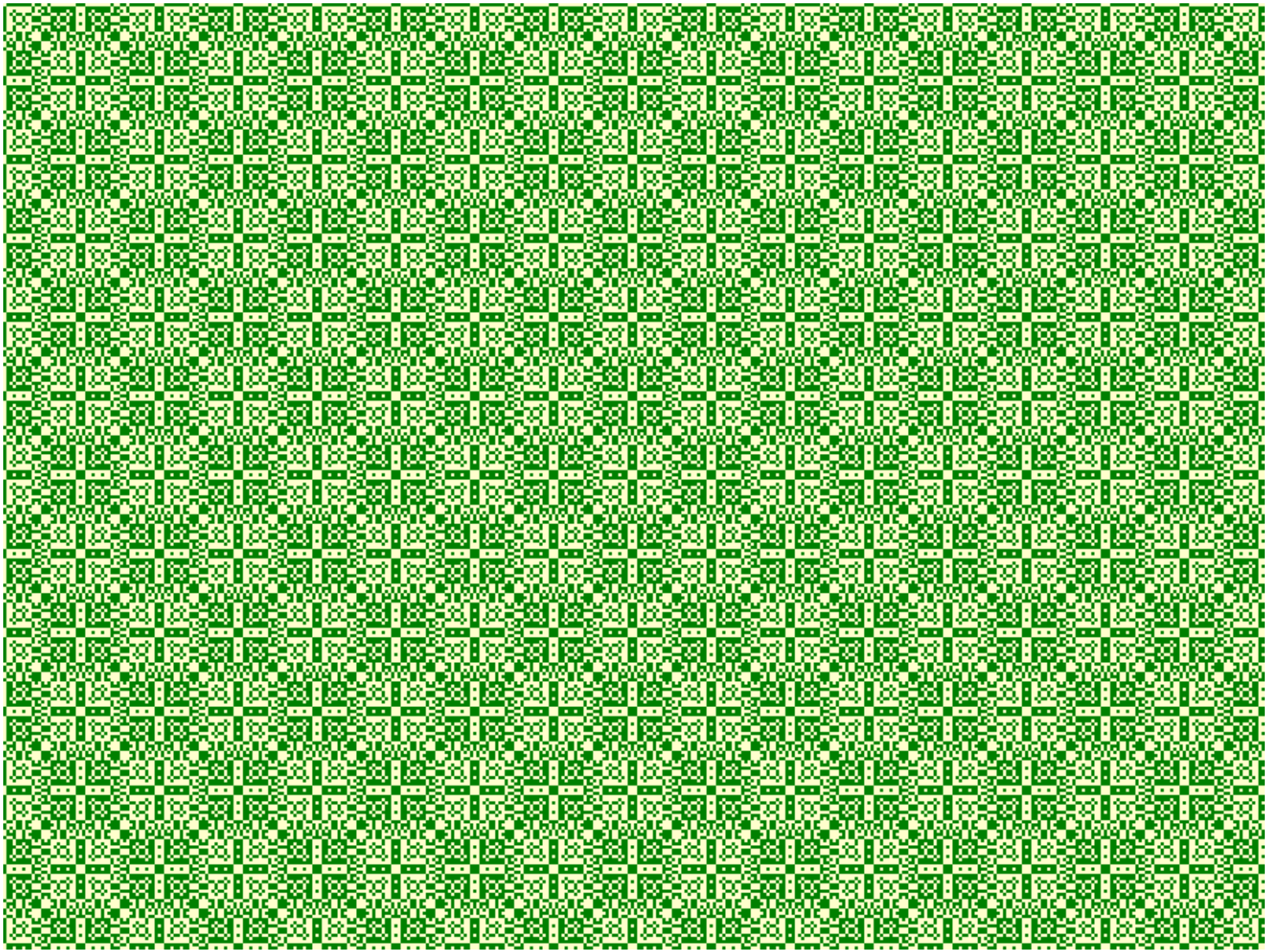
$x^2 - y^2$ 0.4 0.4 2

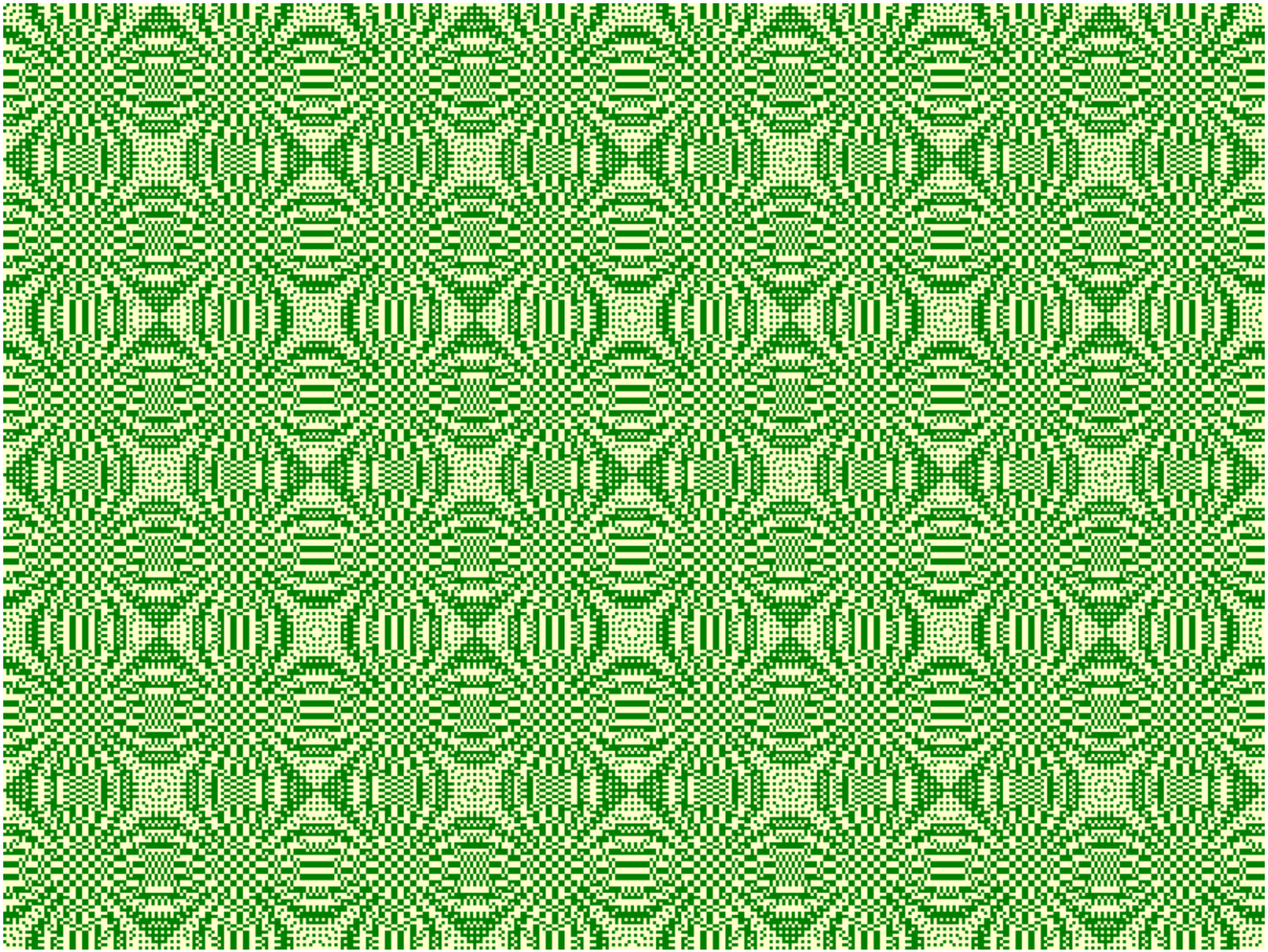


$x^2 - y^2$ 0.5 0.5 2

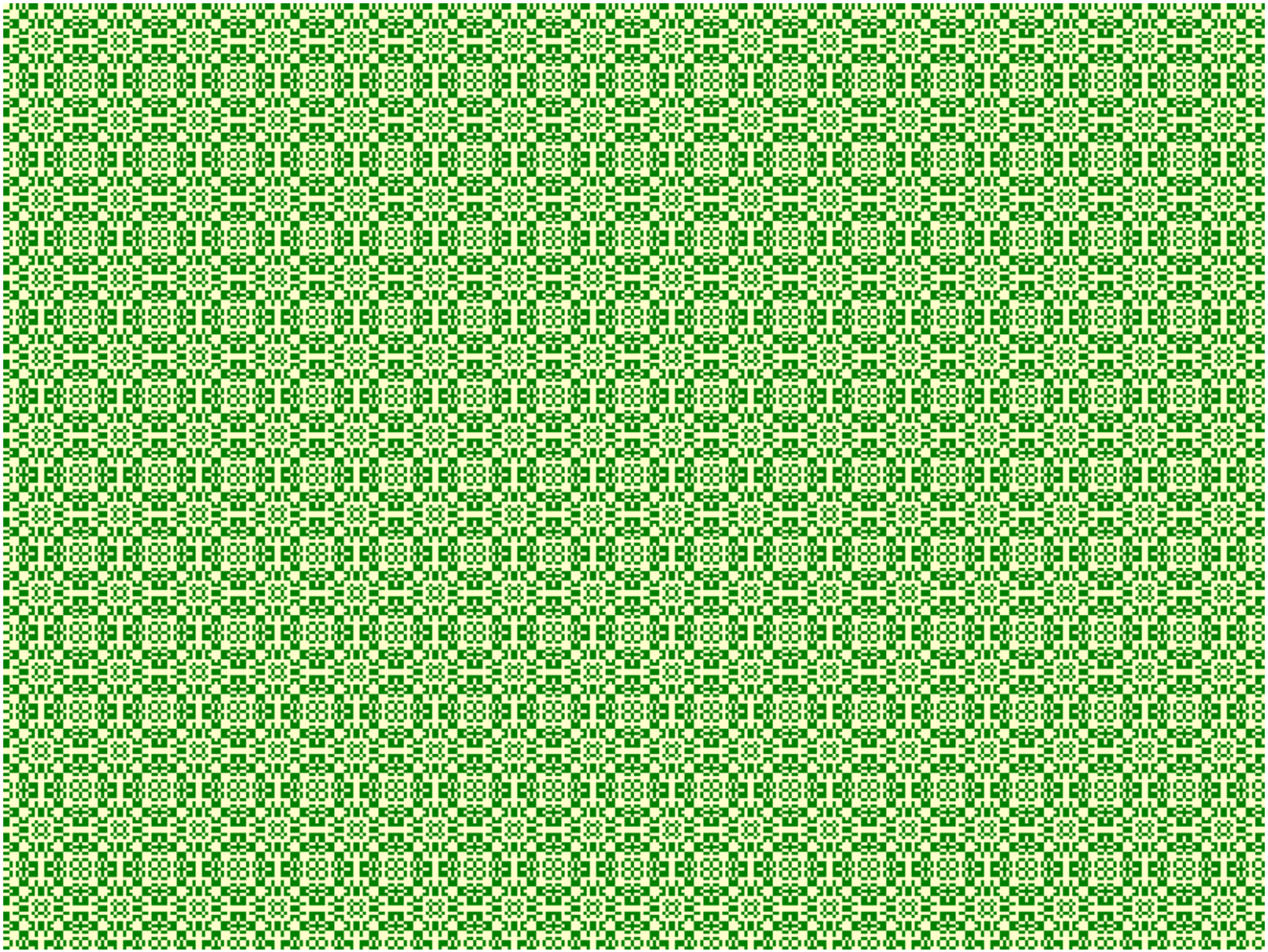


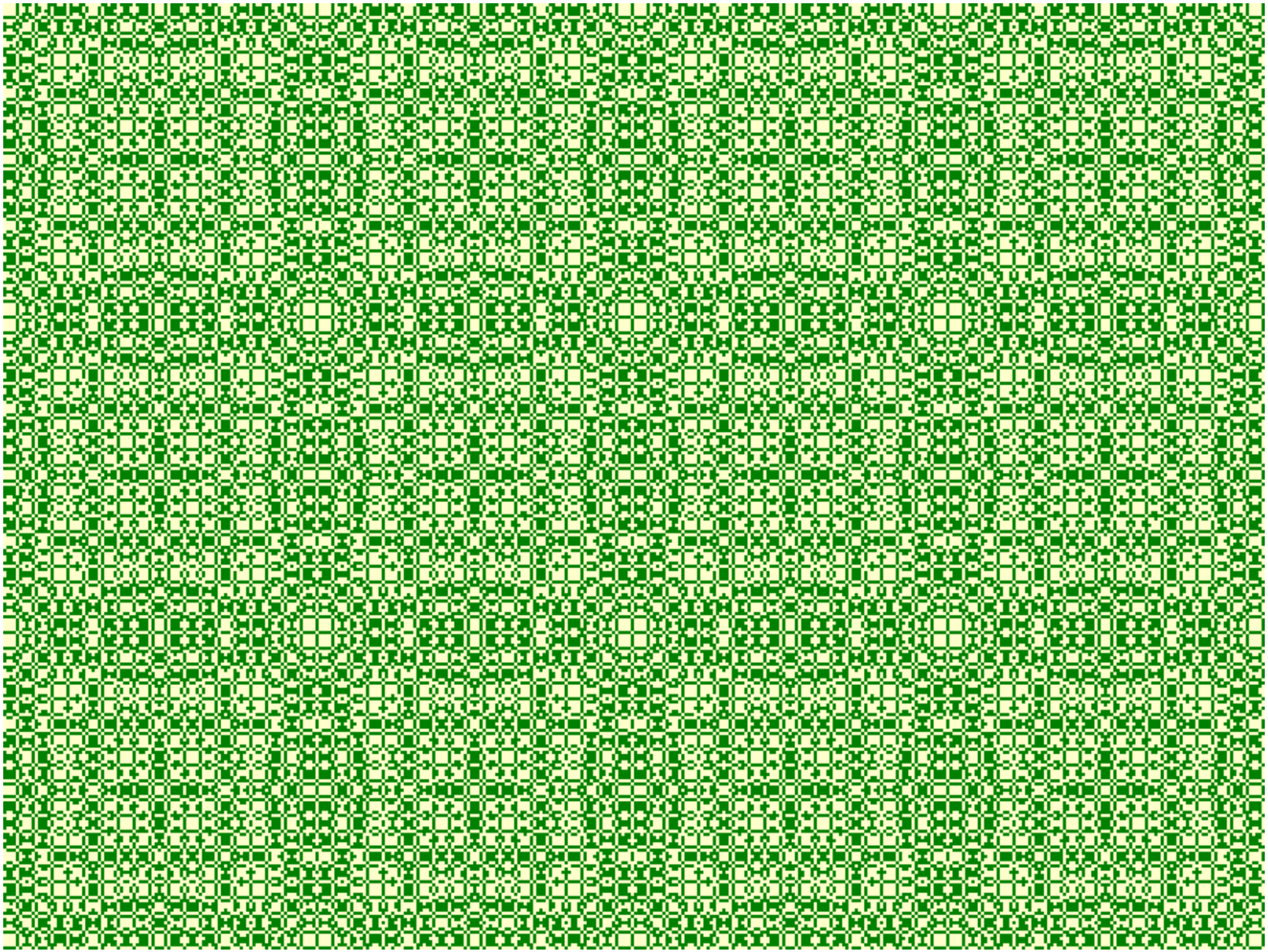
$x^2 - y^2$ 0.6 0.6 2



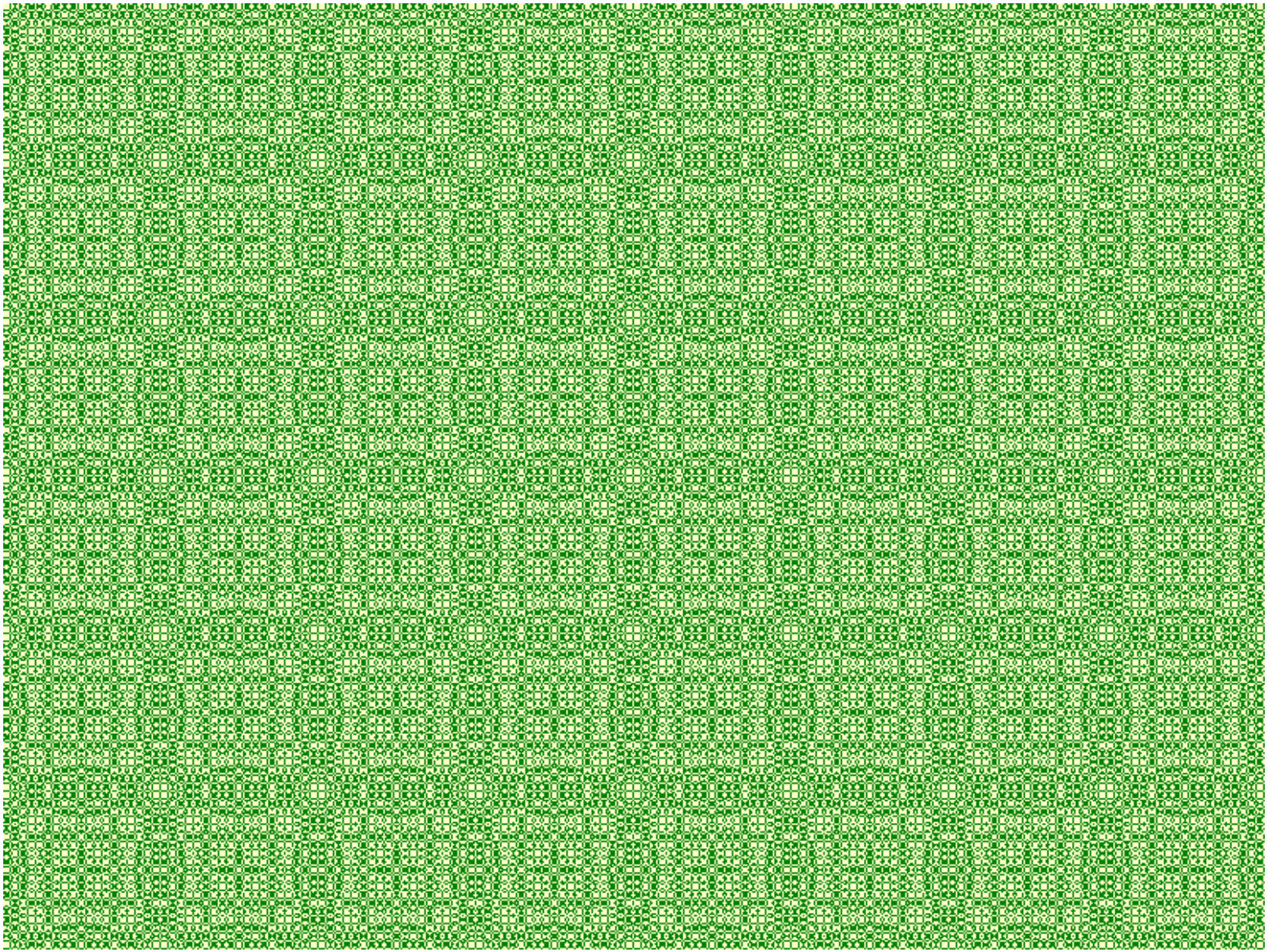


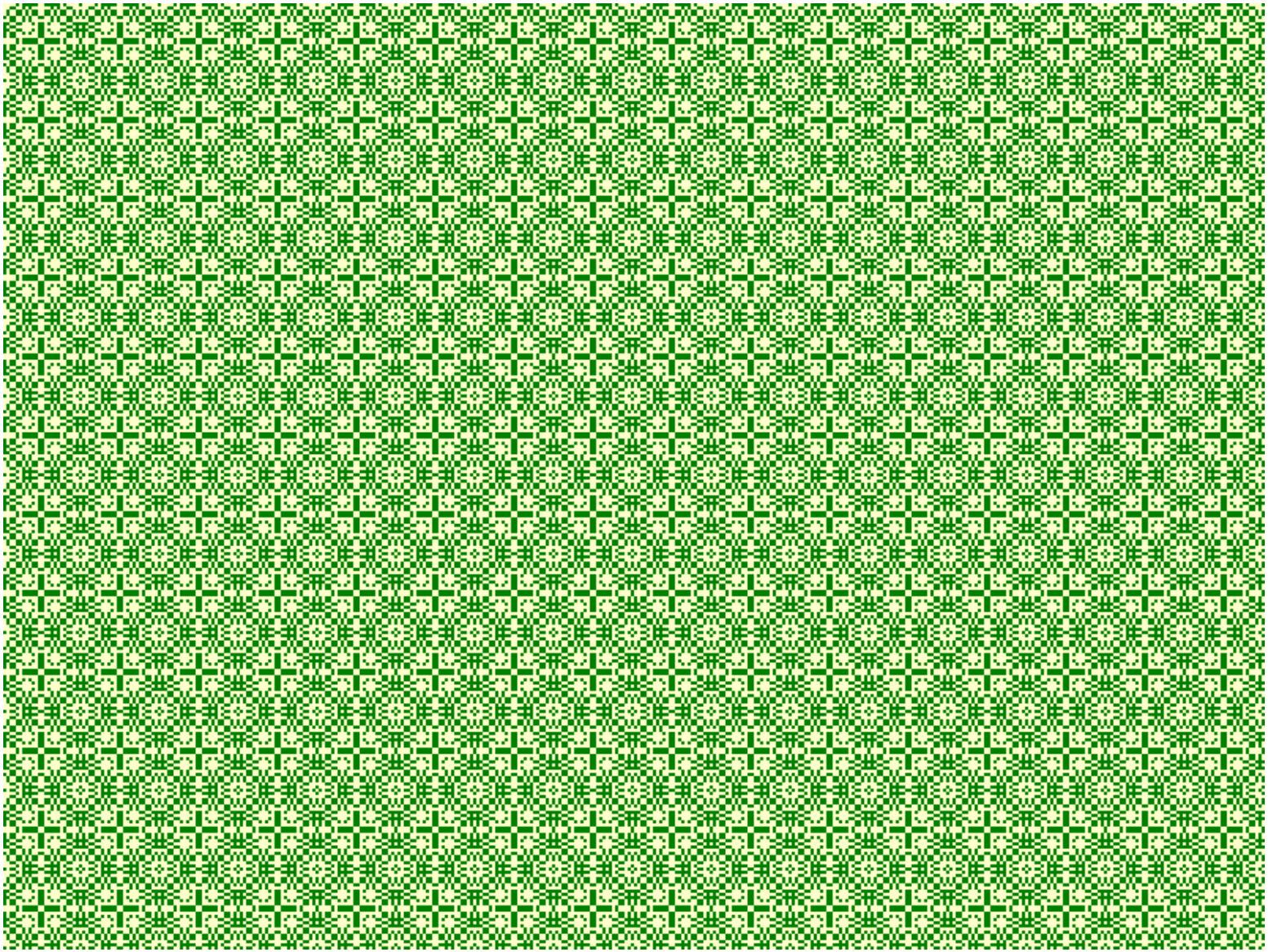
$x^2 - y^2$ 0.8 0.8 2

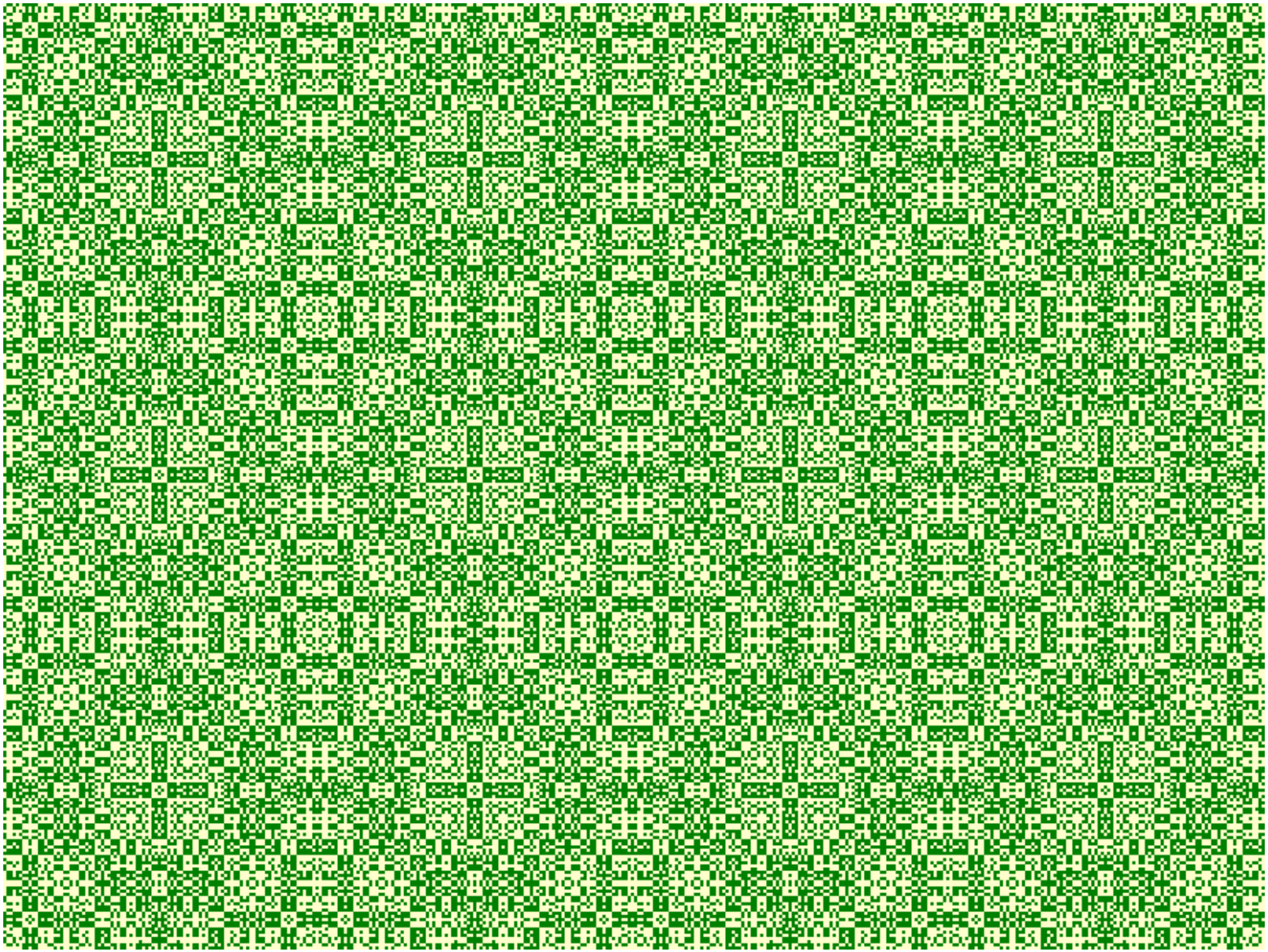


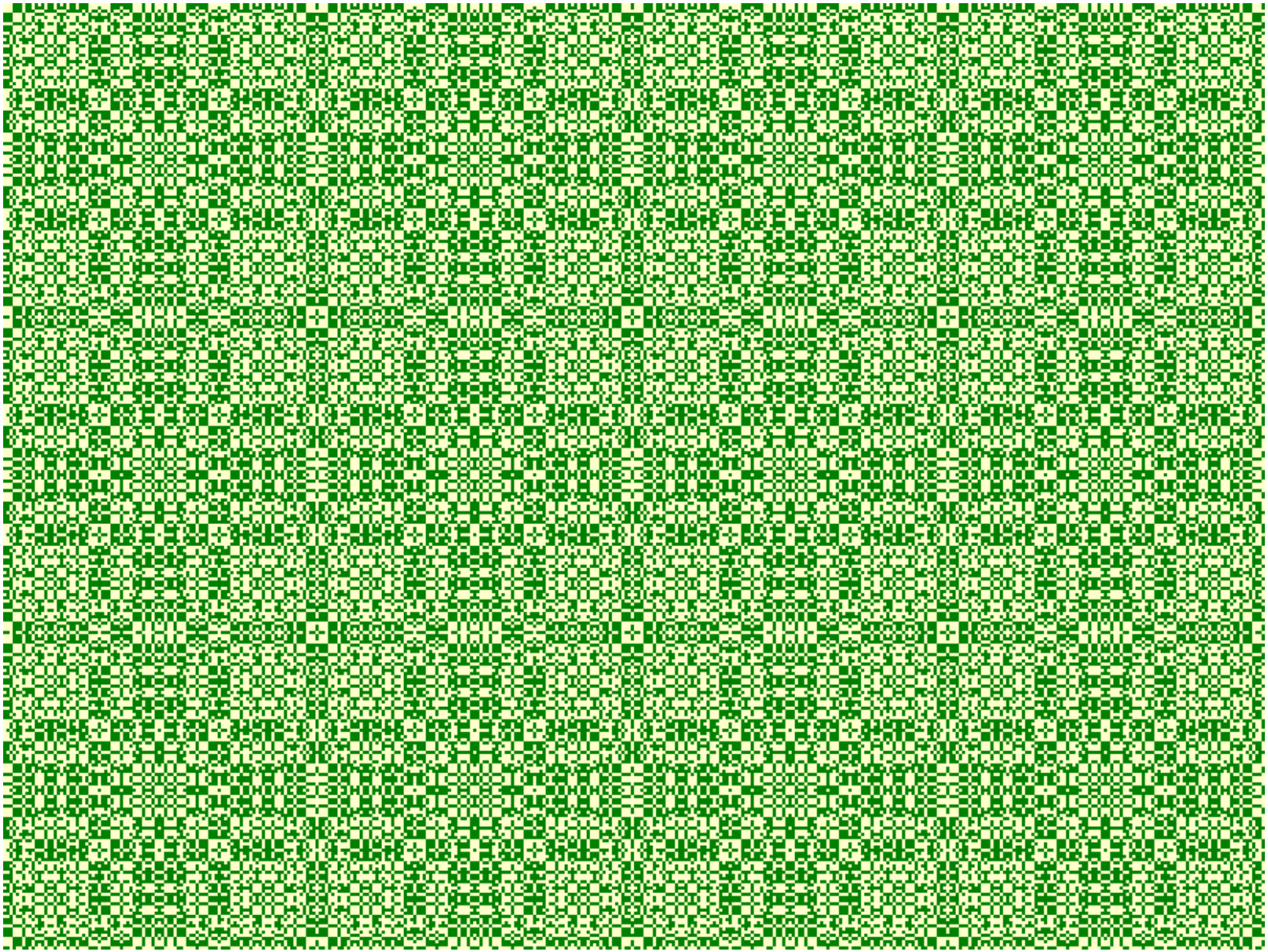


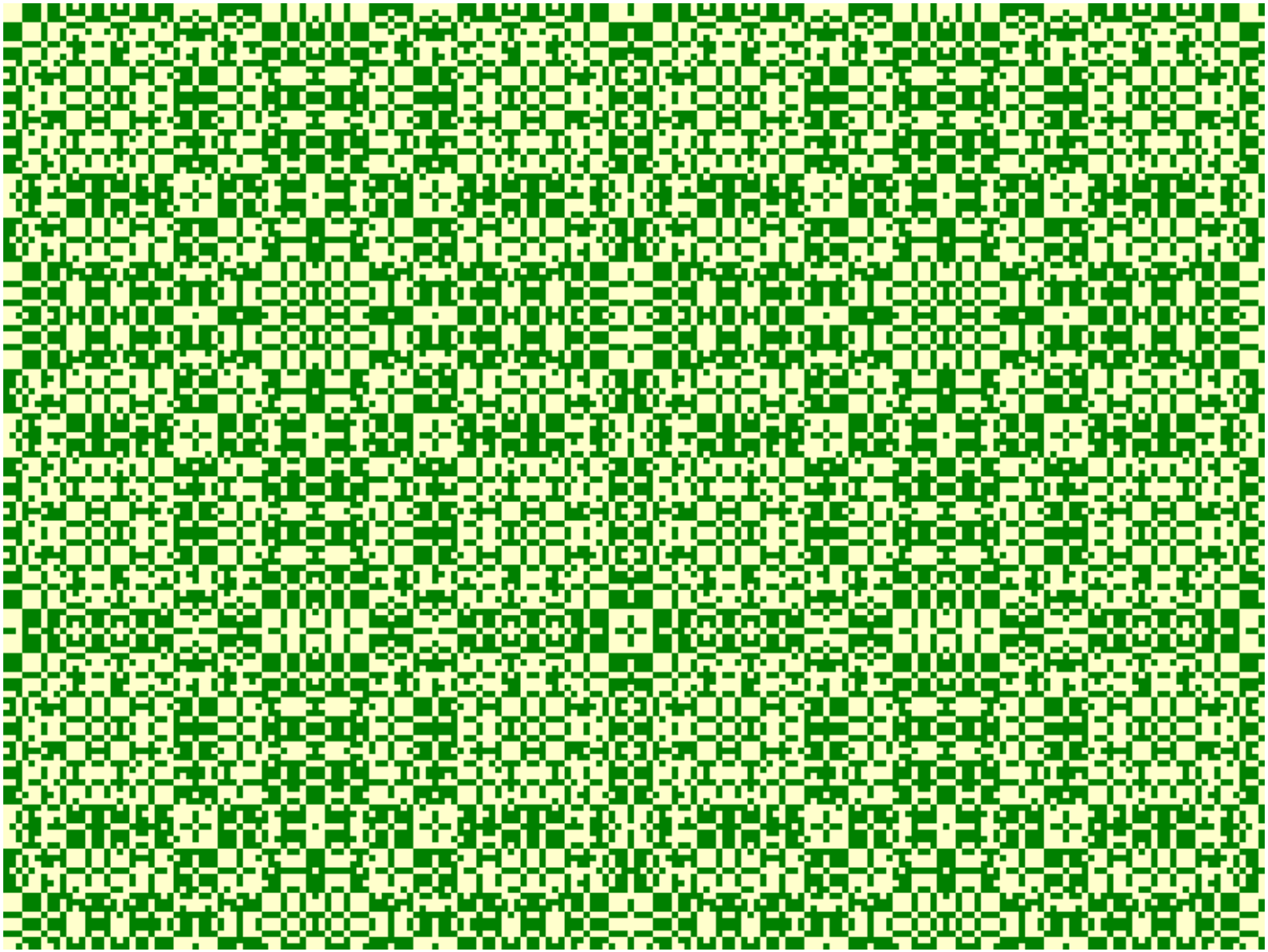
$x^2 - y^2$ 0.9 0.9 1

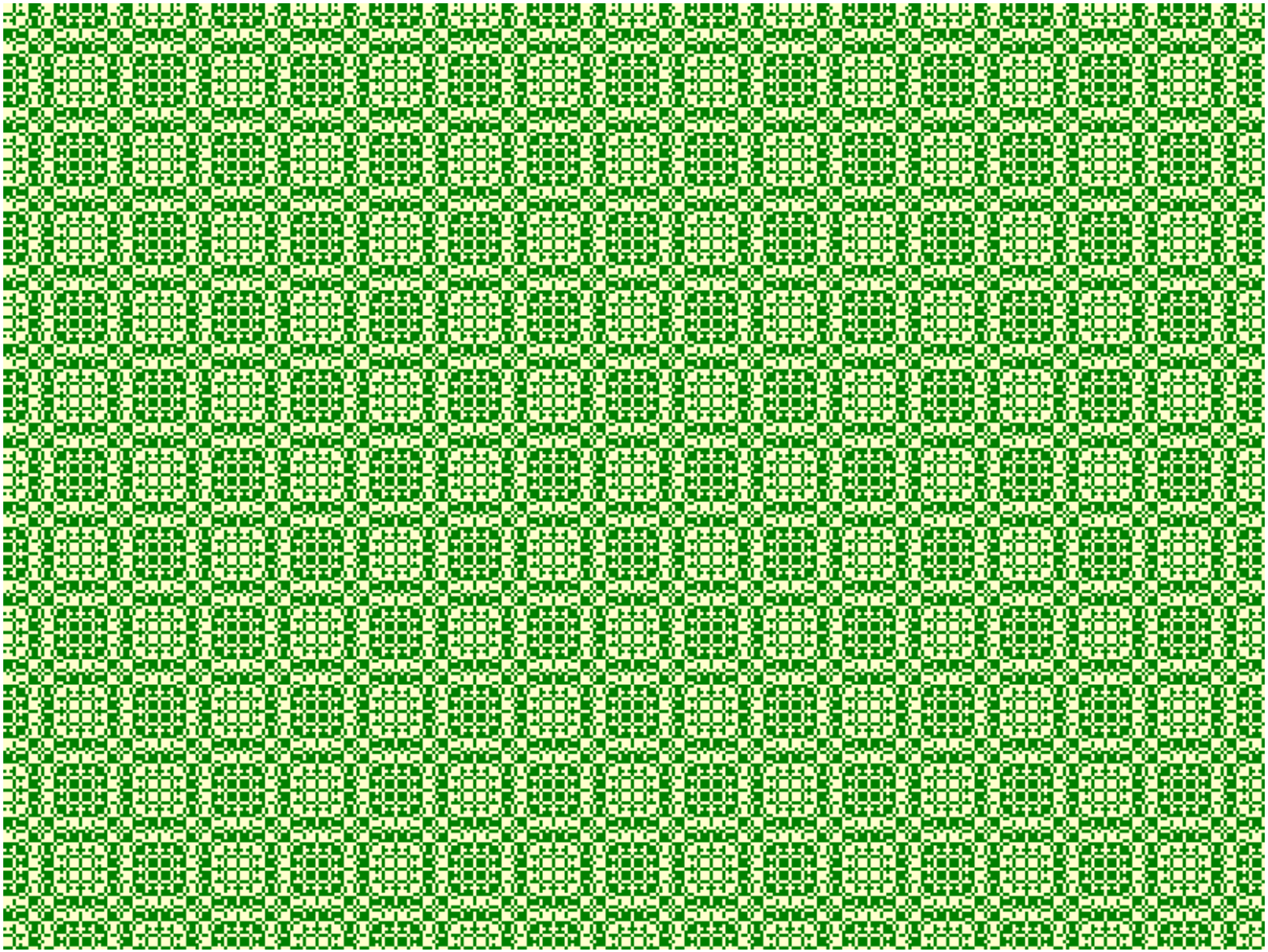


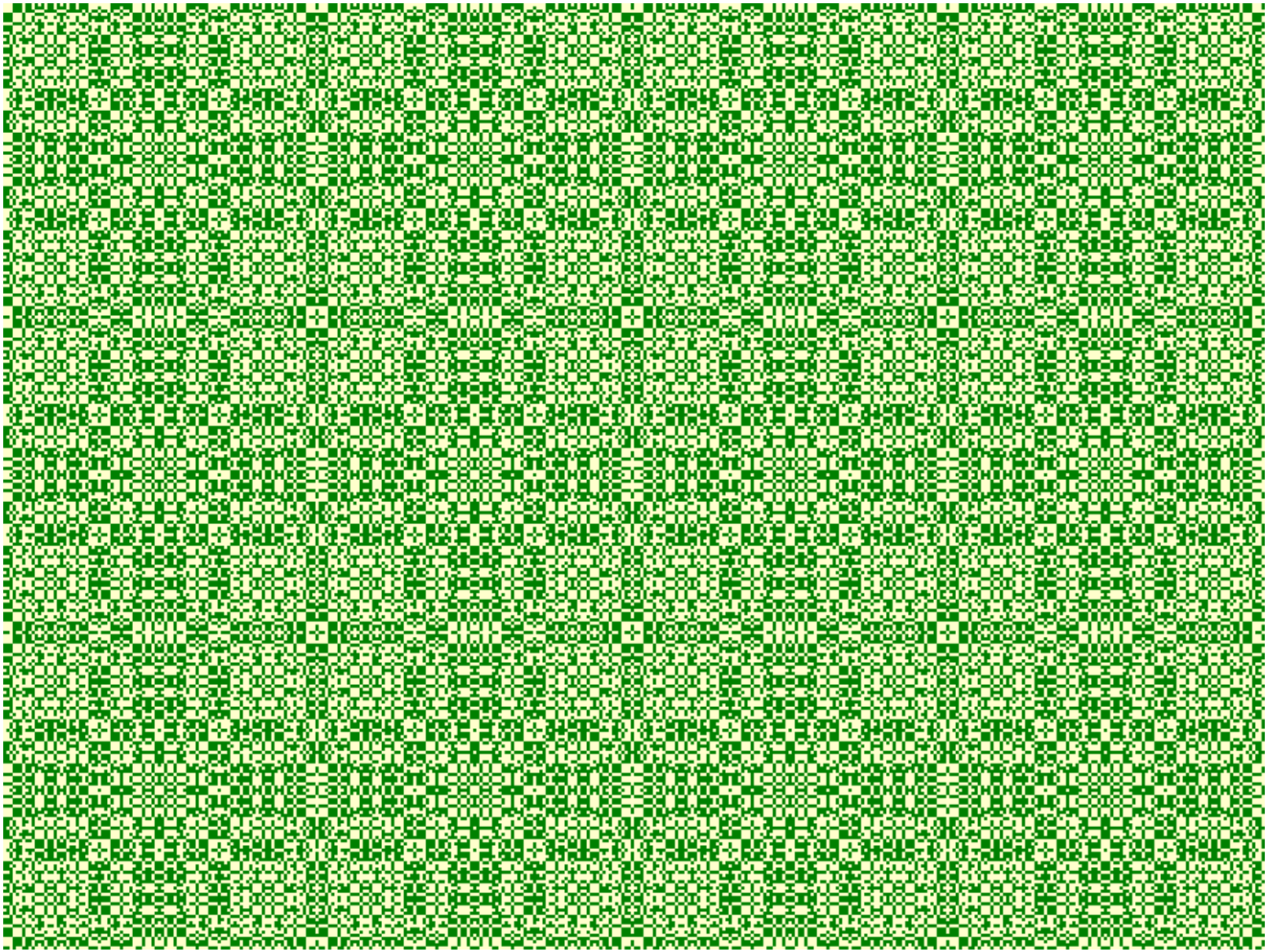


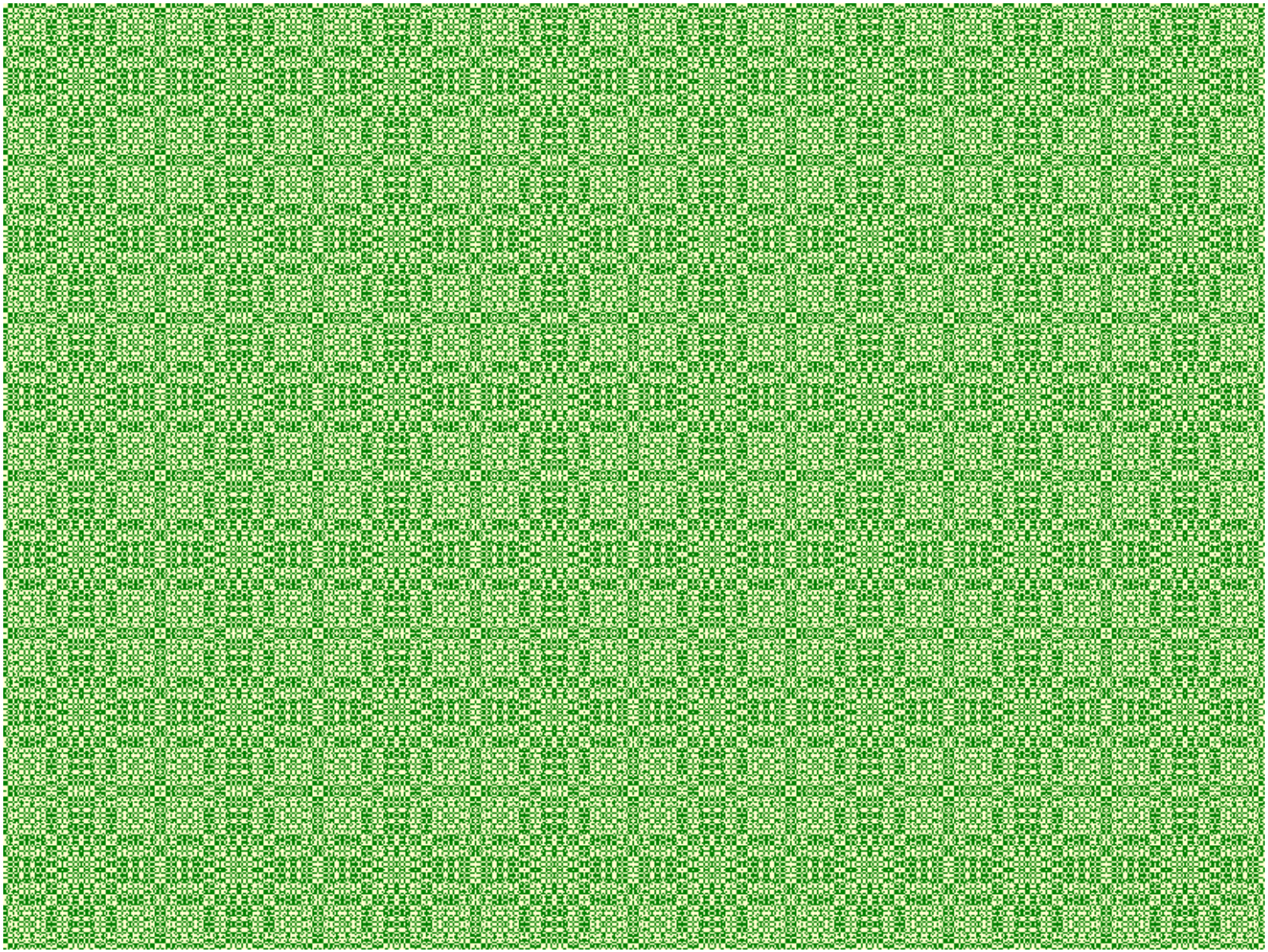


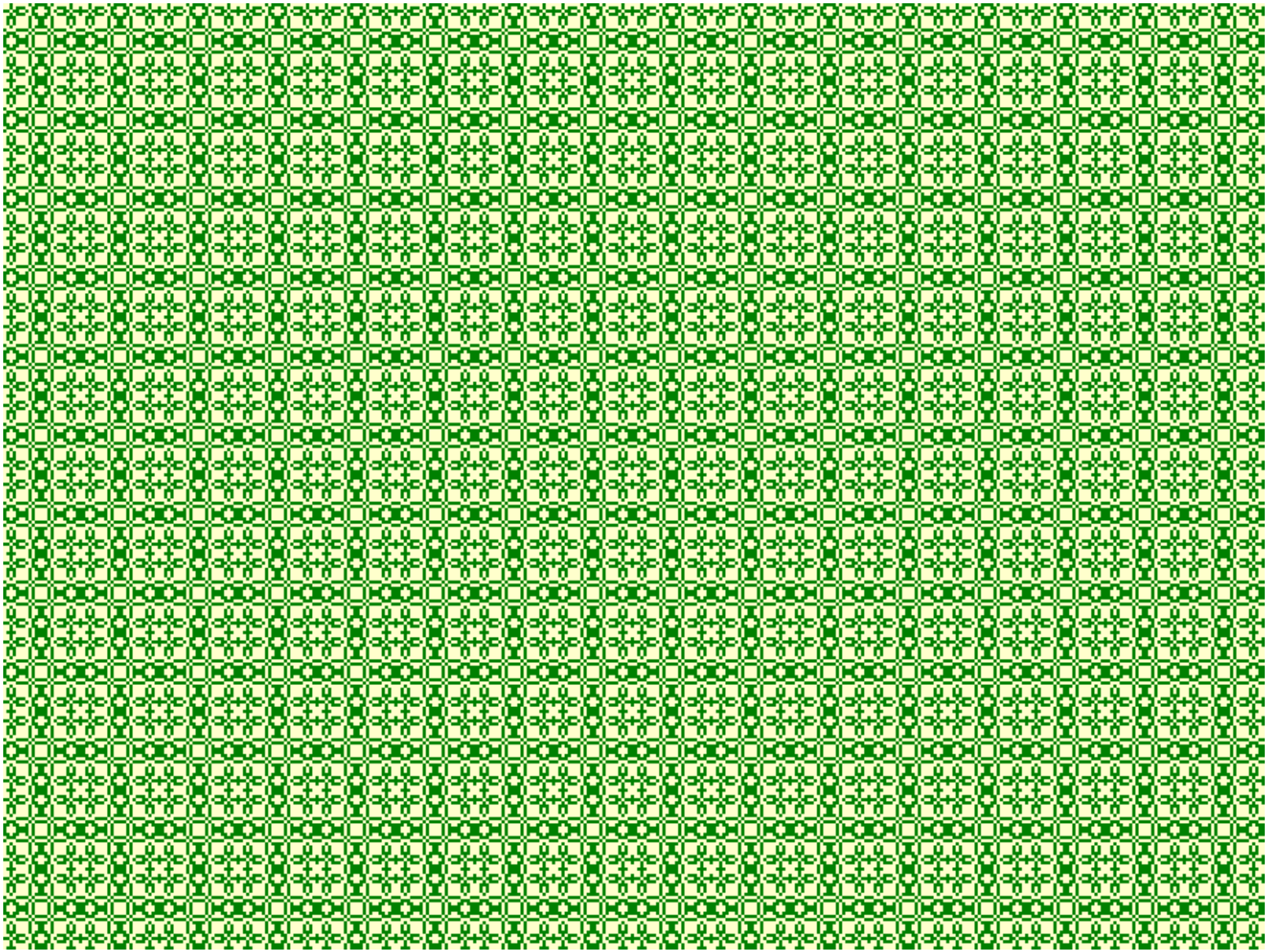


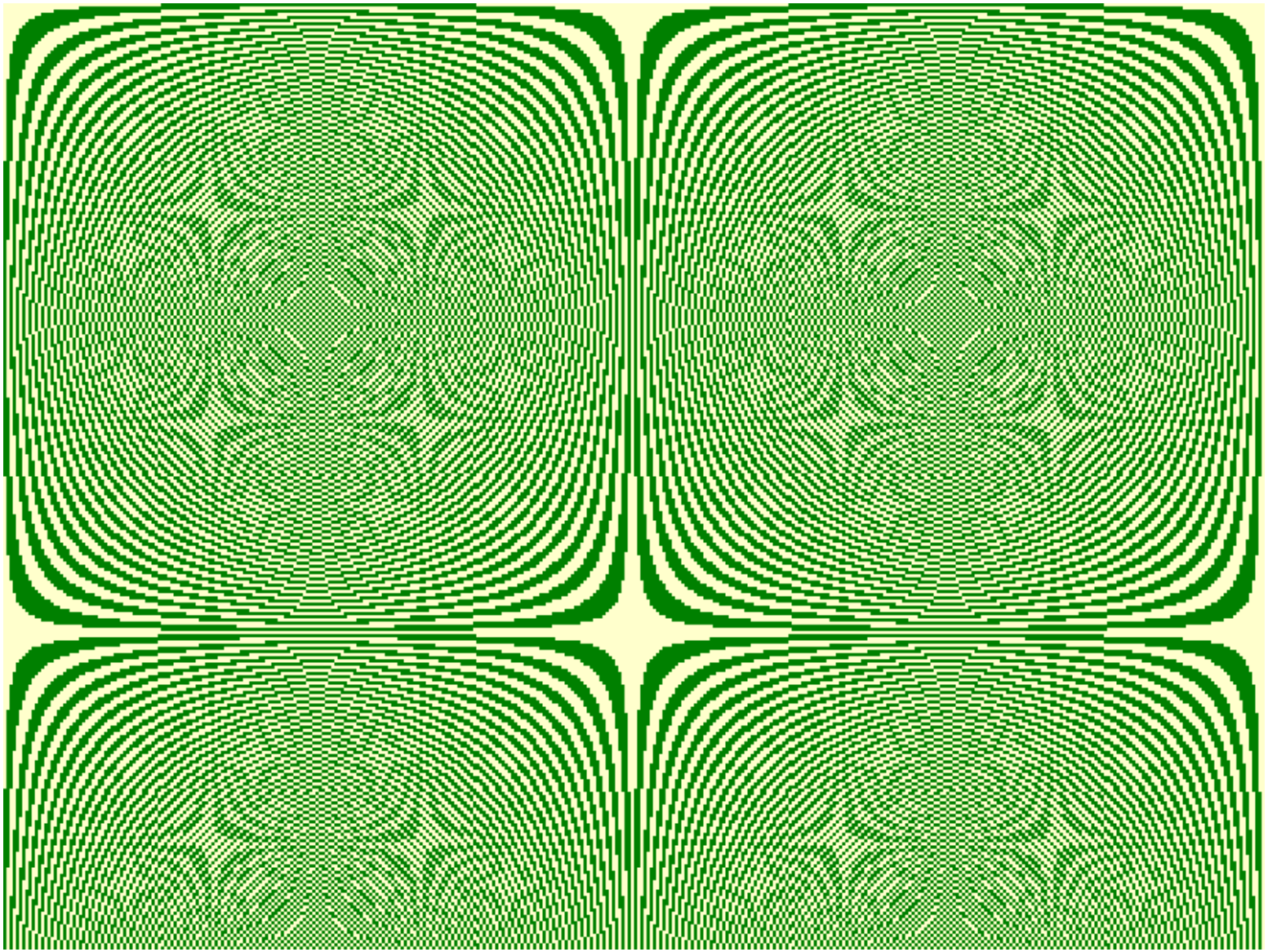


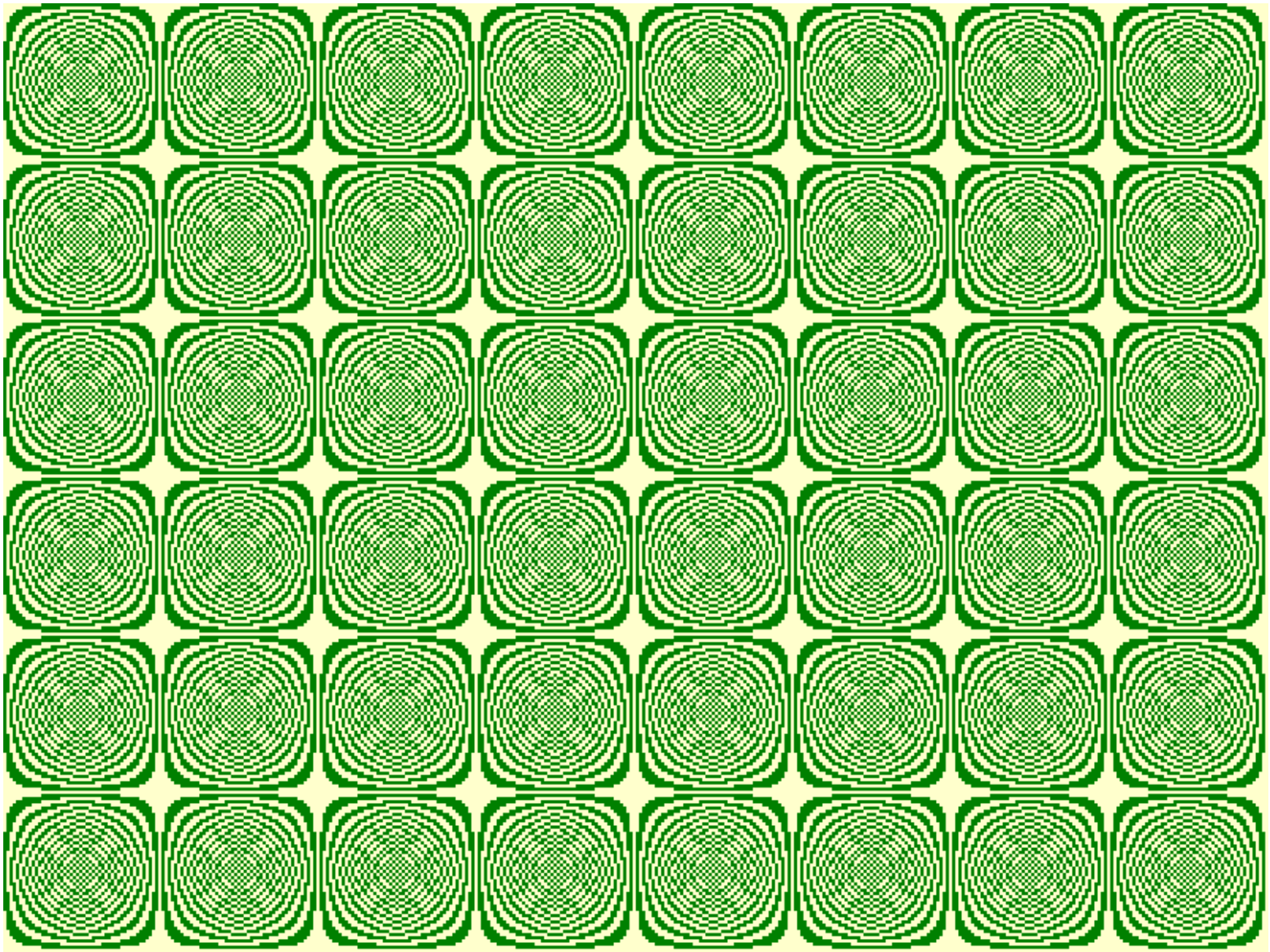


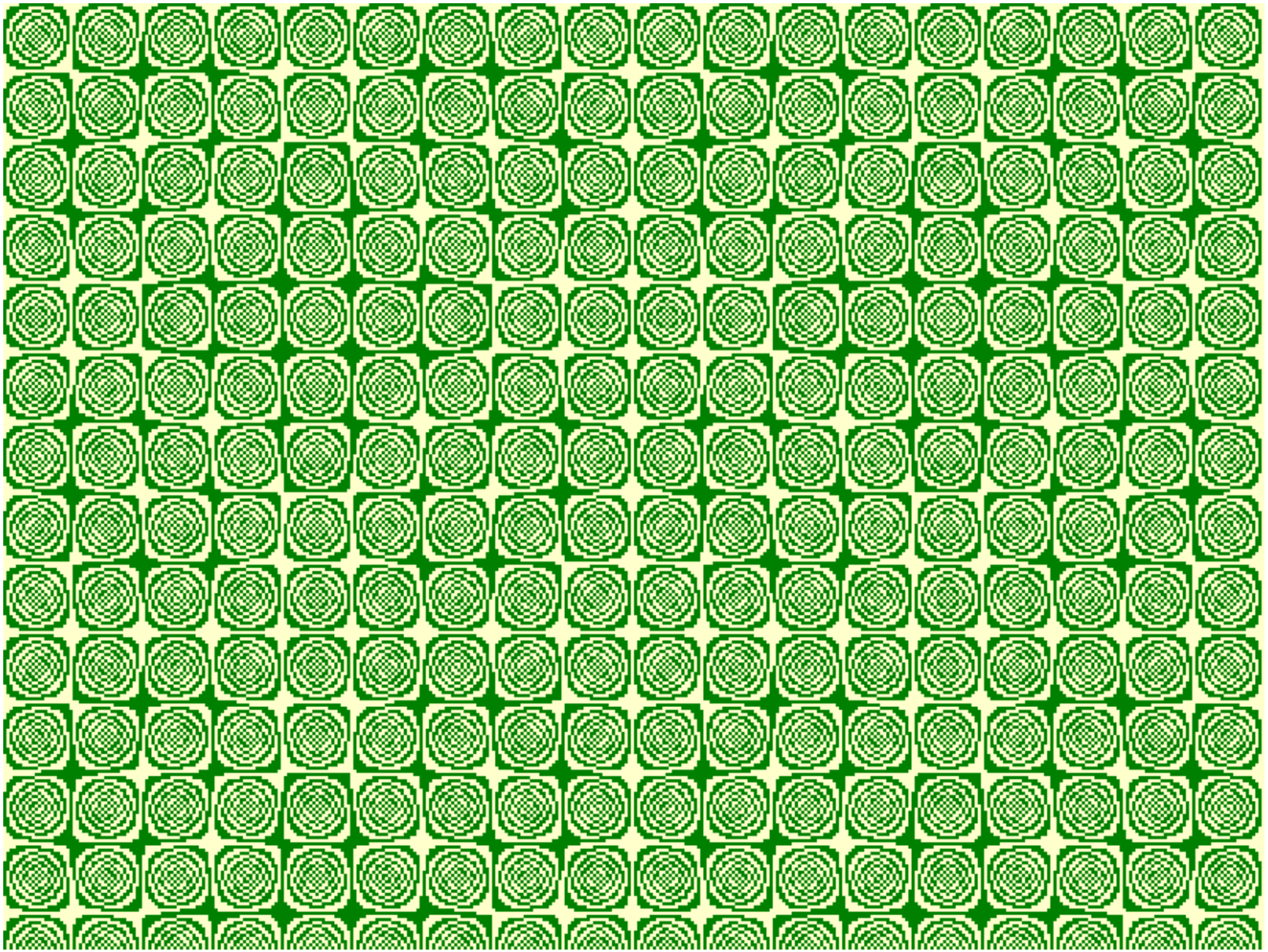


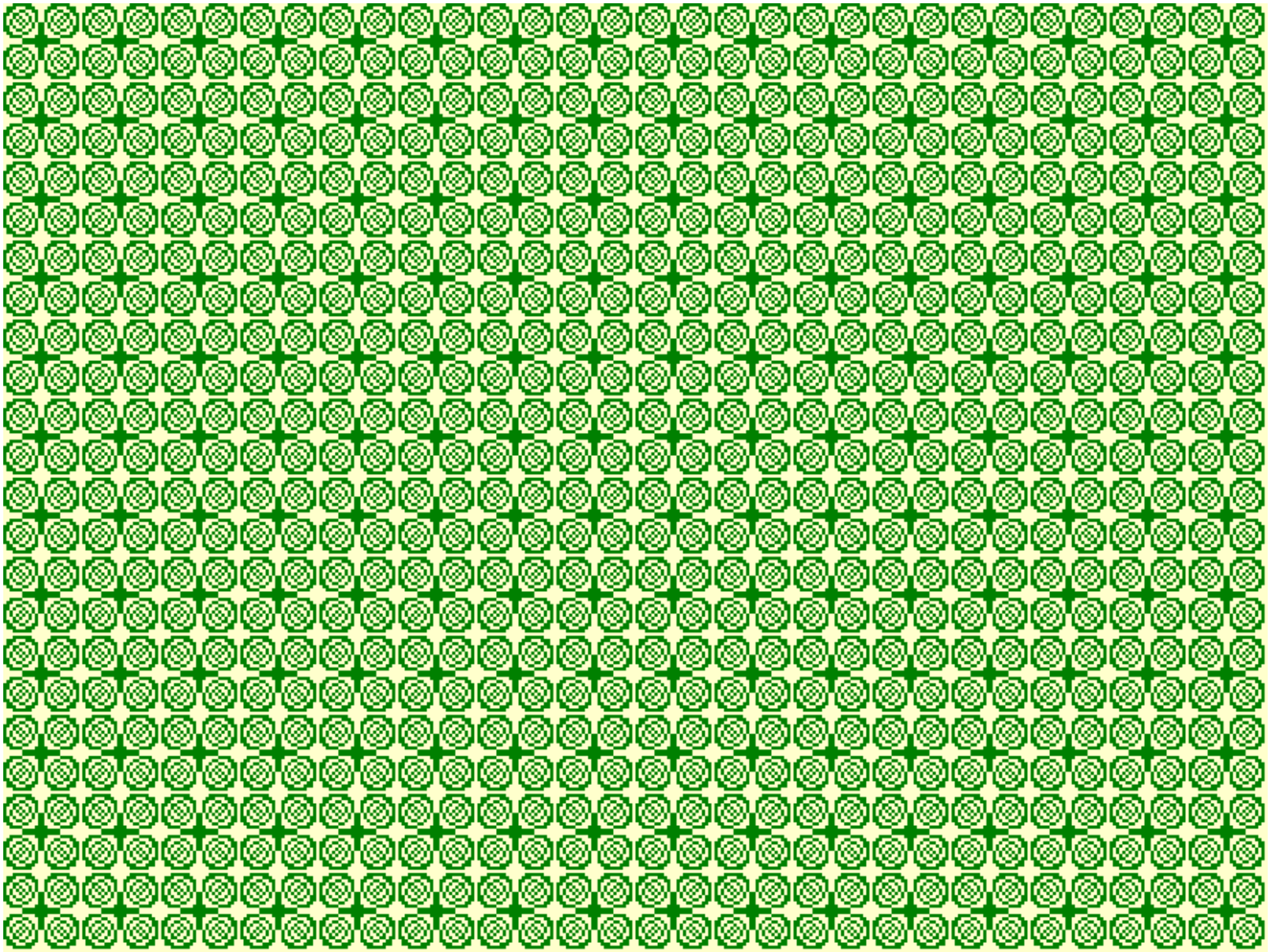


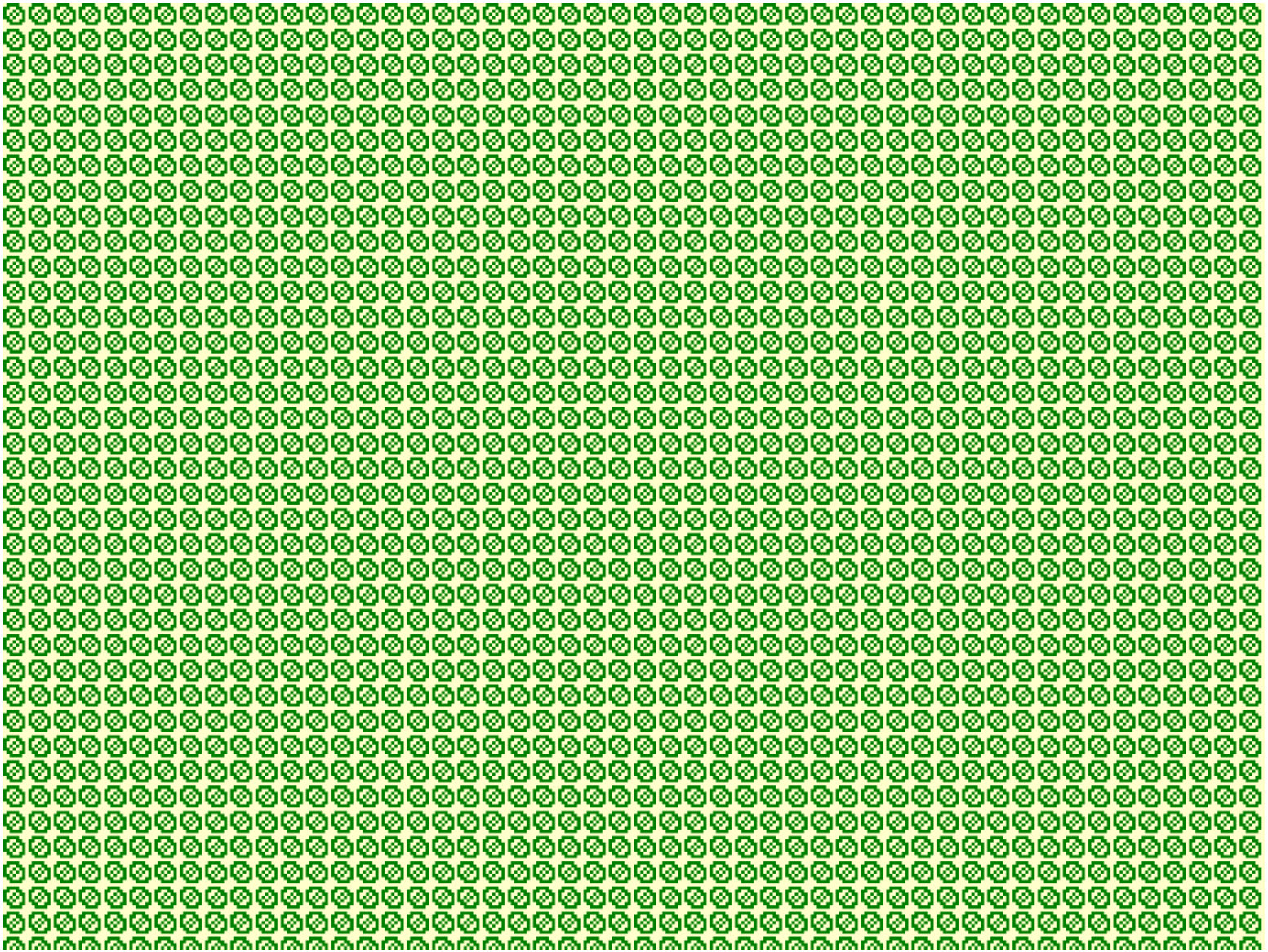


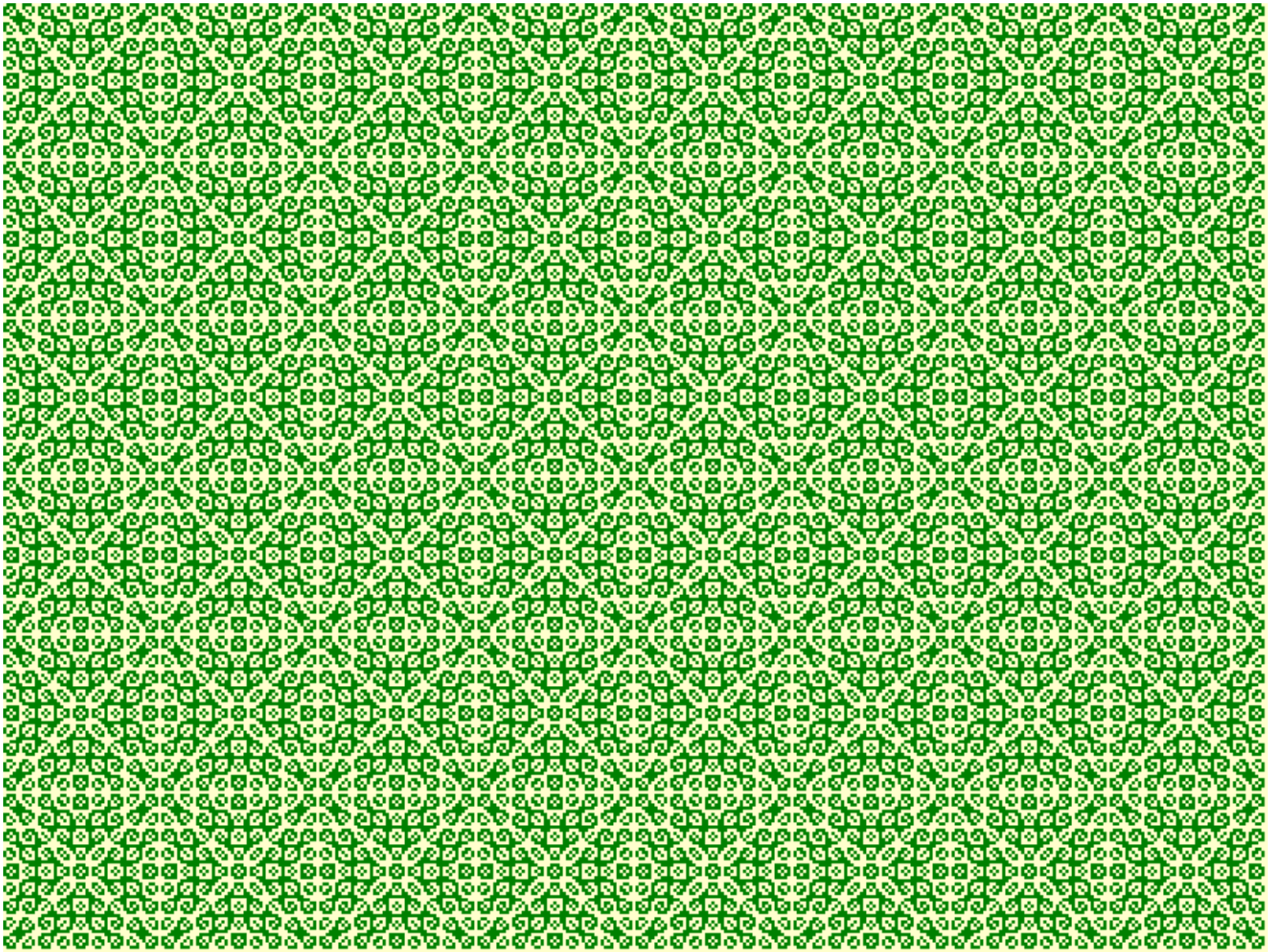


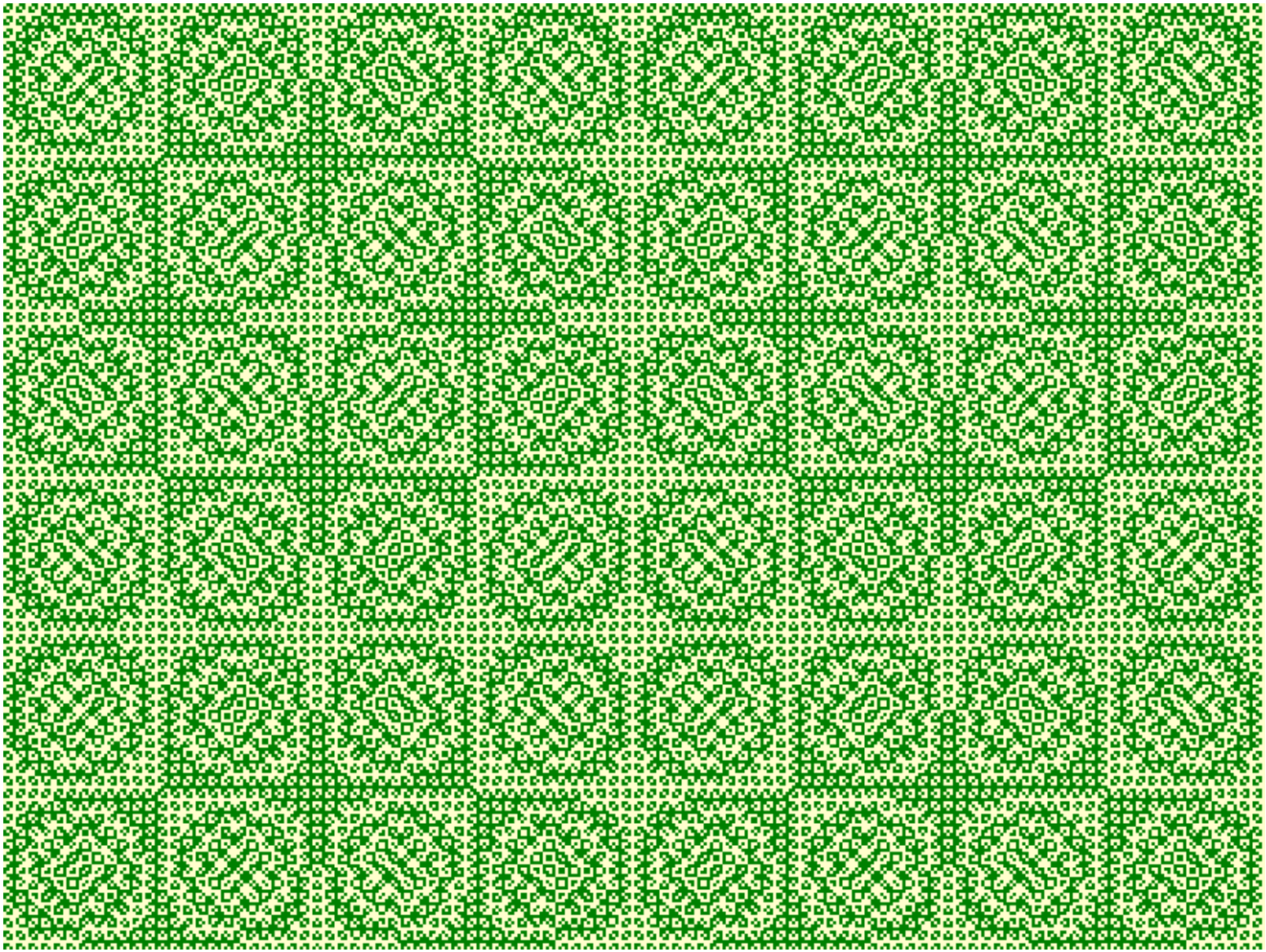


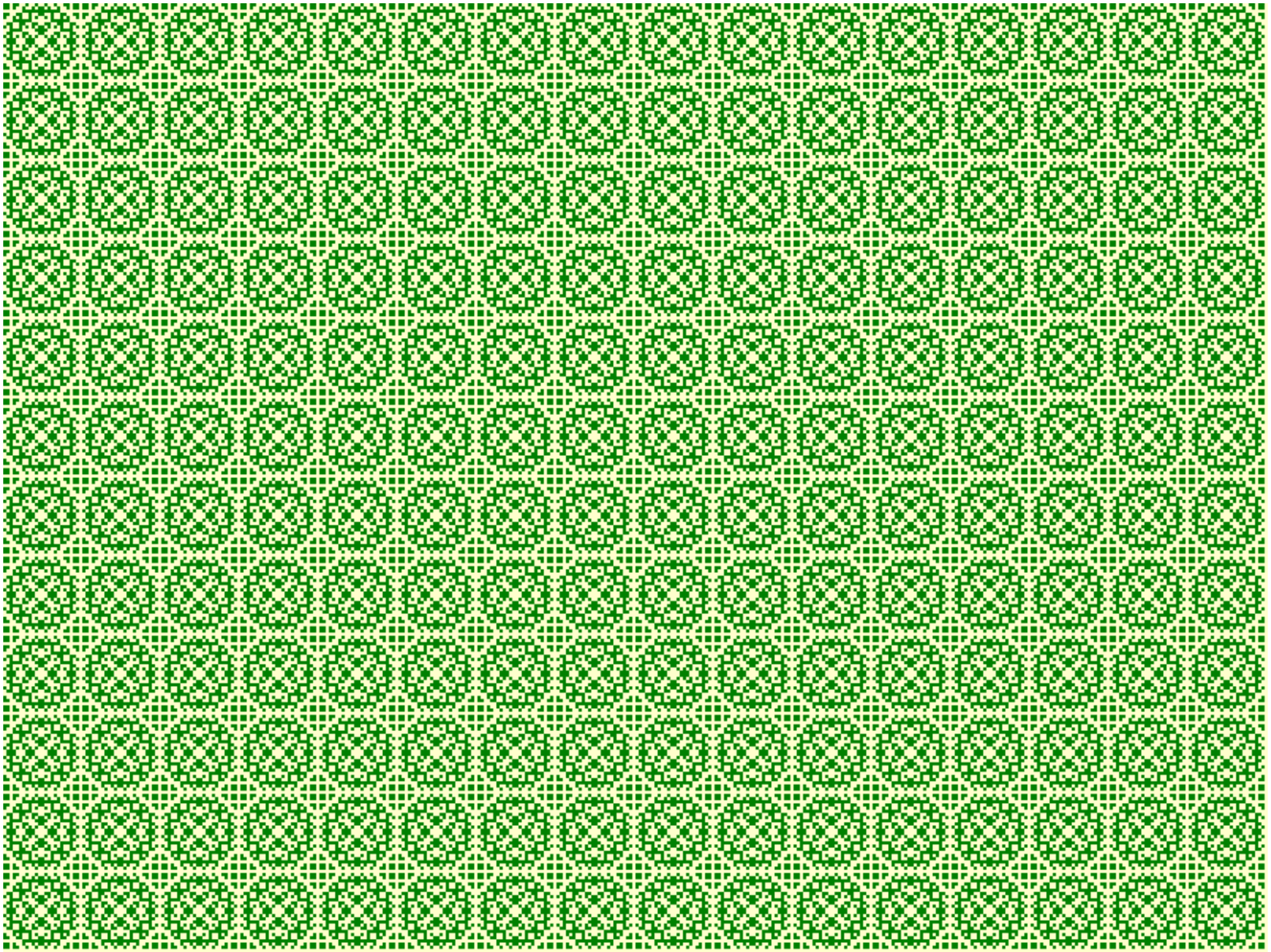


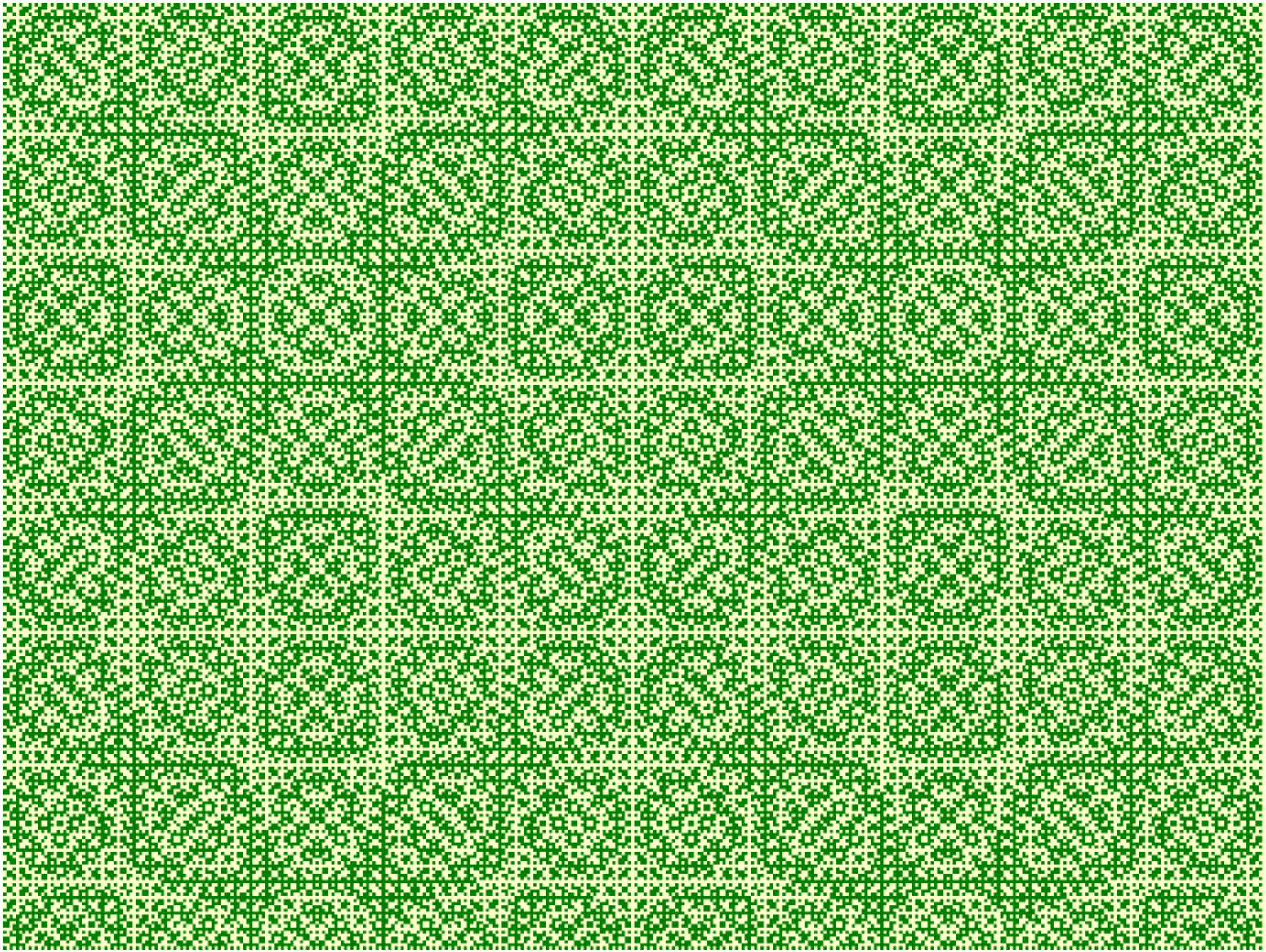


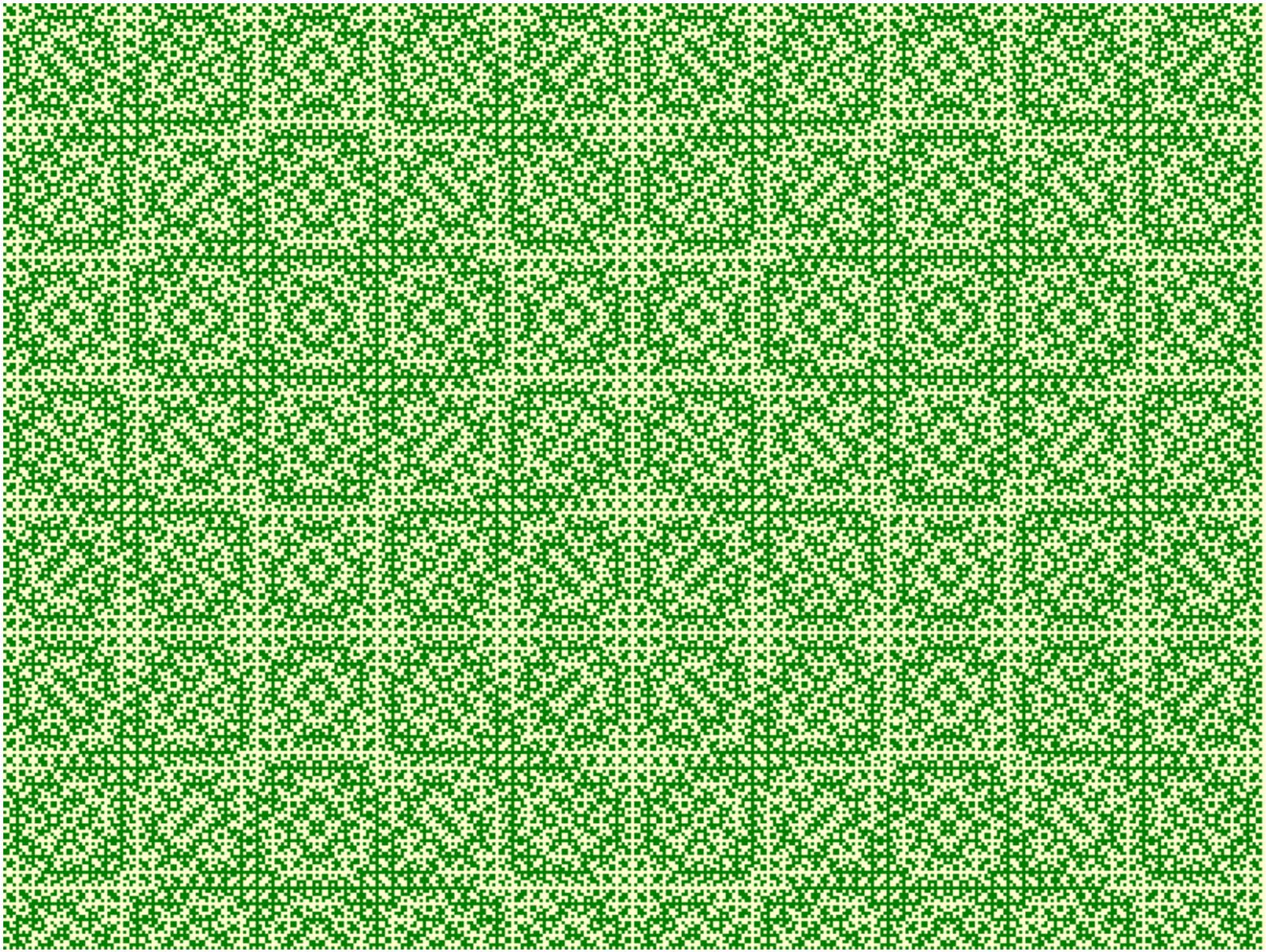


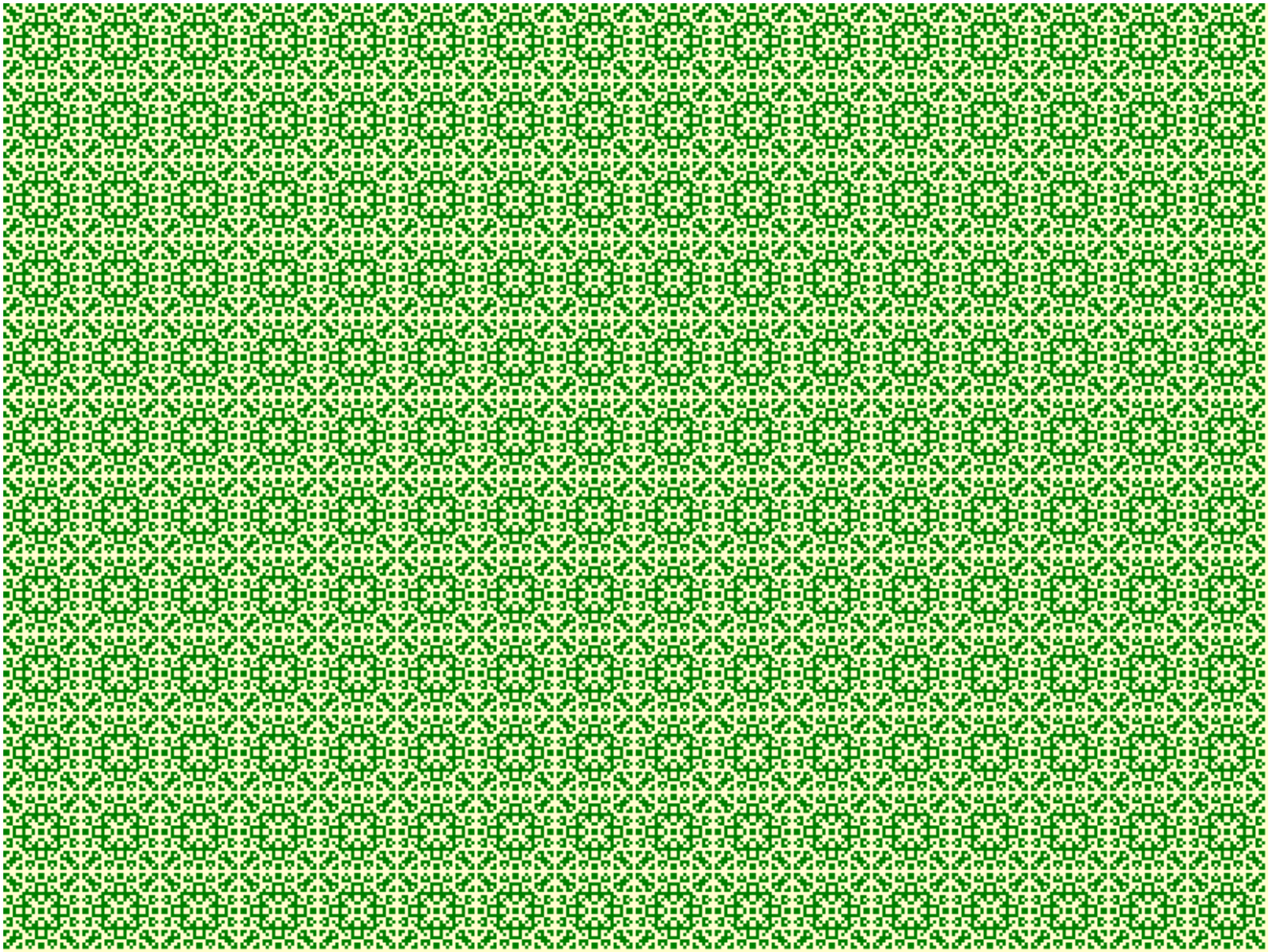


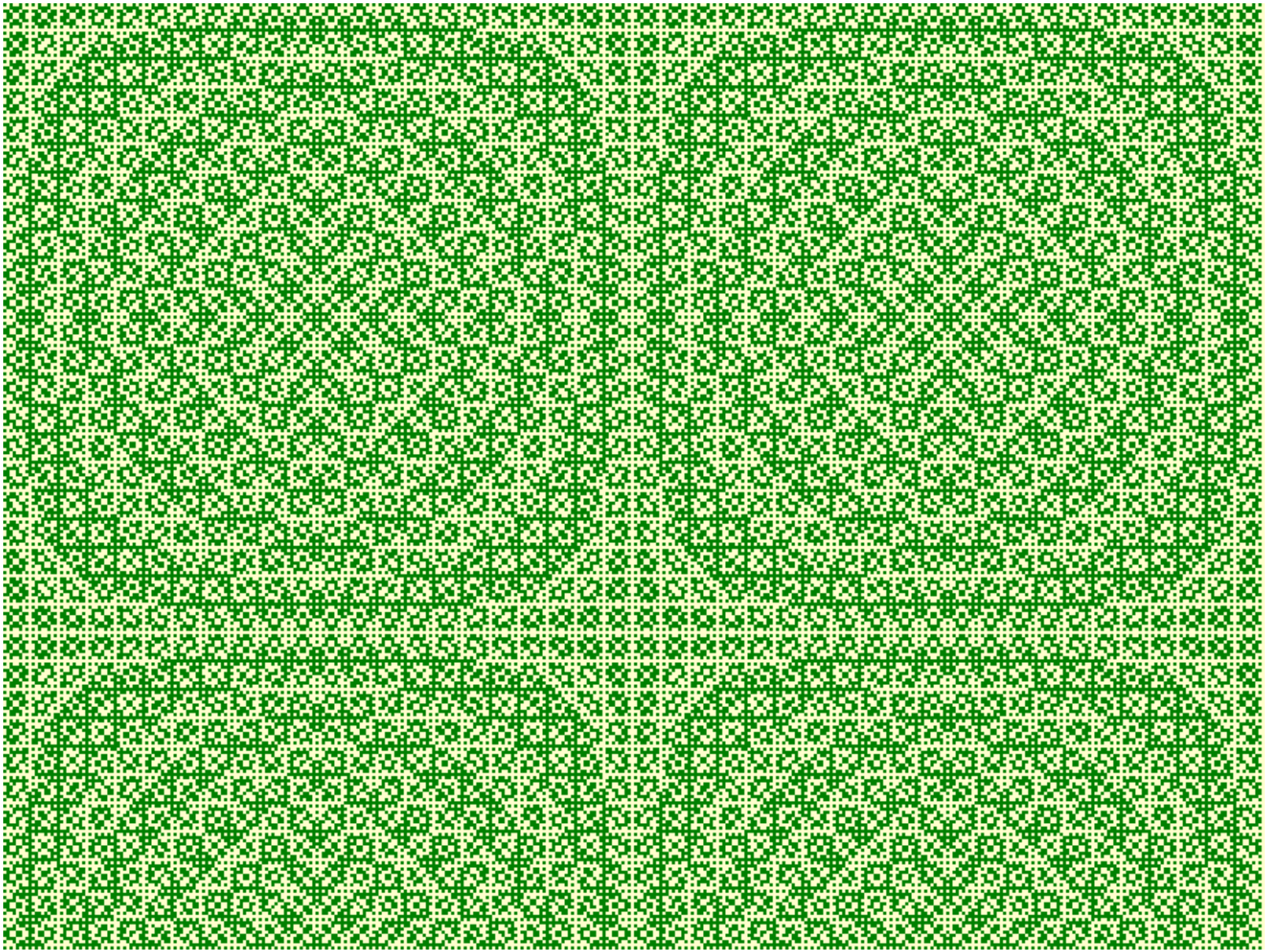


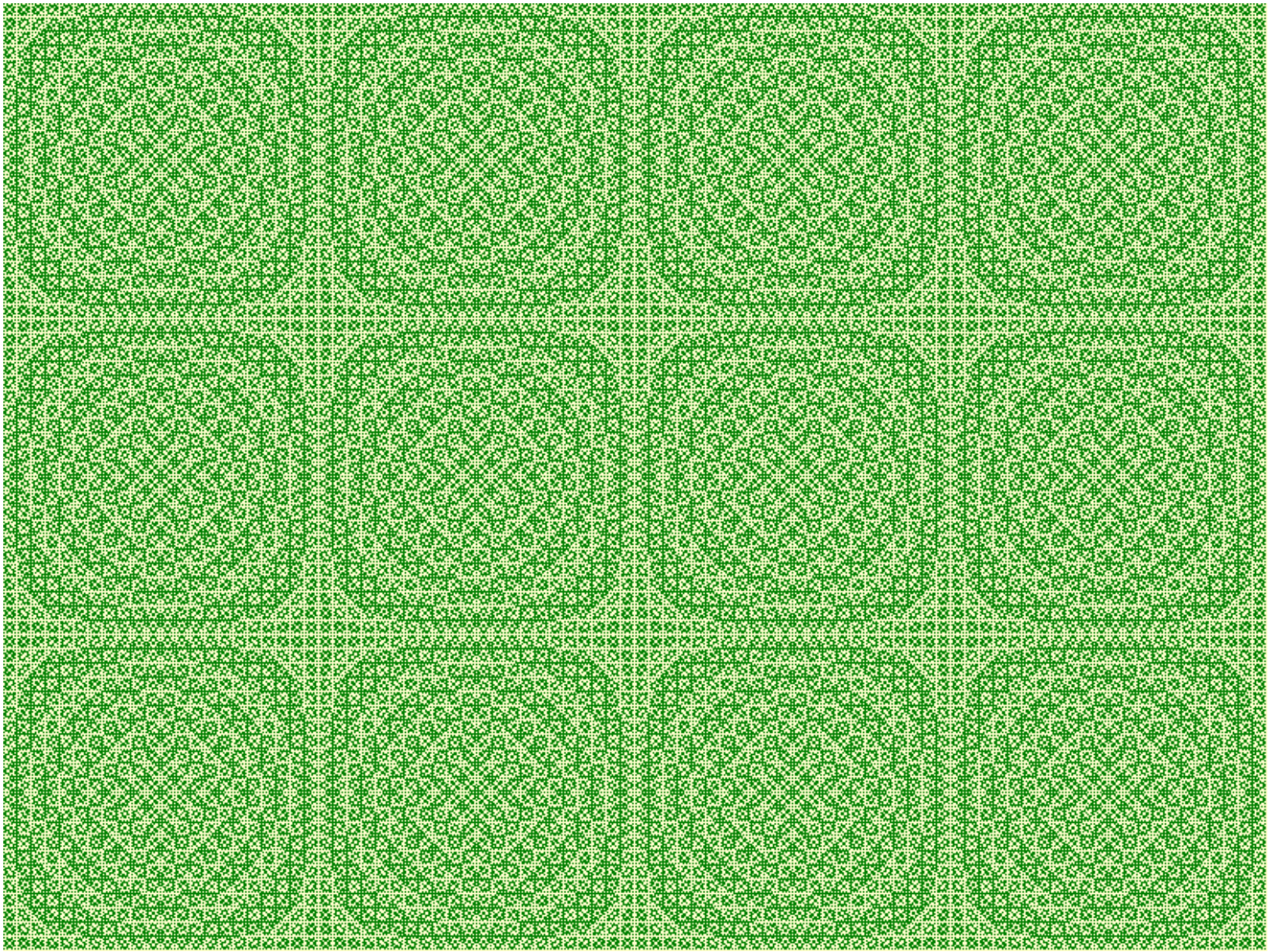


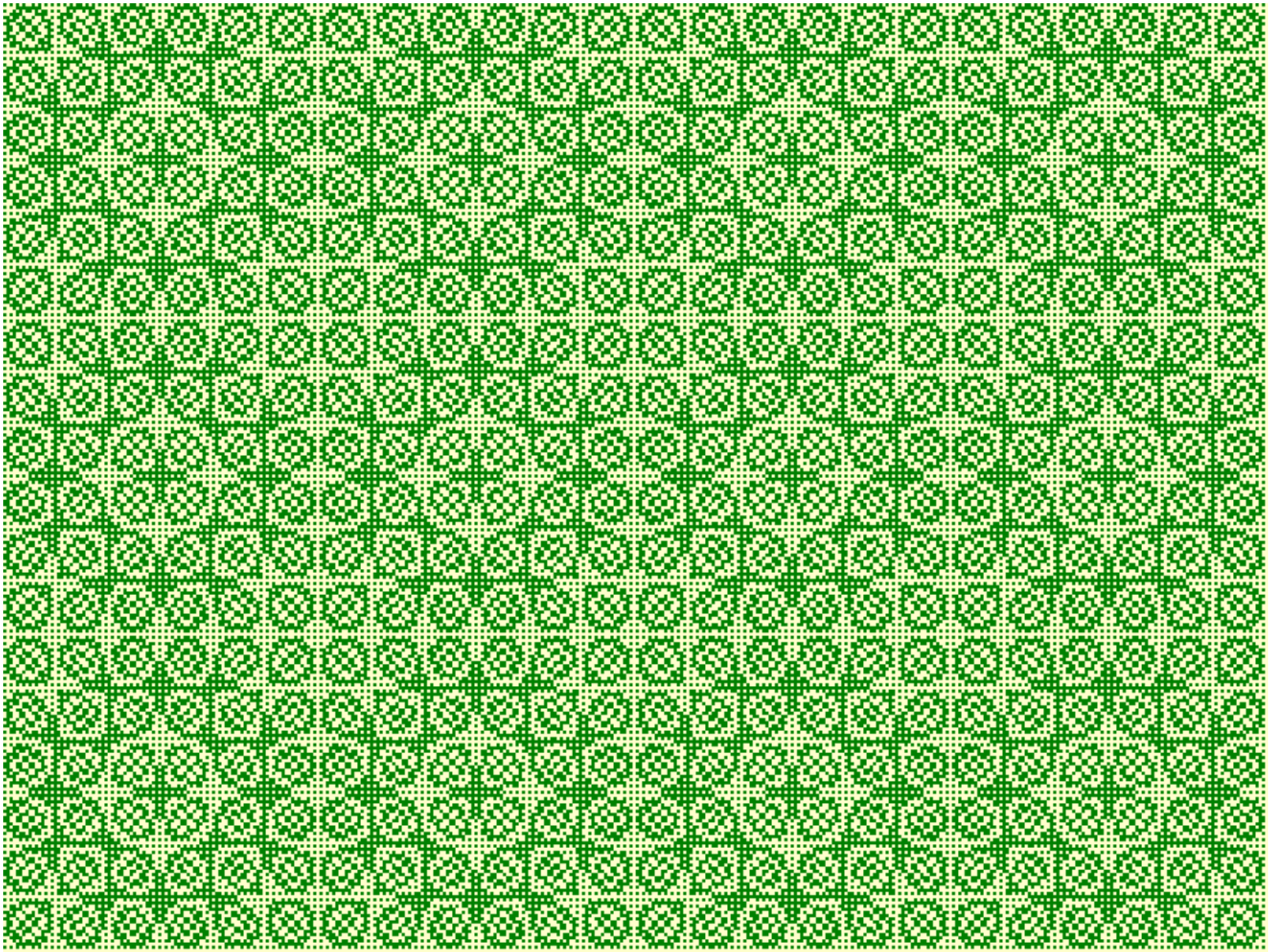


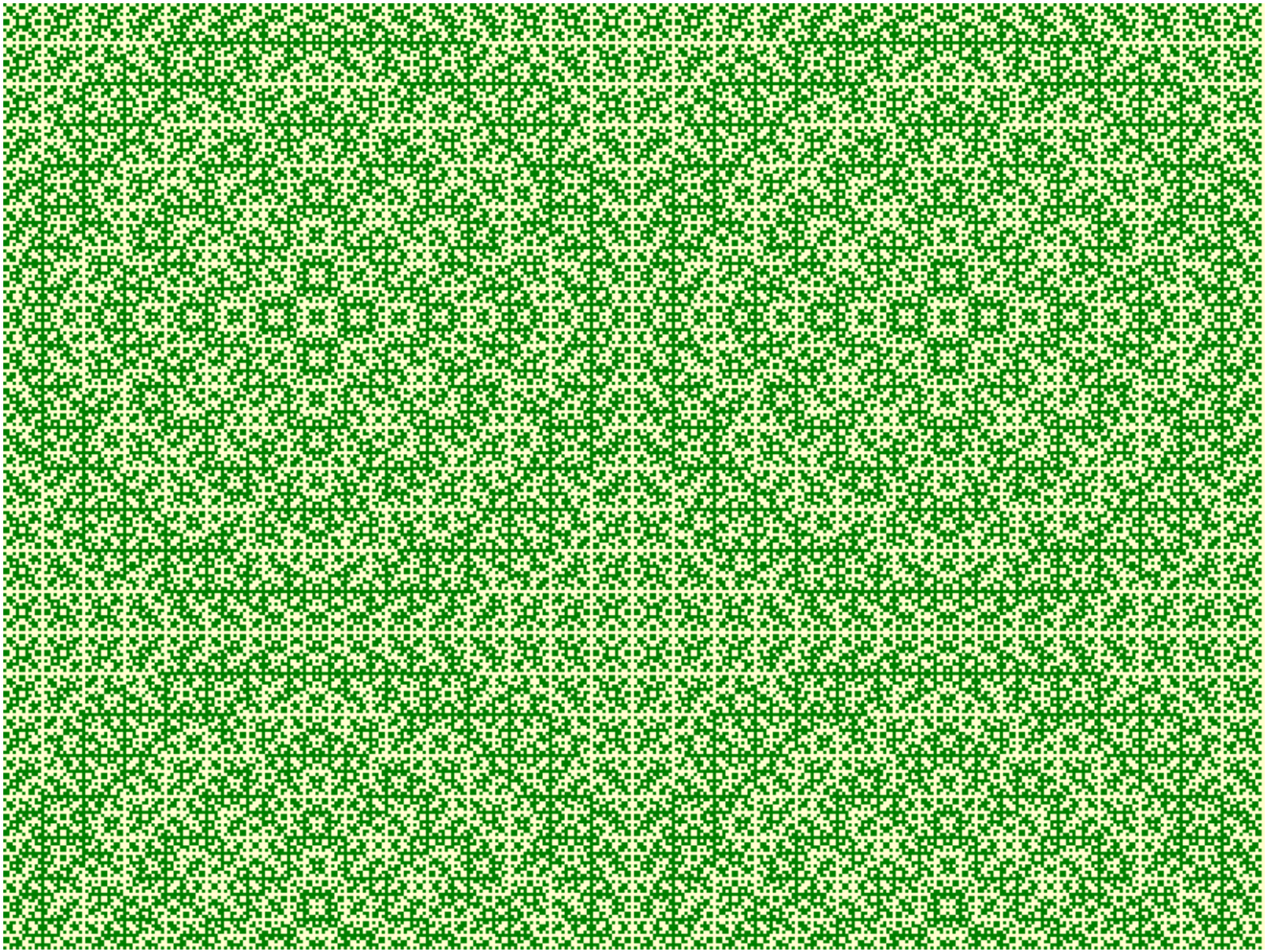


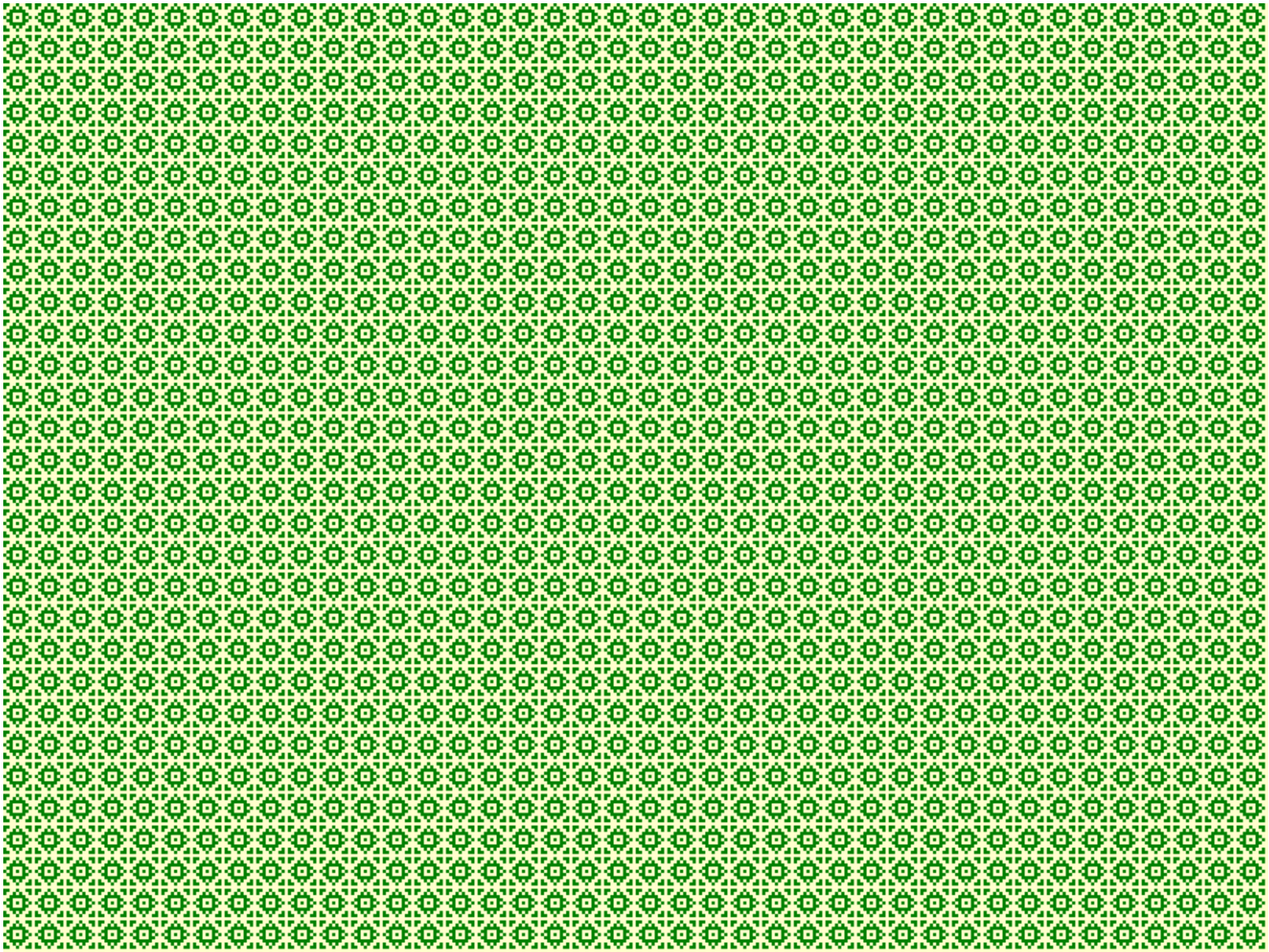


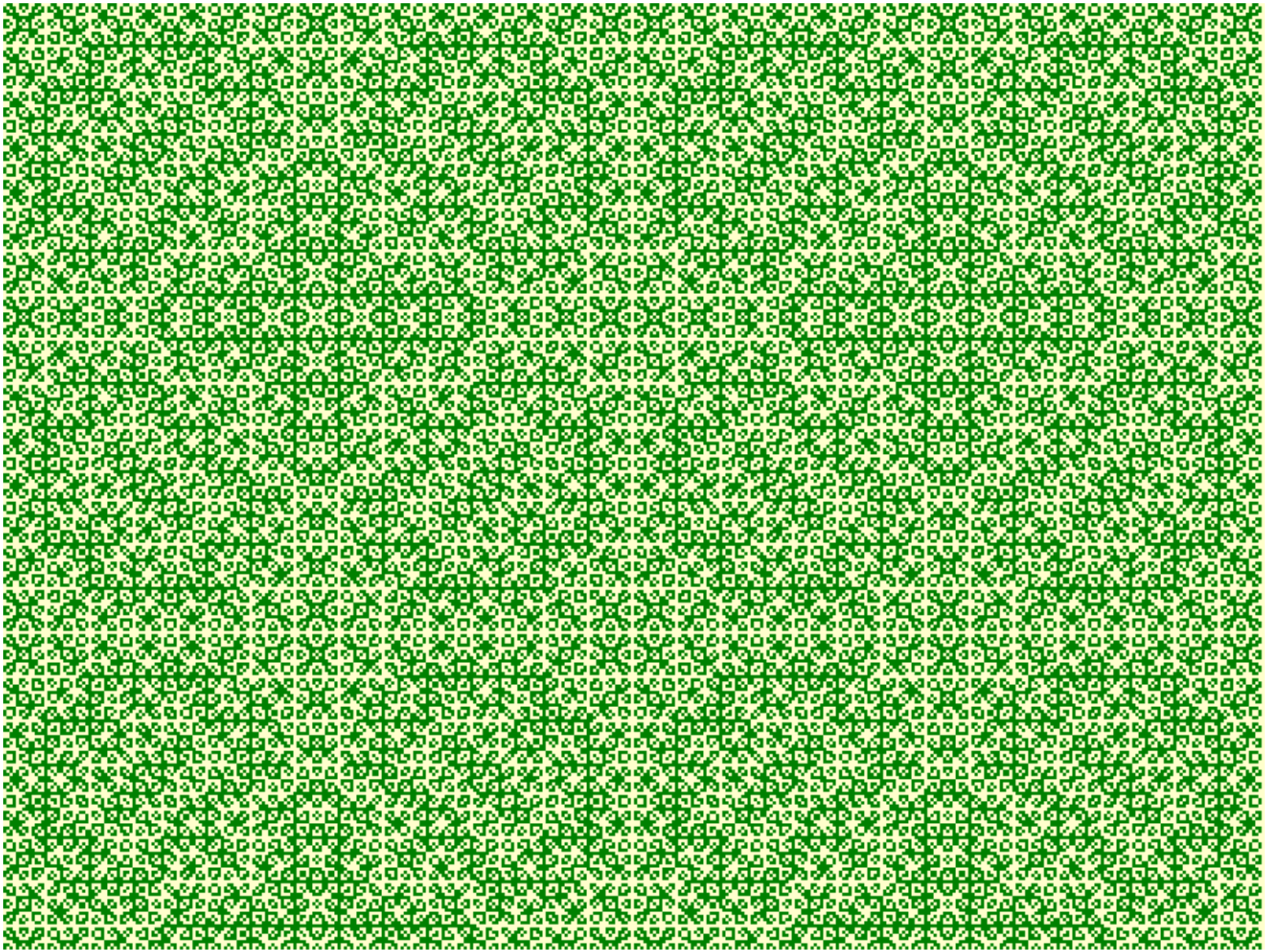


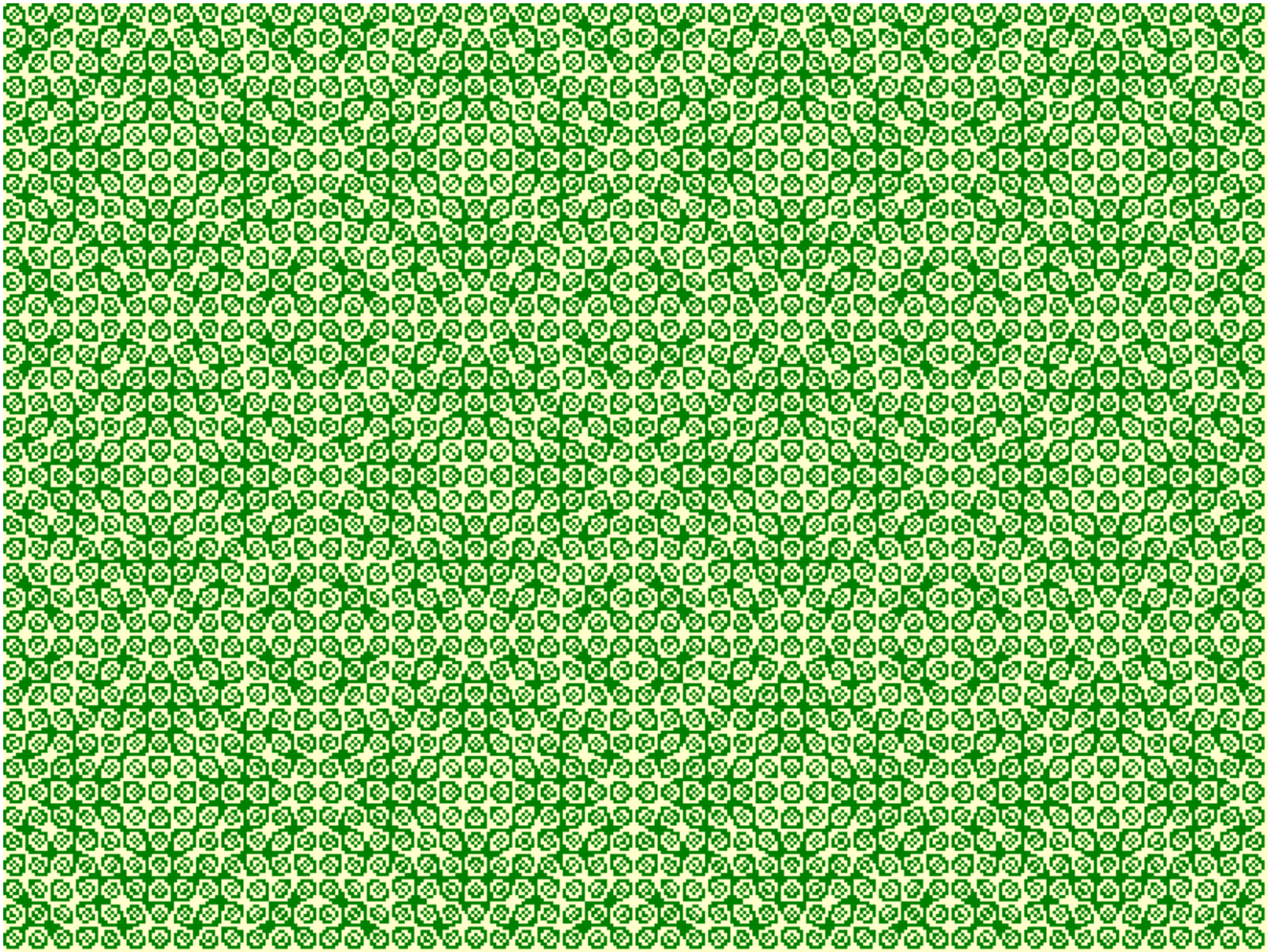


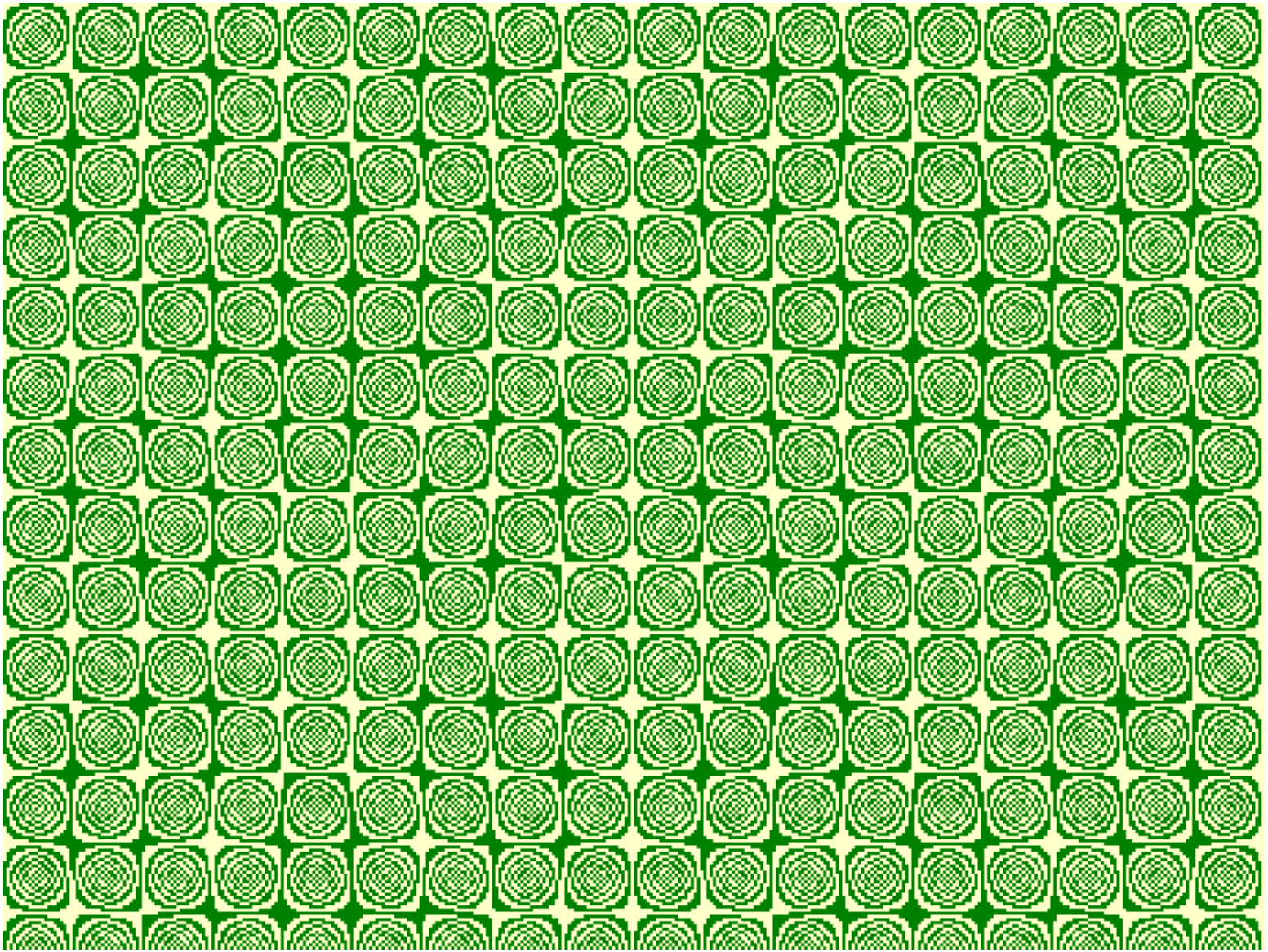


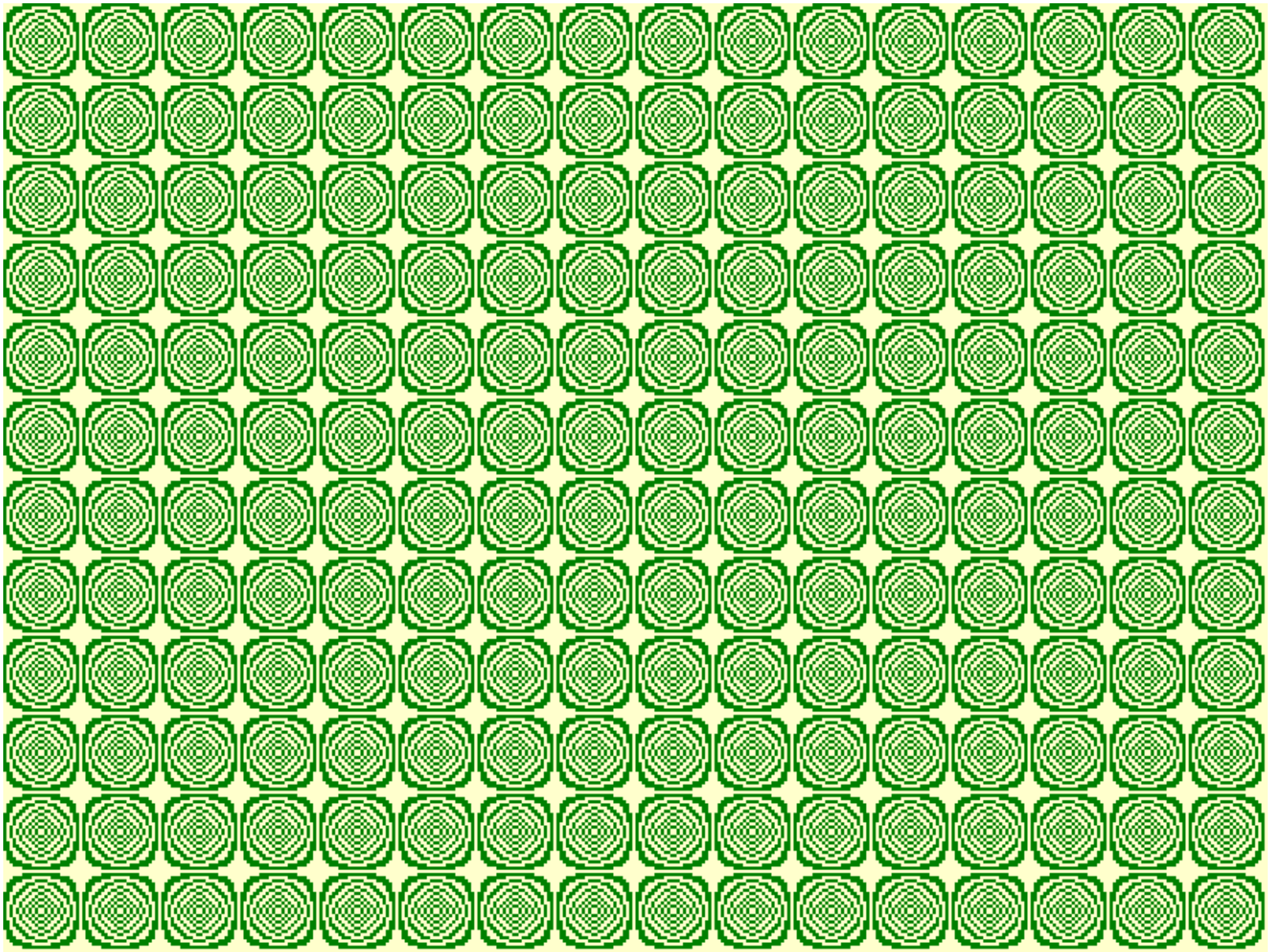


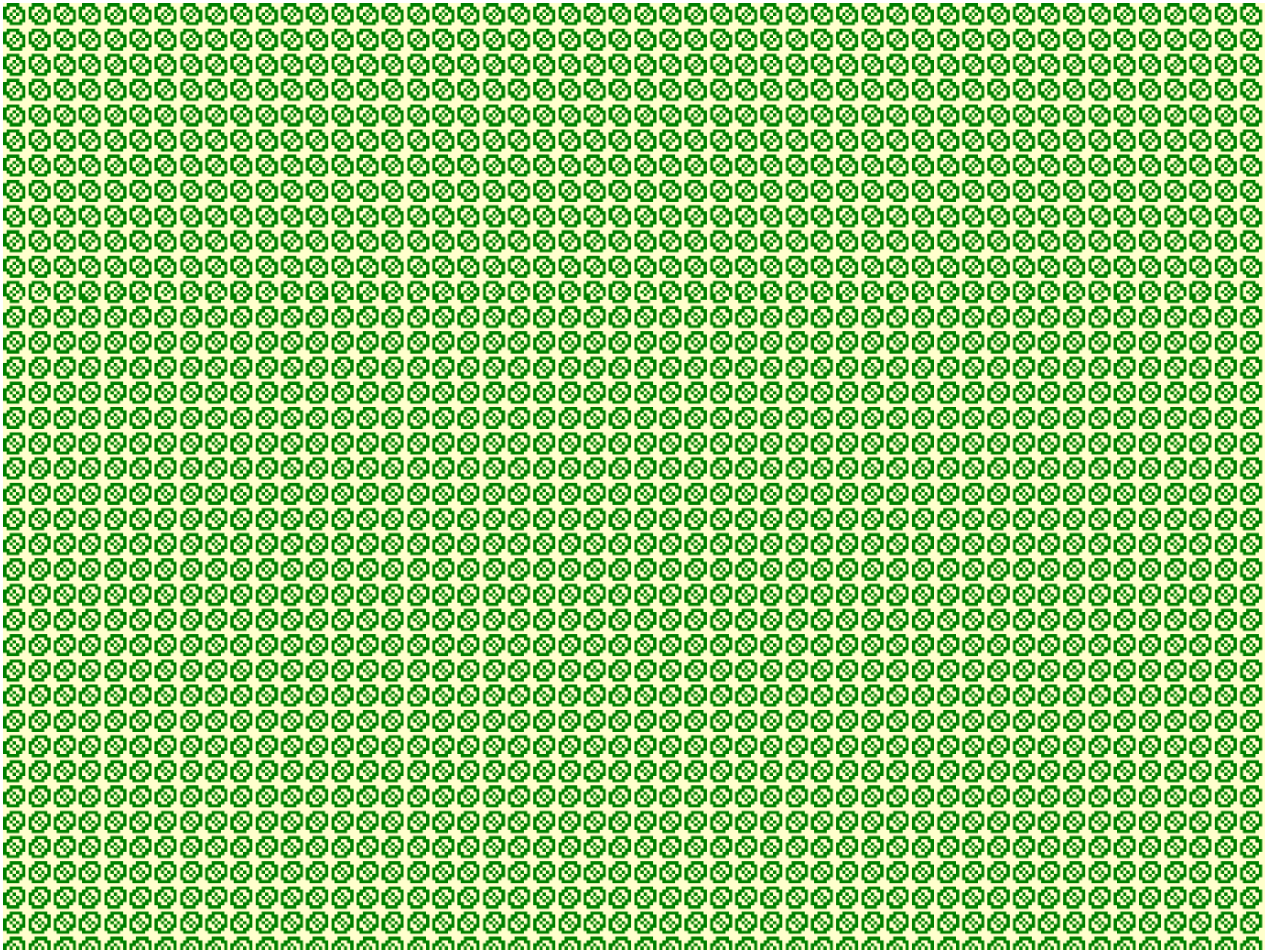


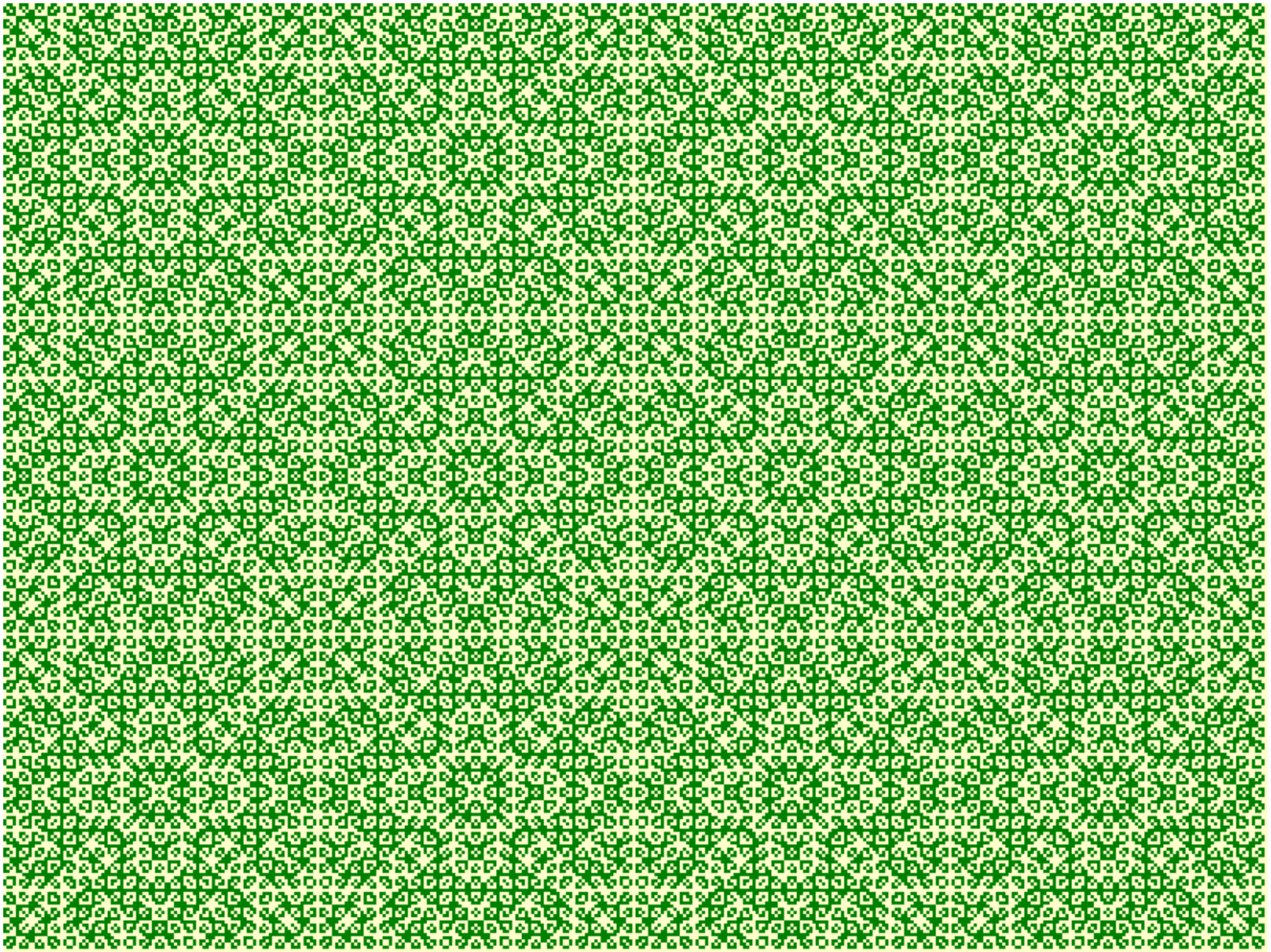


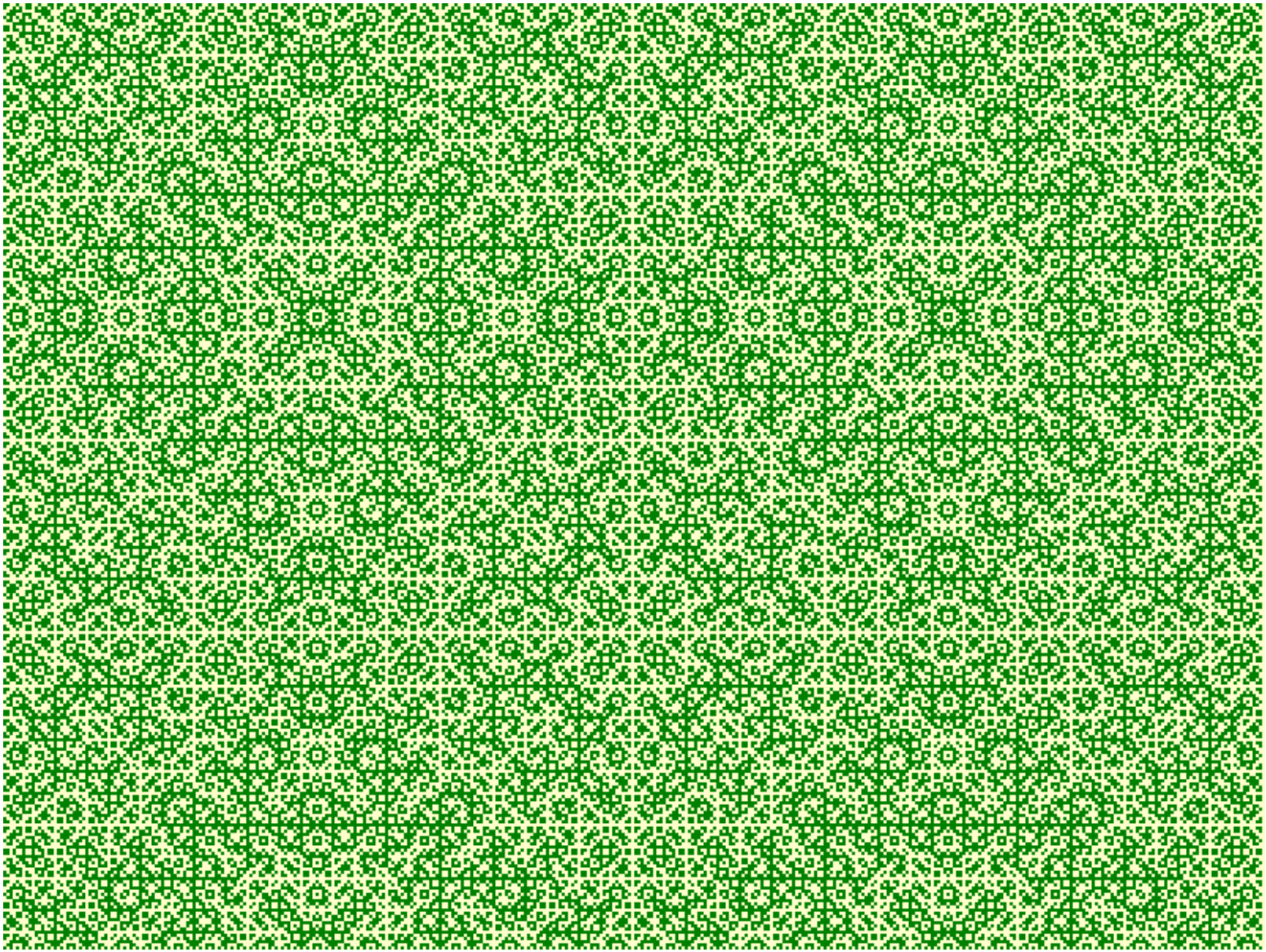


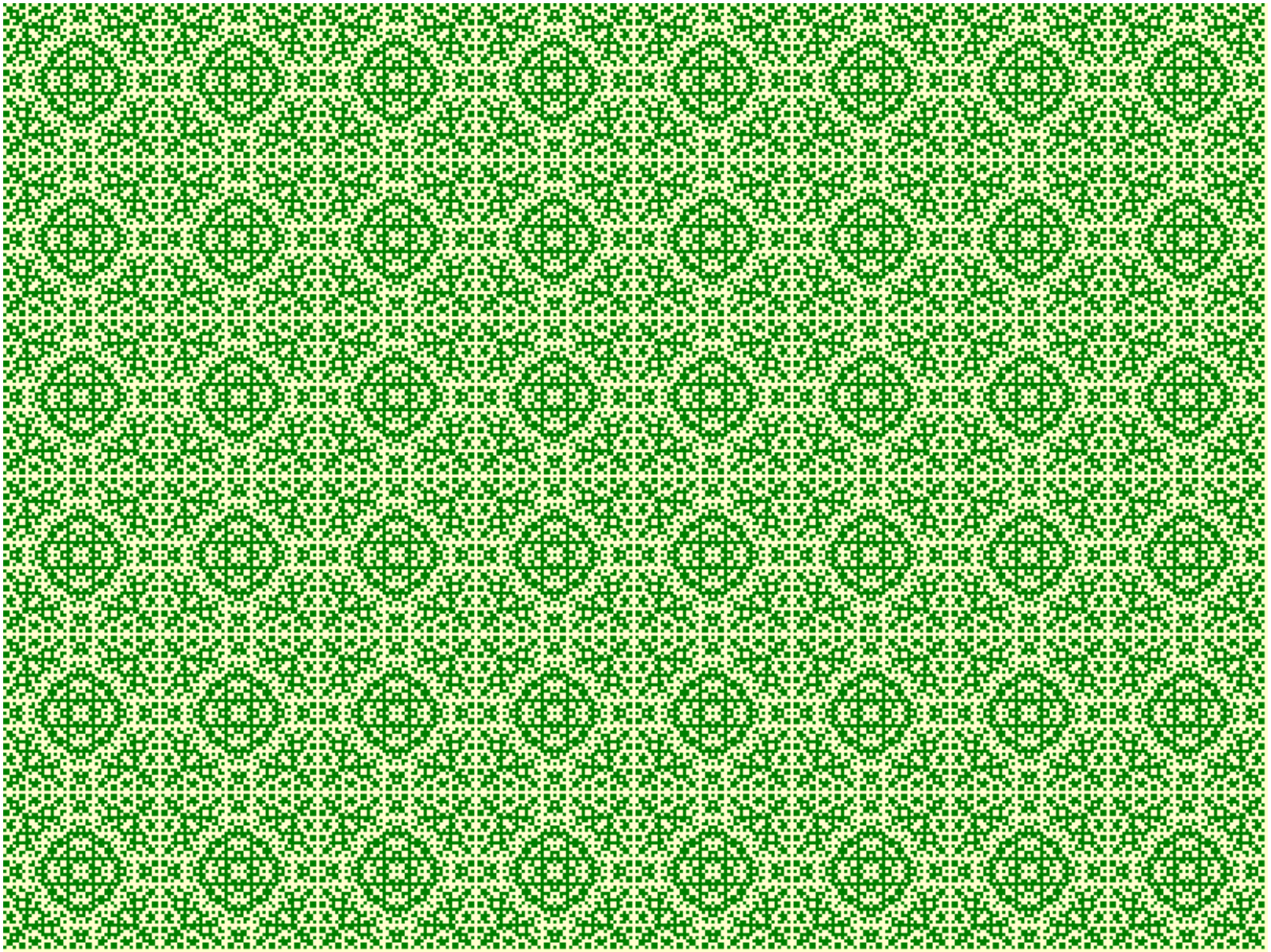


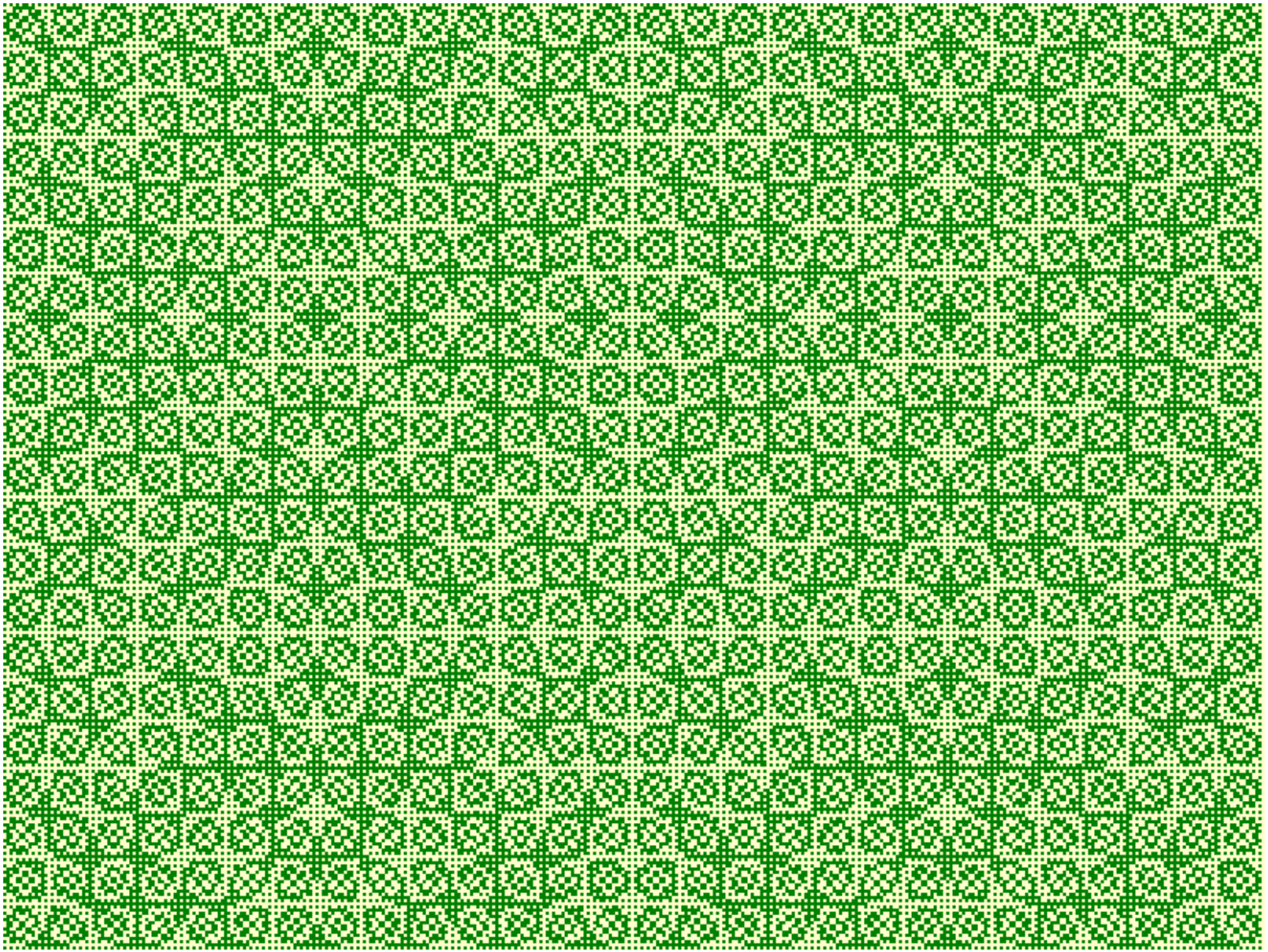


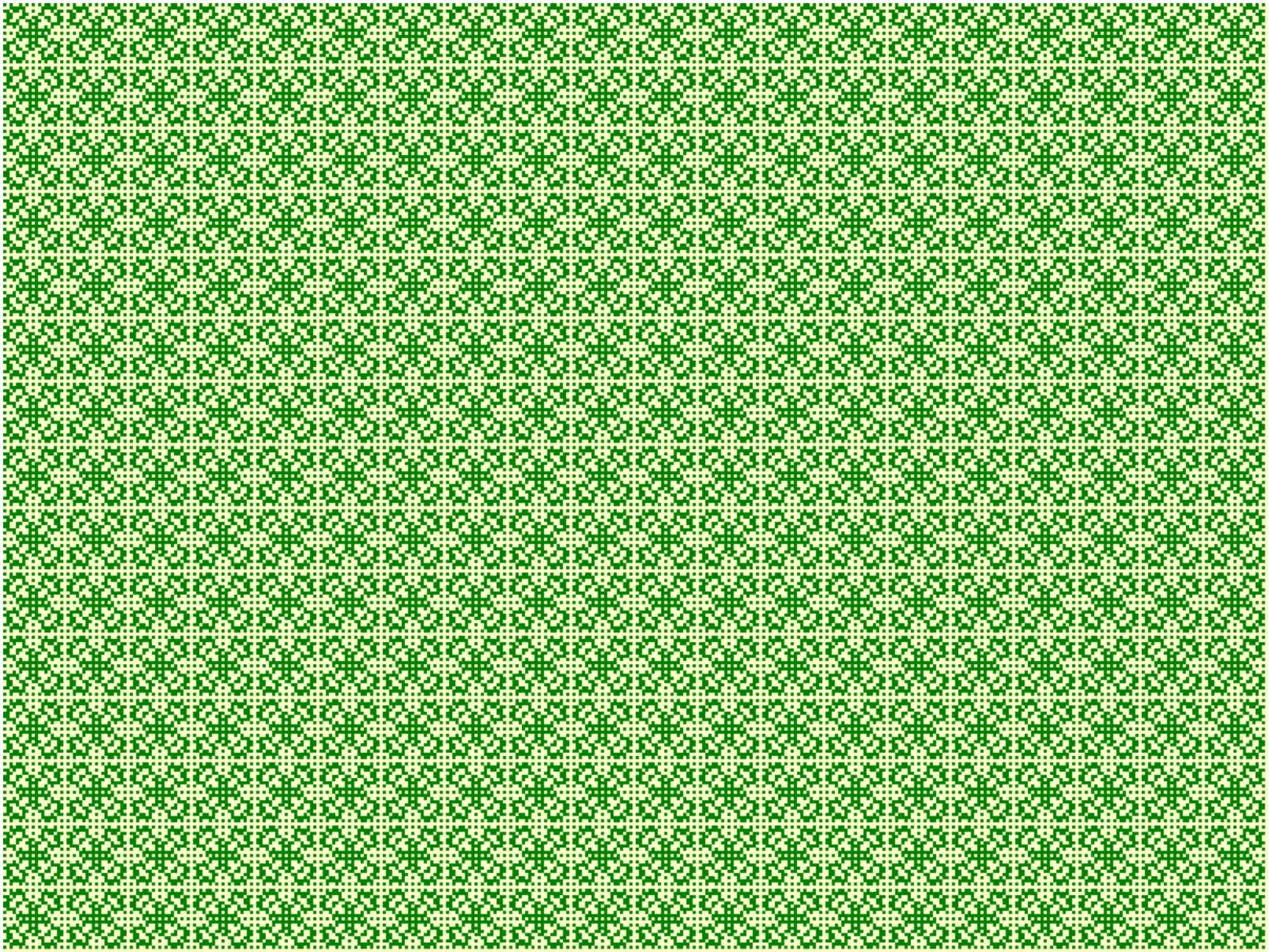


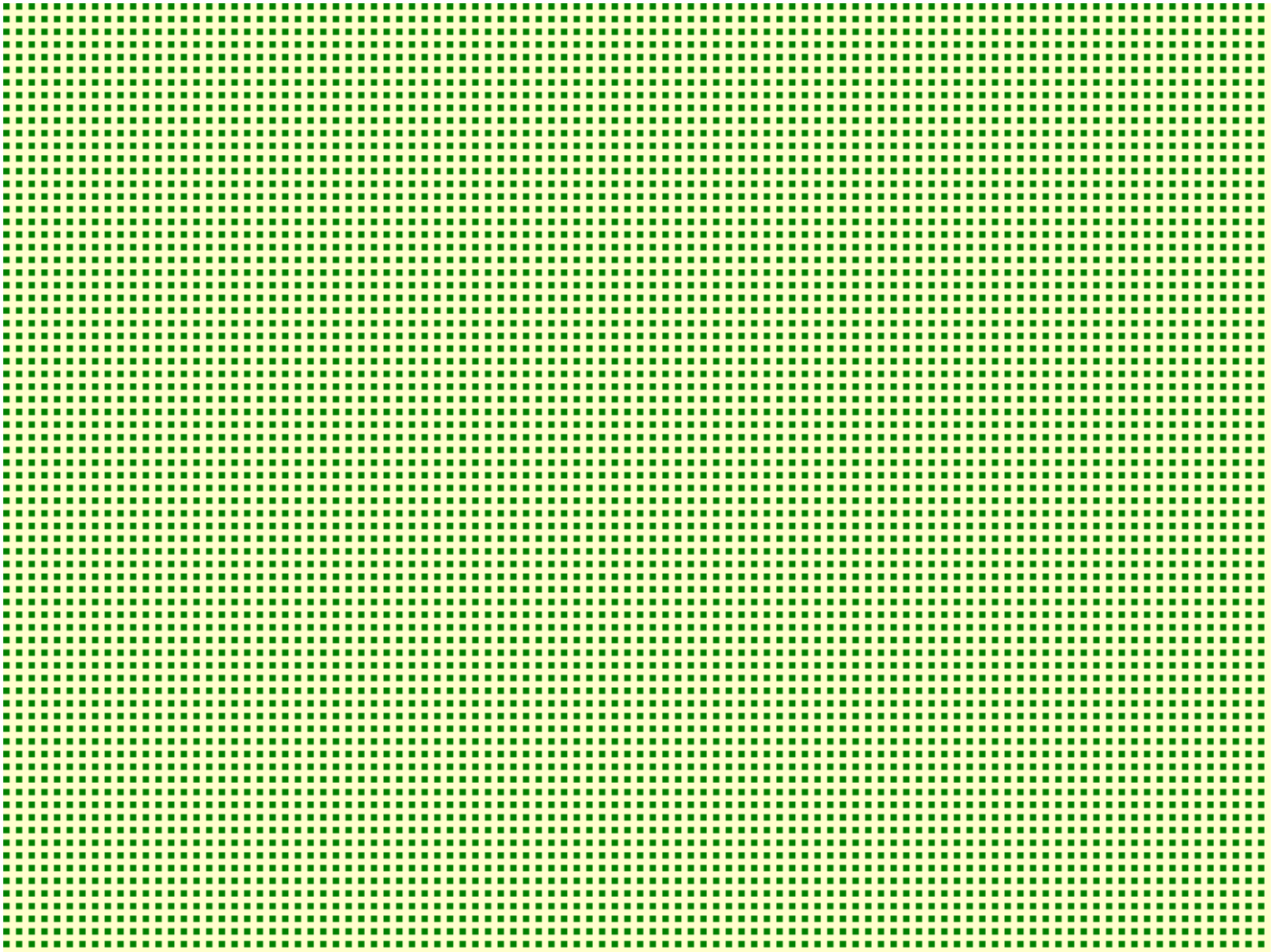


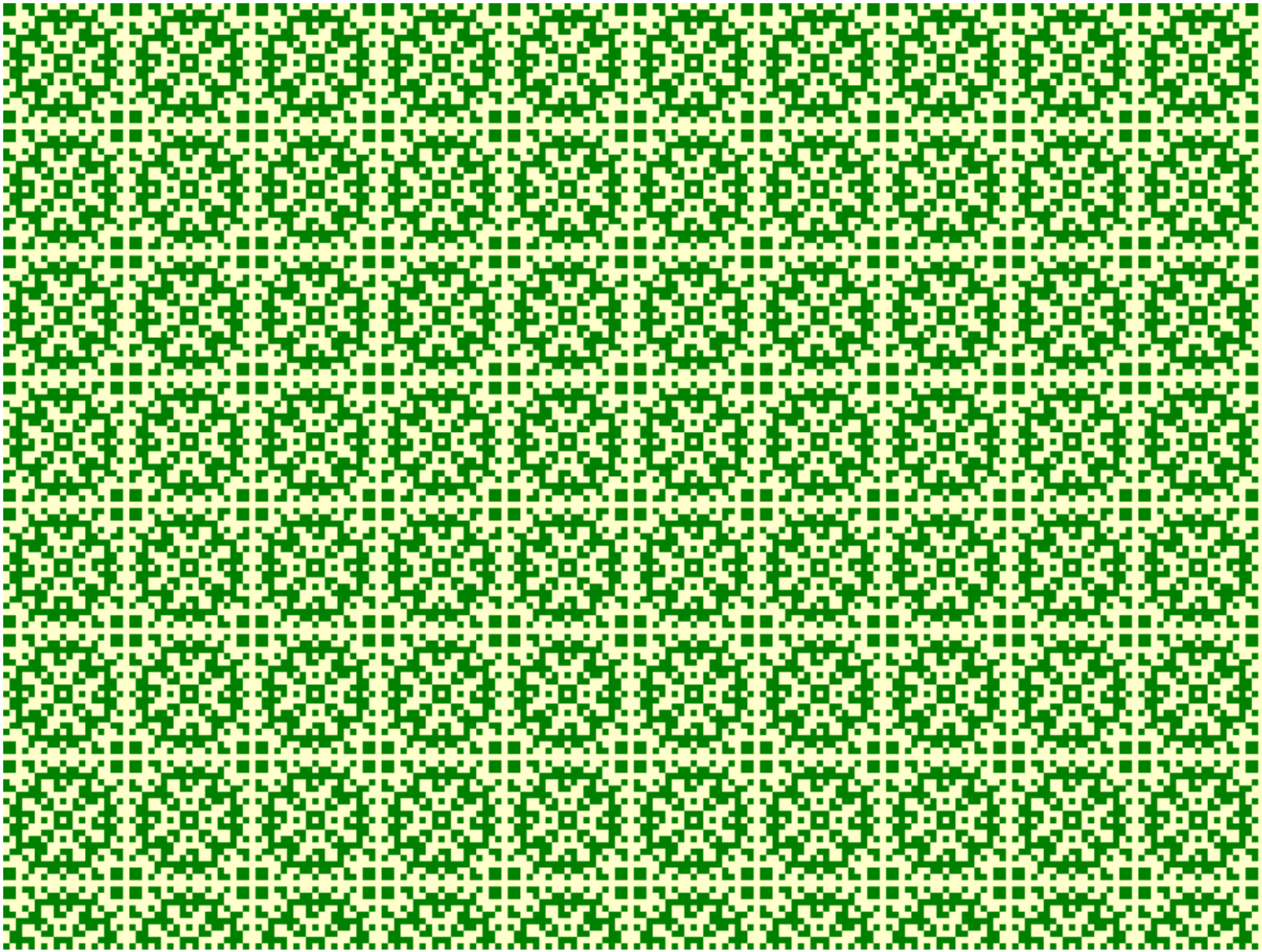


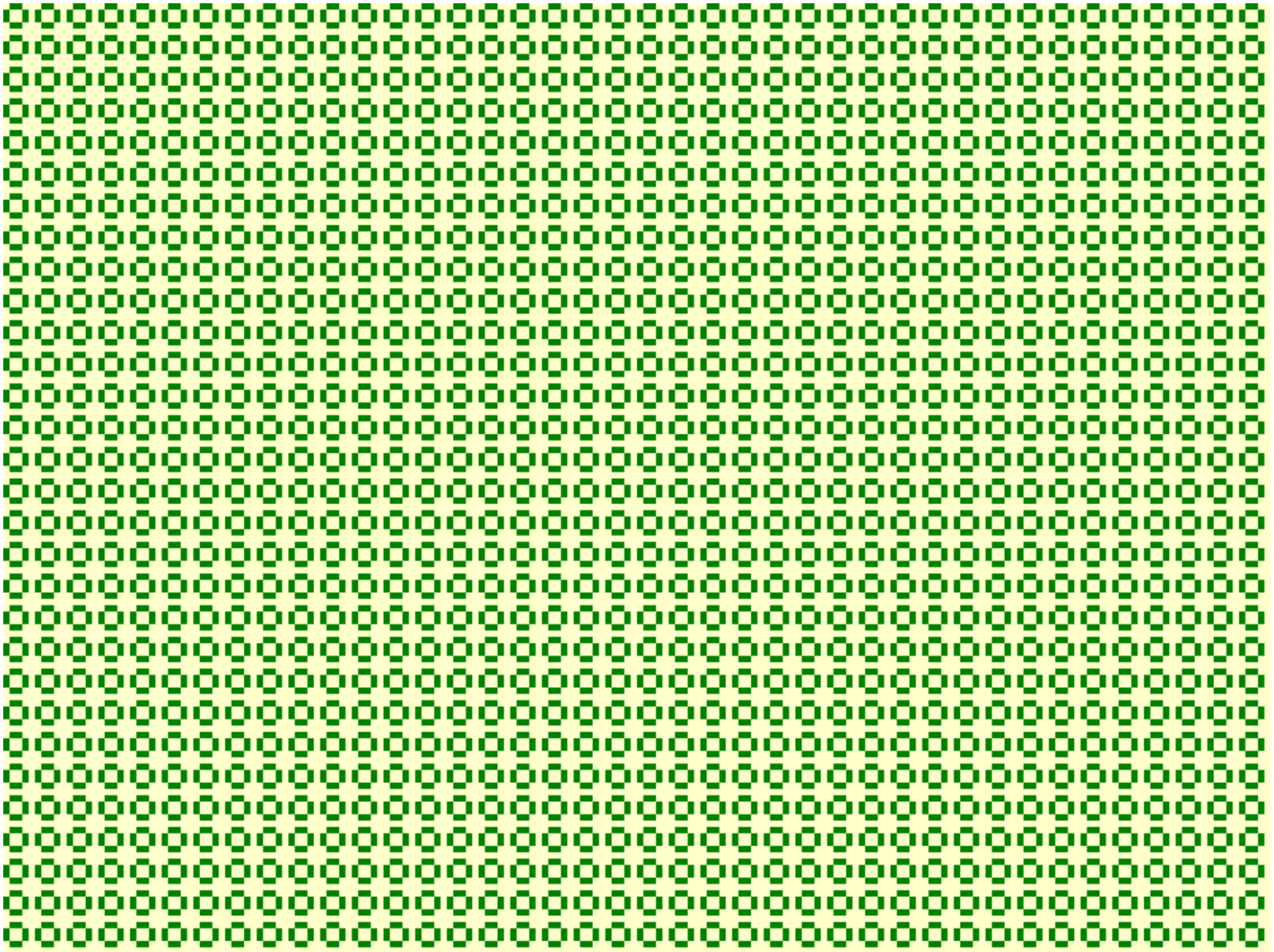


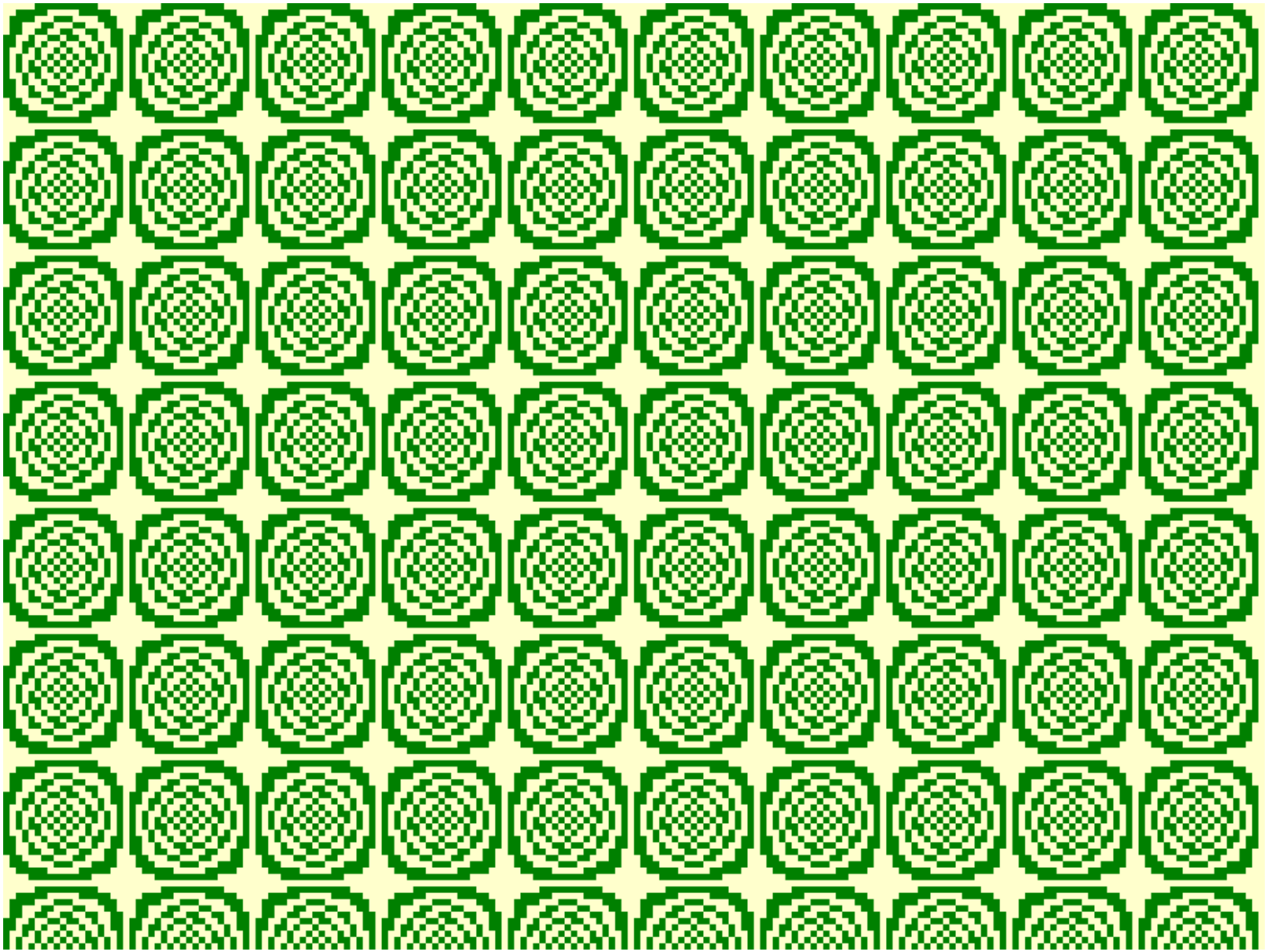


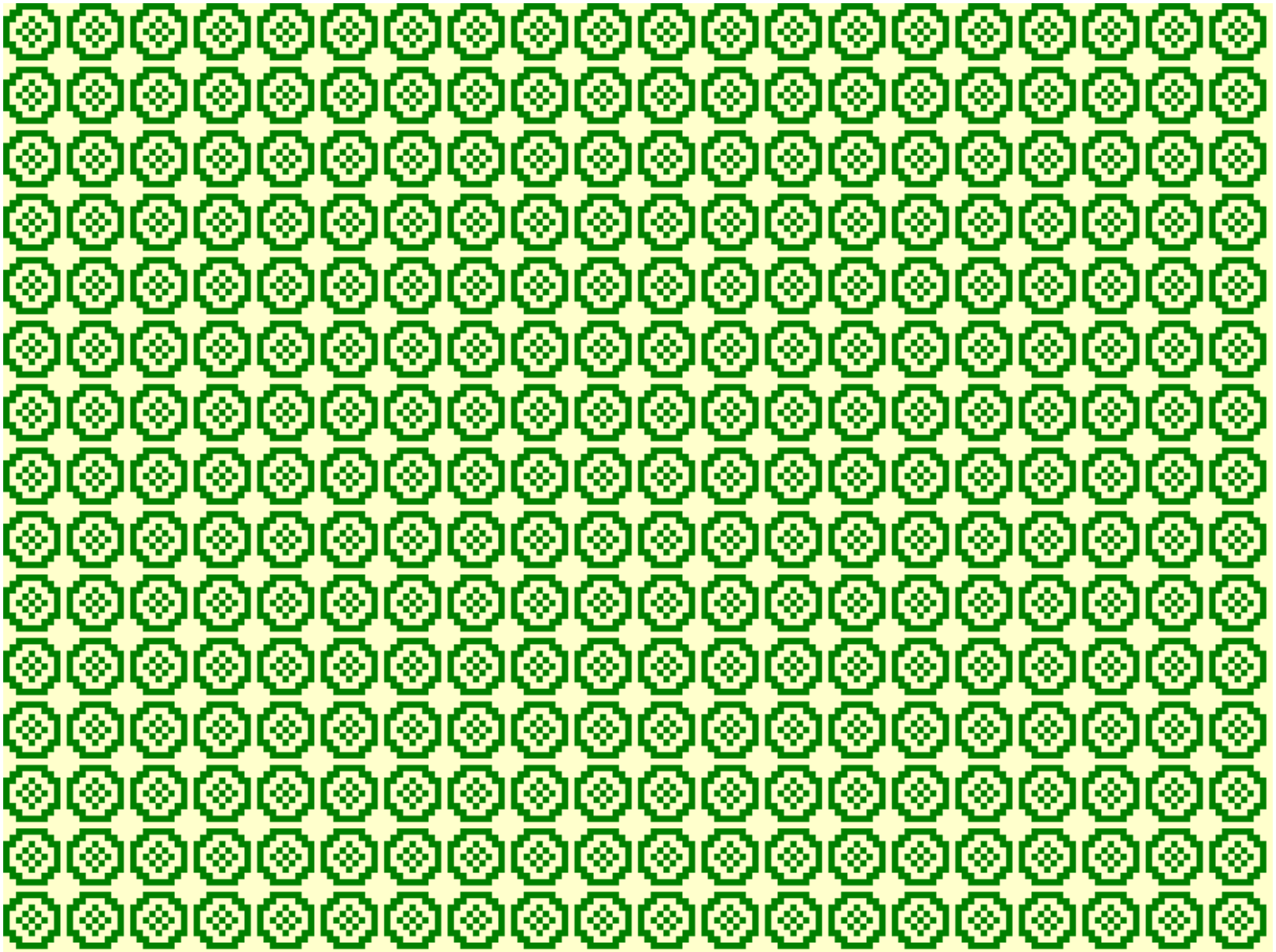


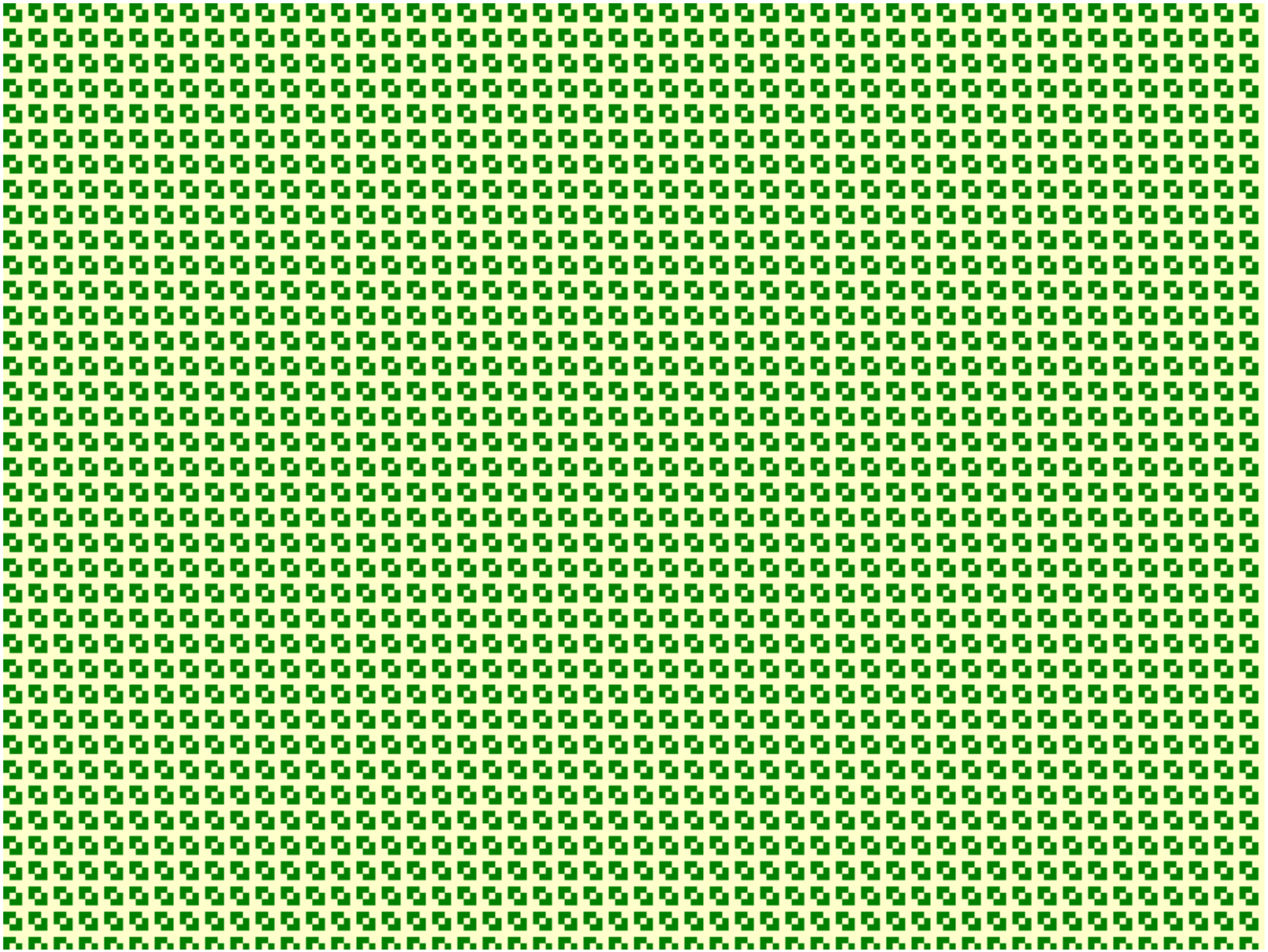


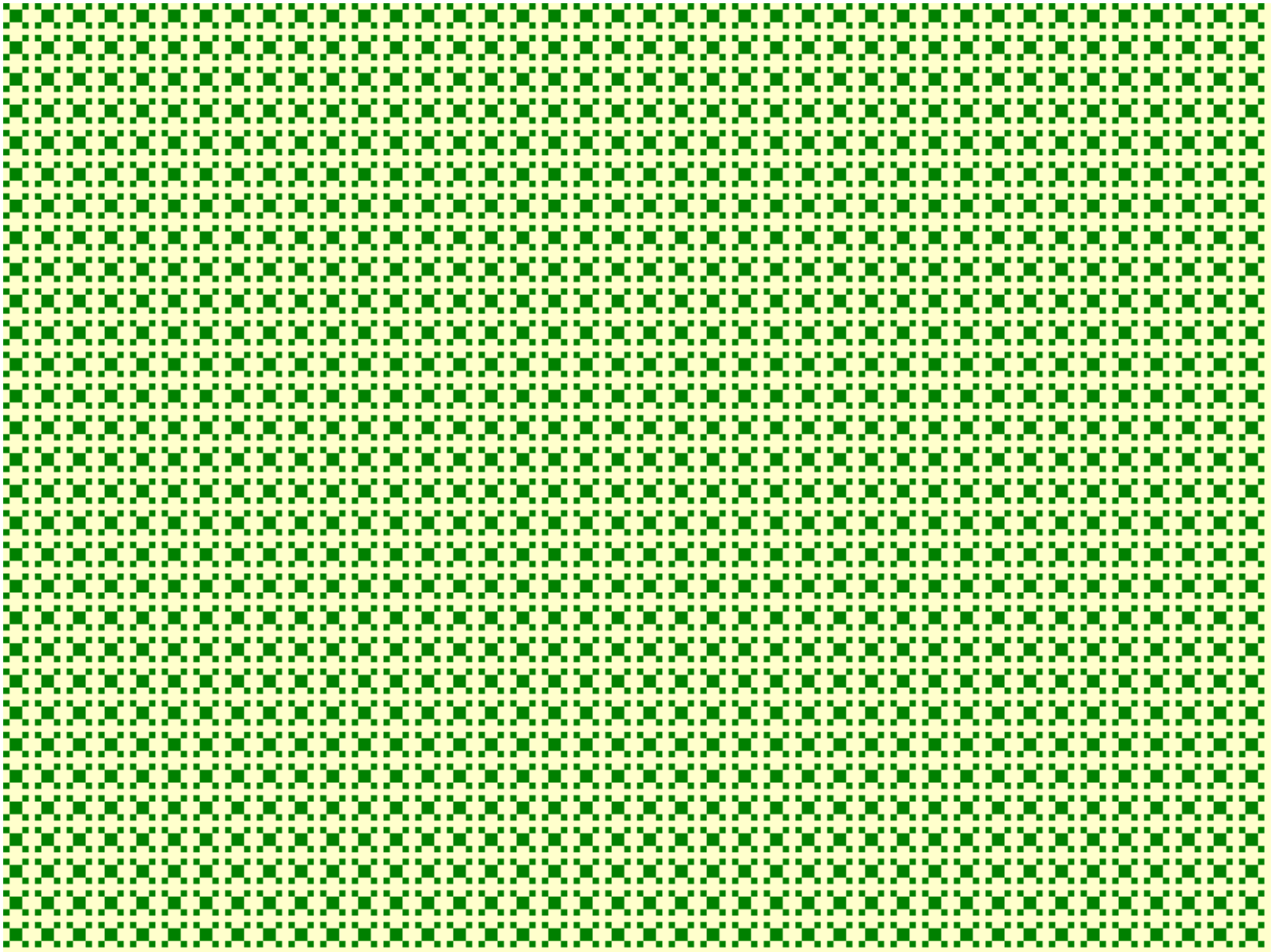


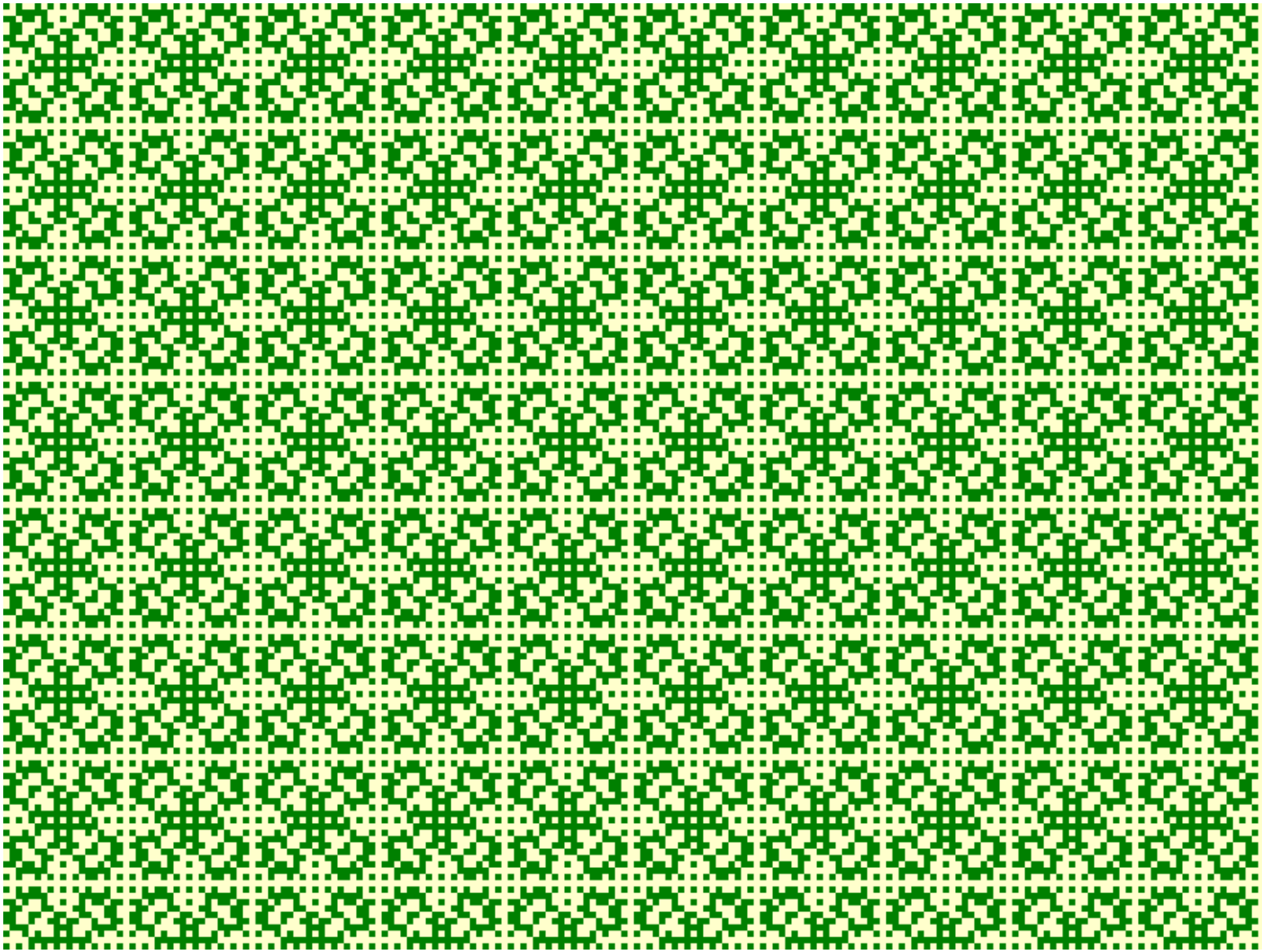


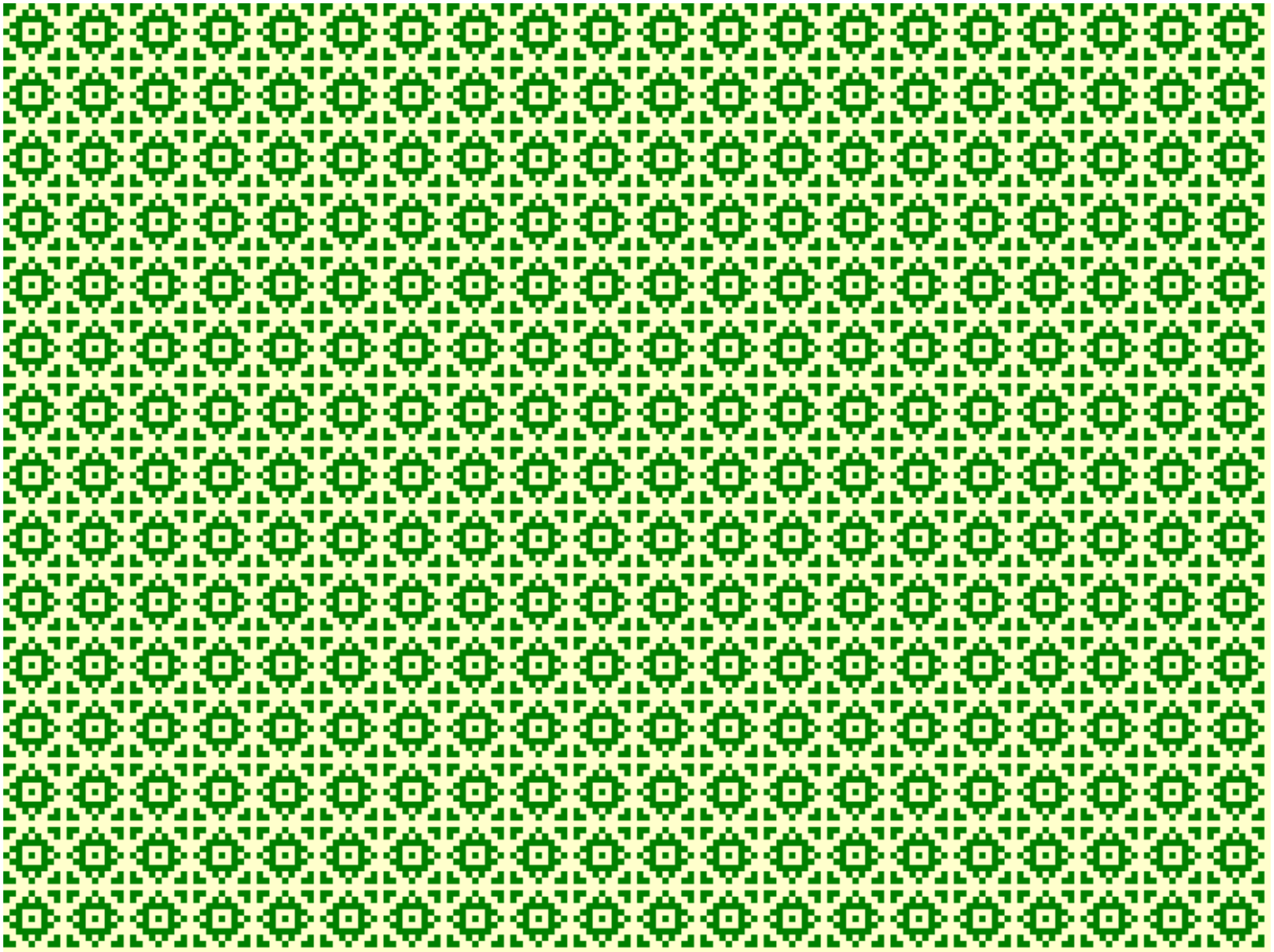


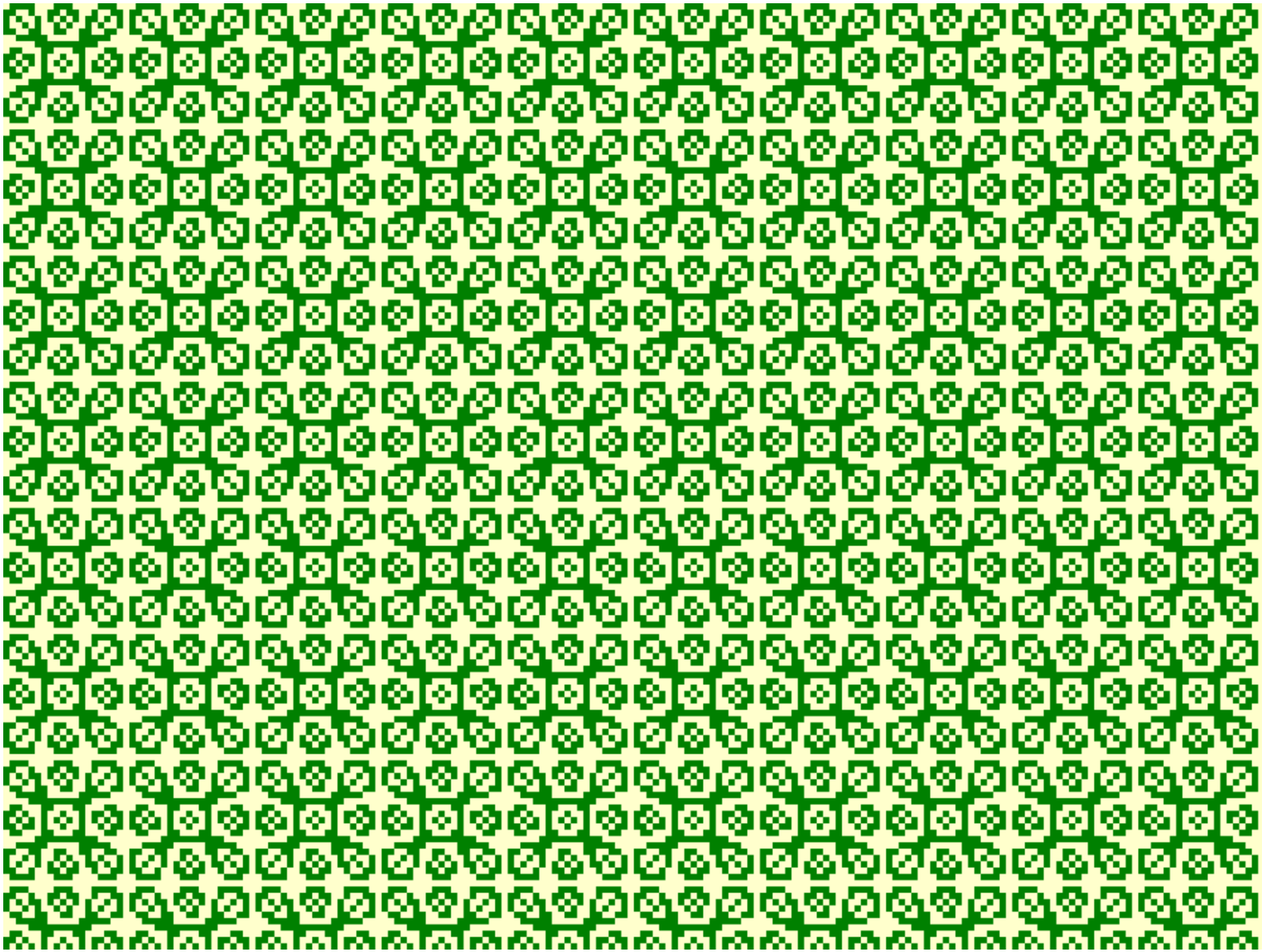


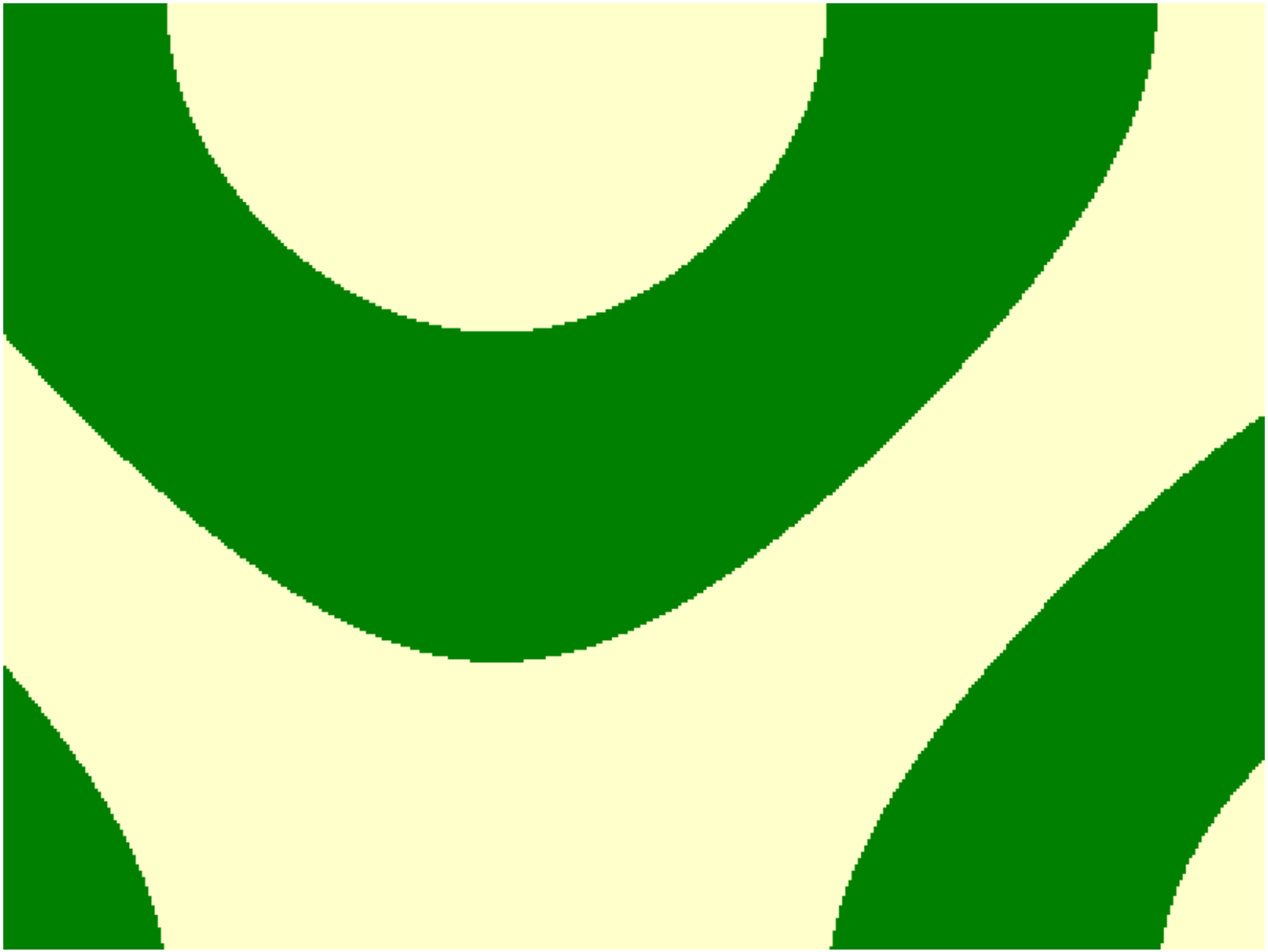


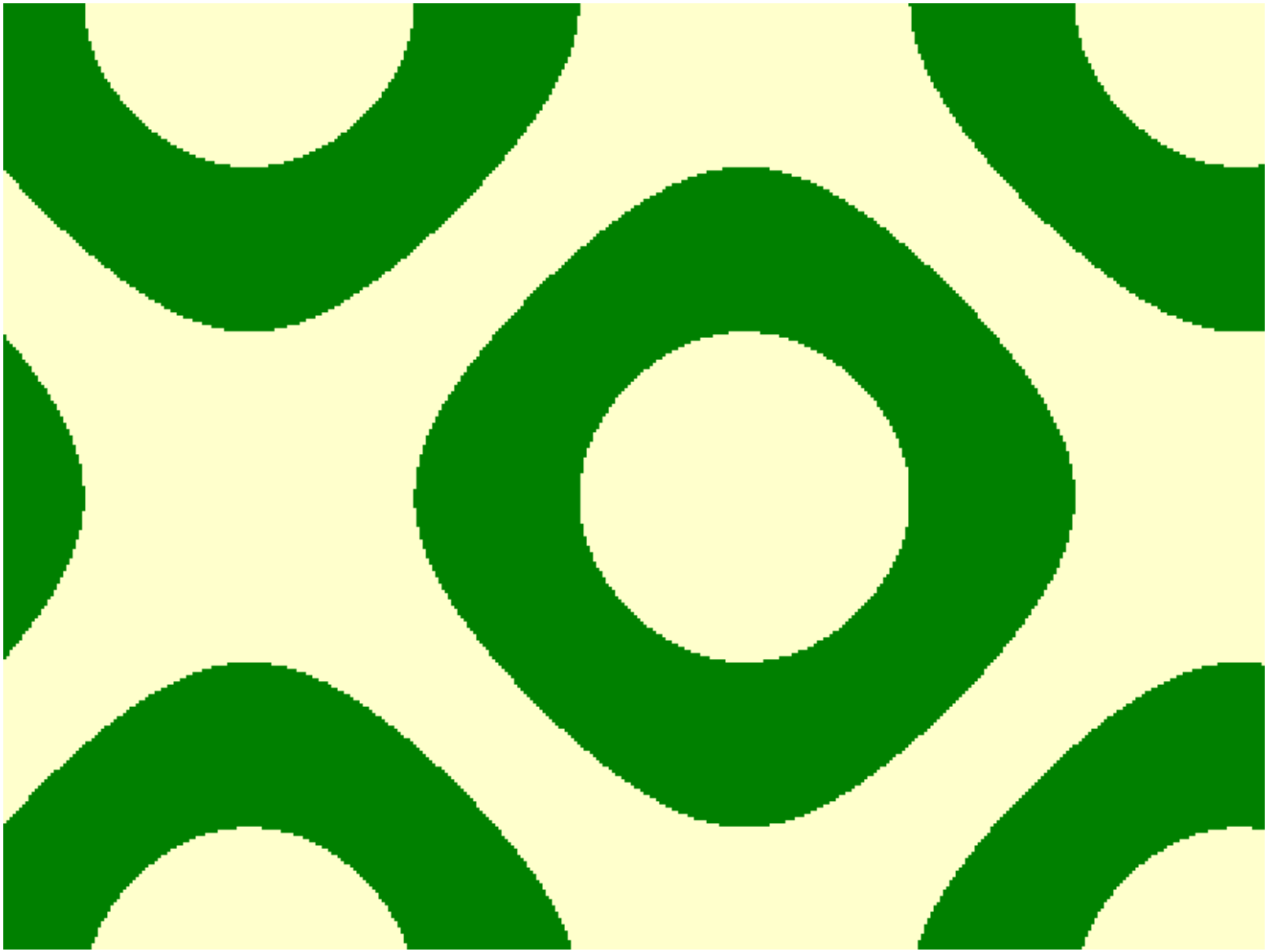


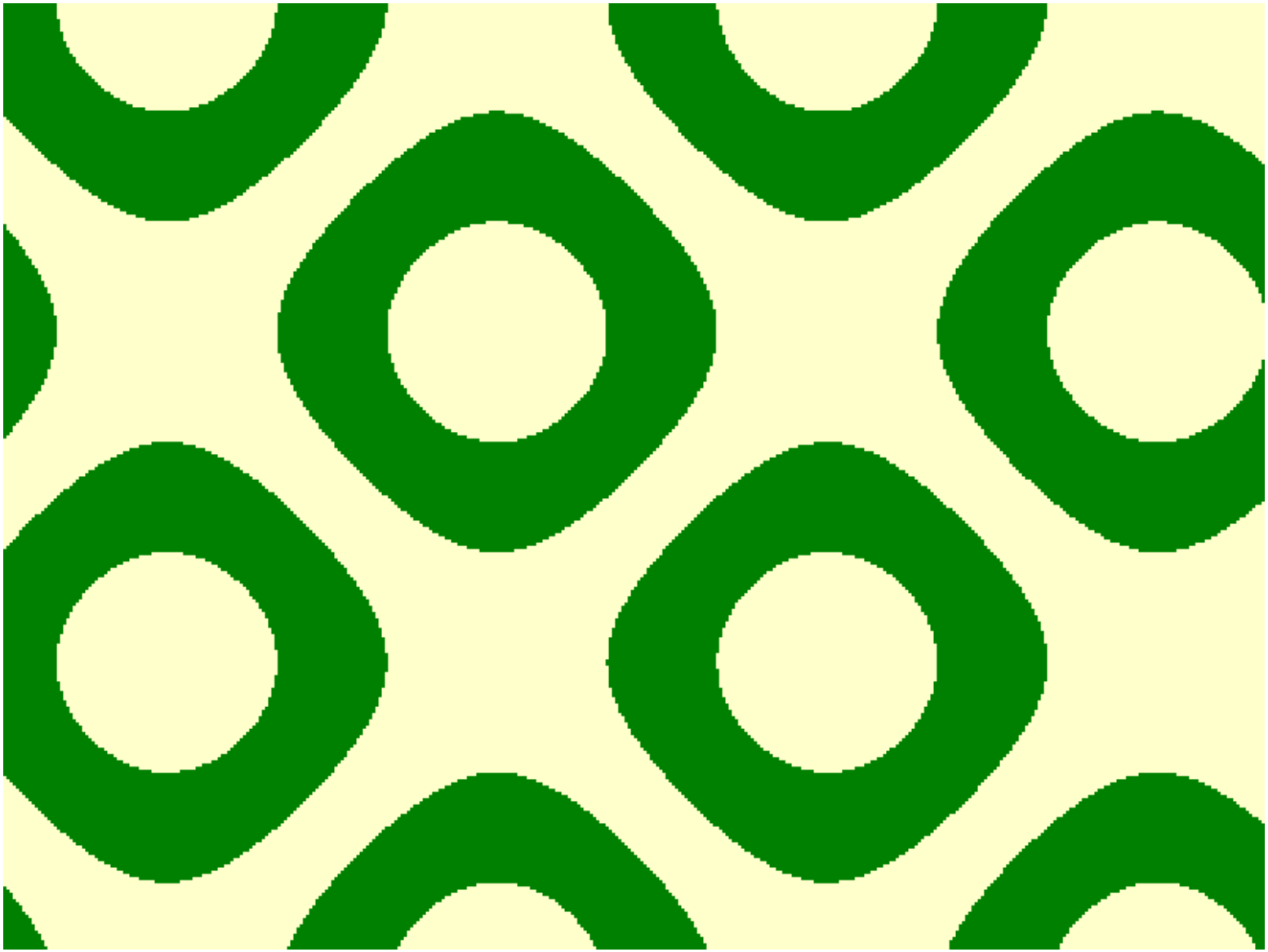


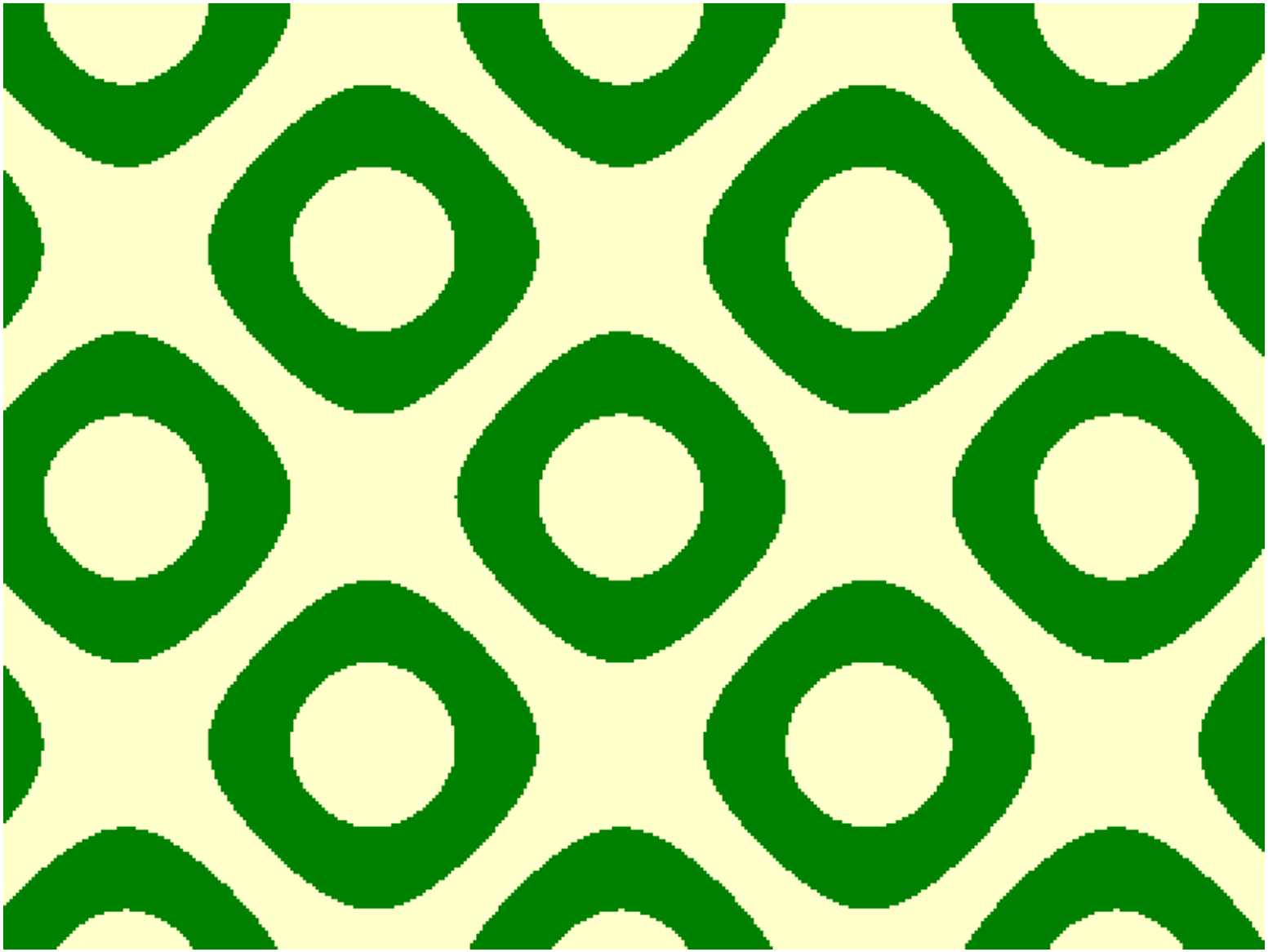


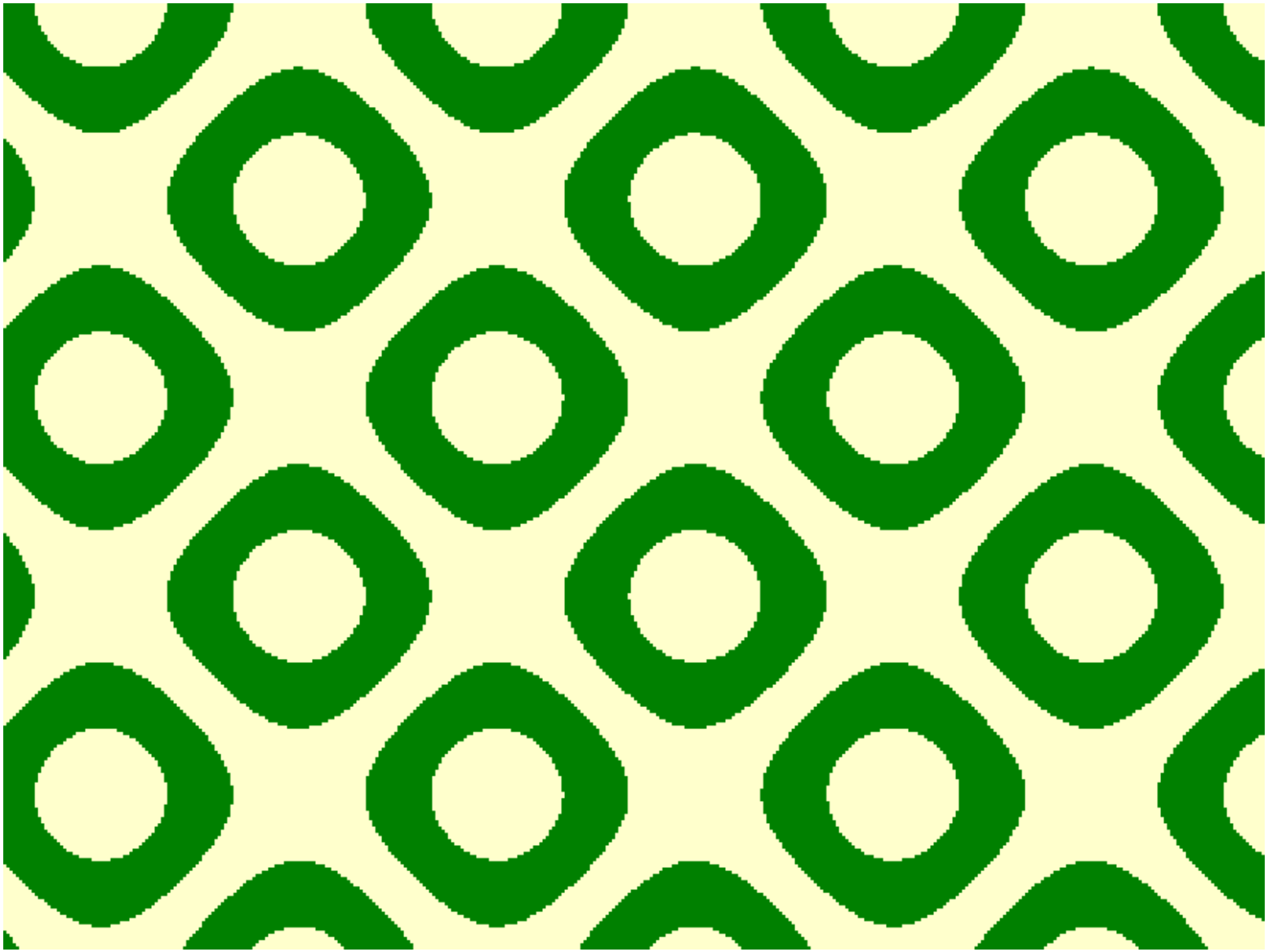


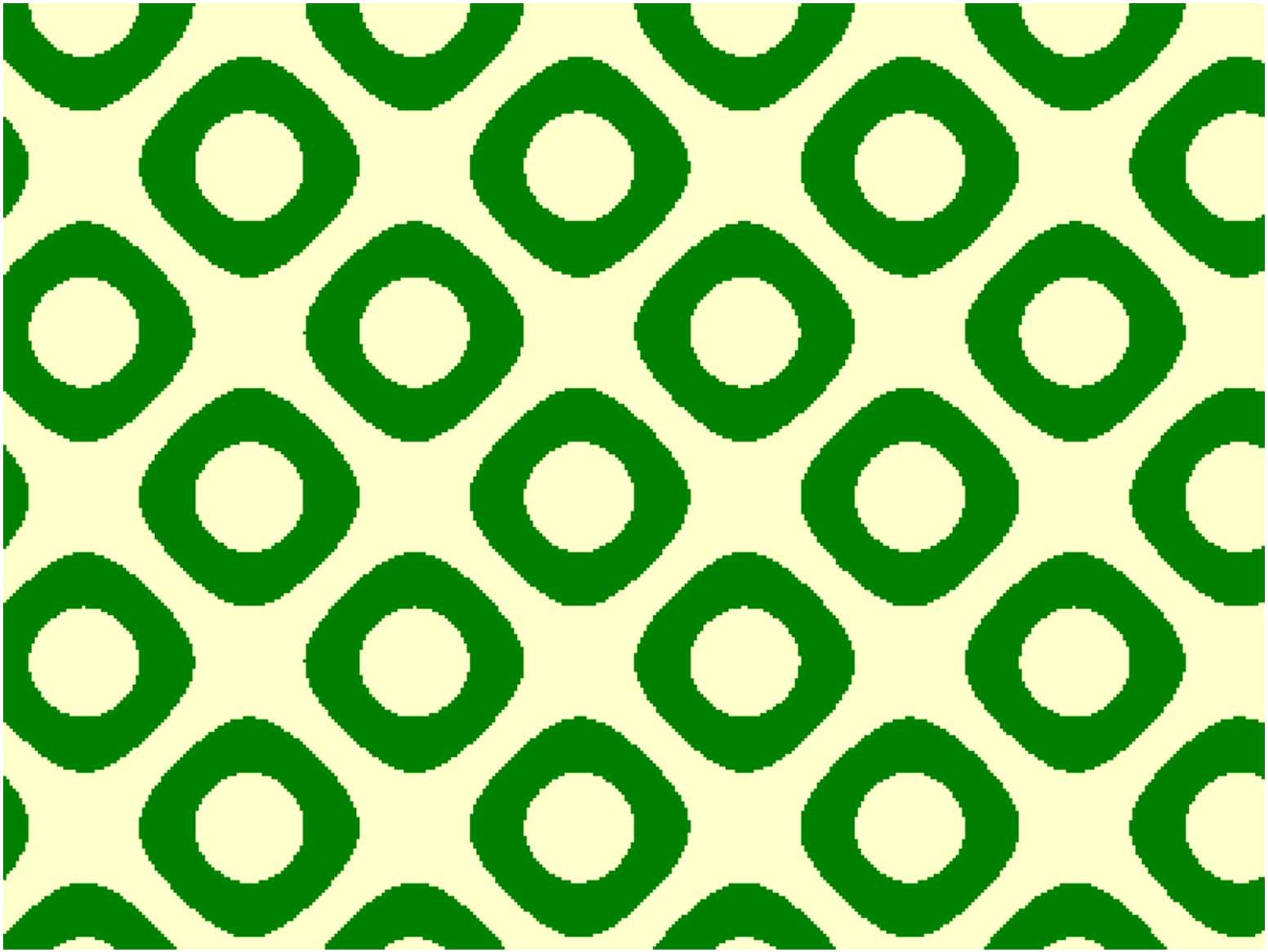


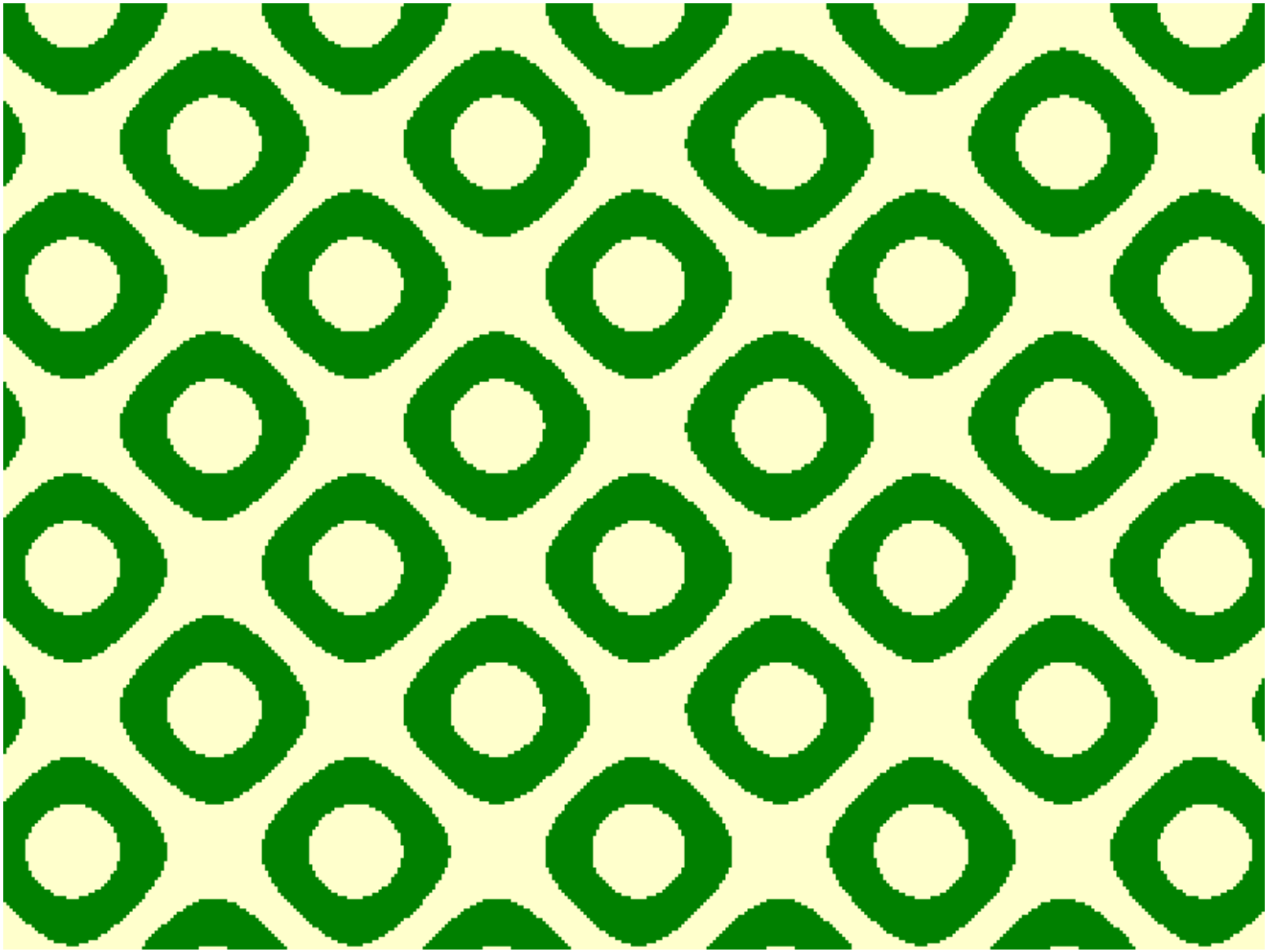


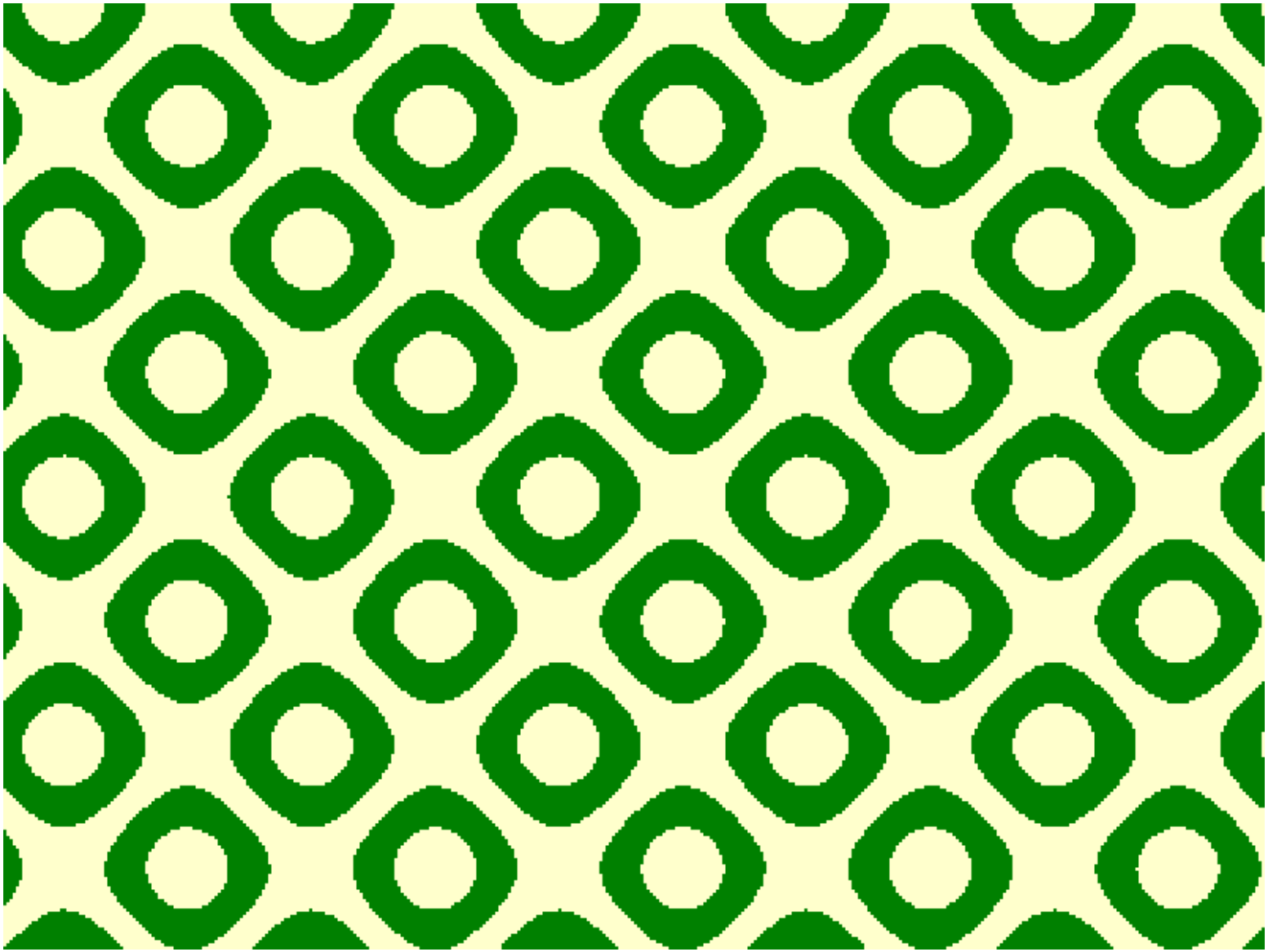


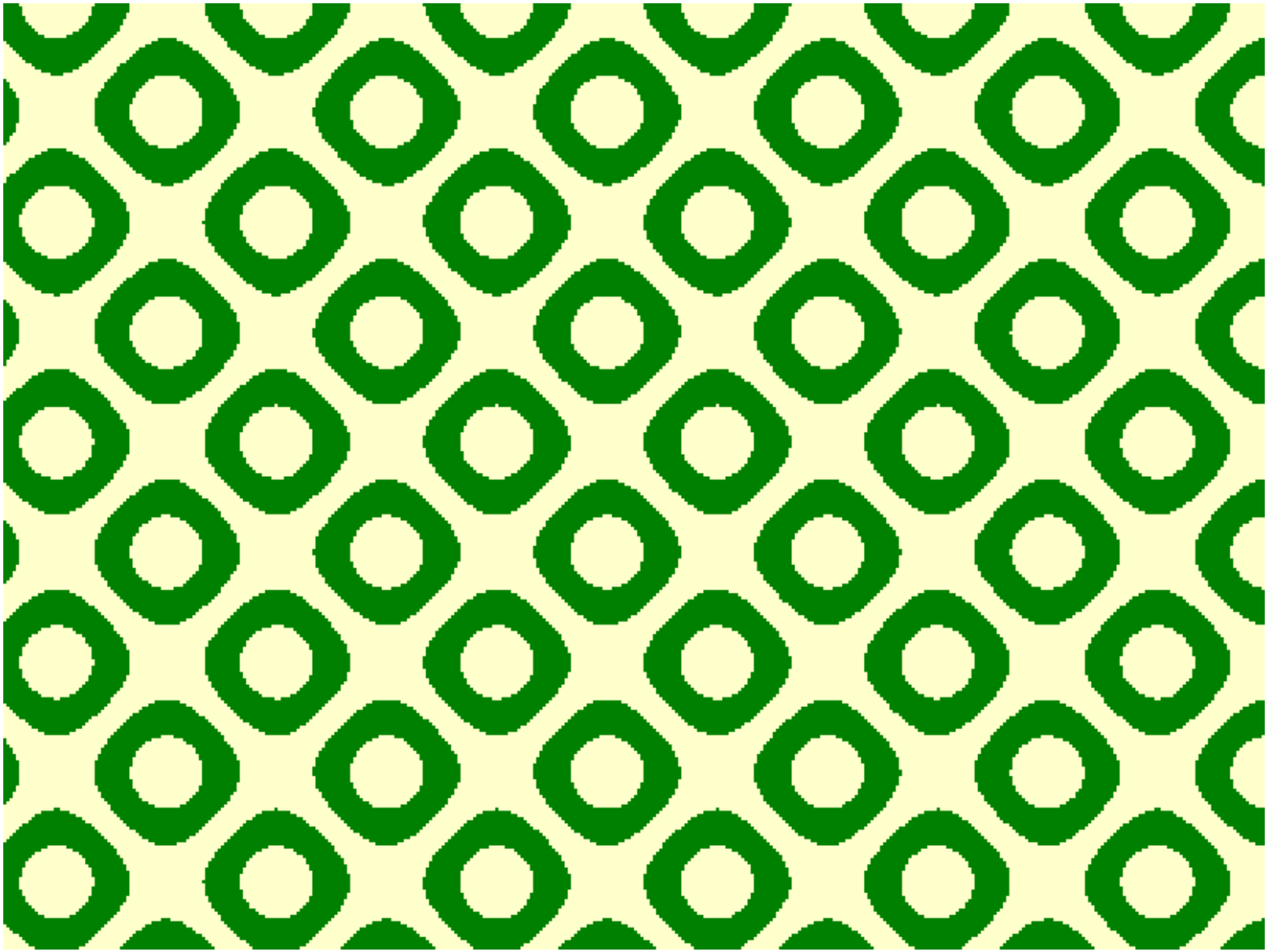


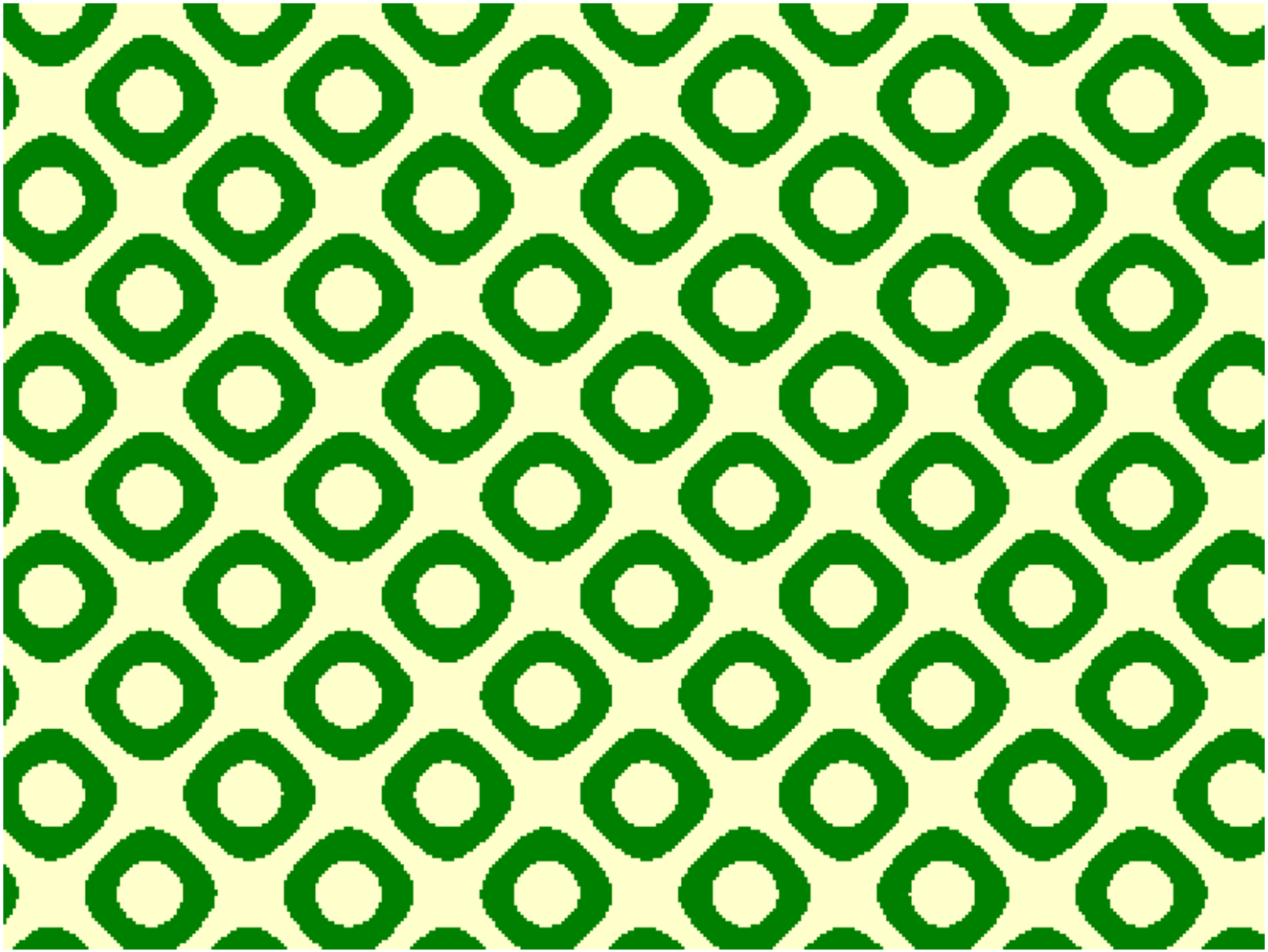


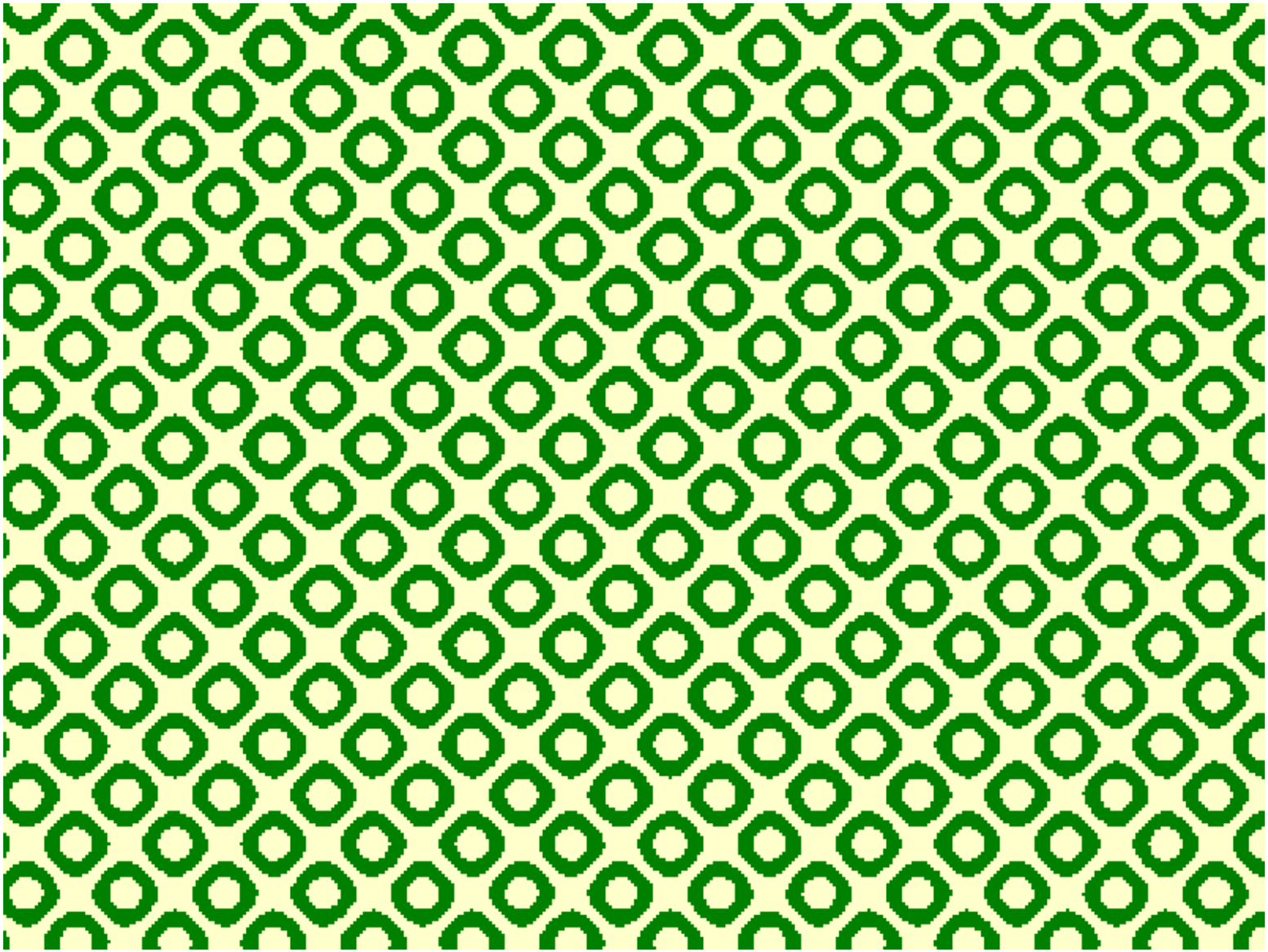


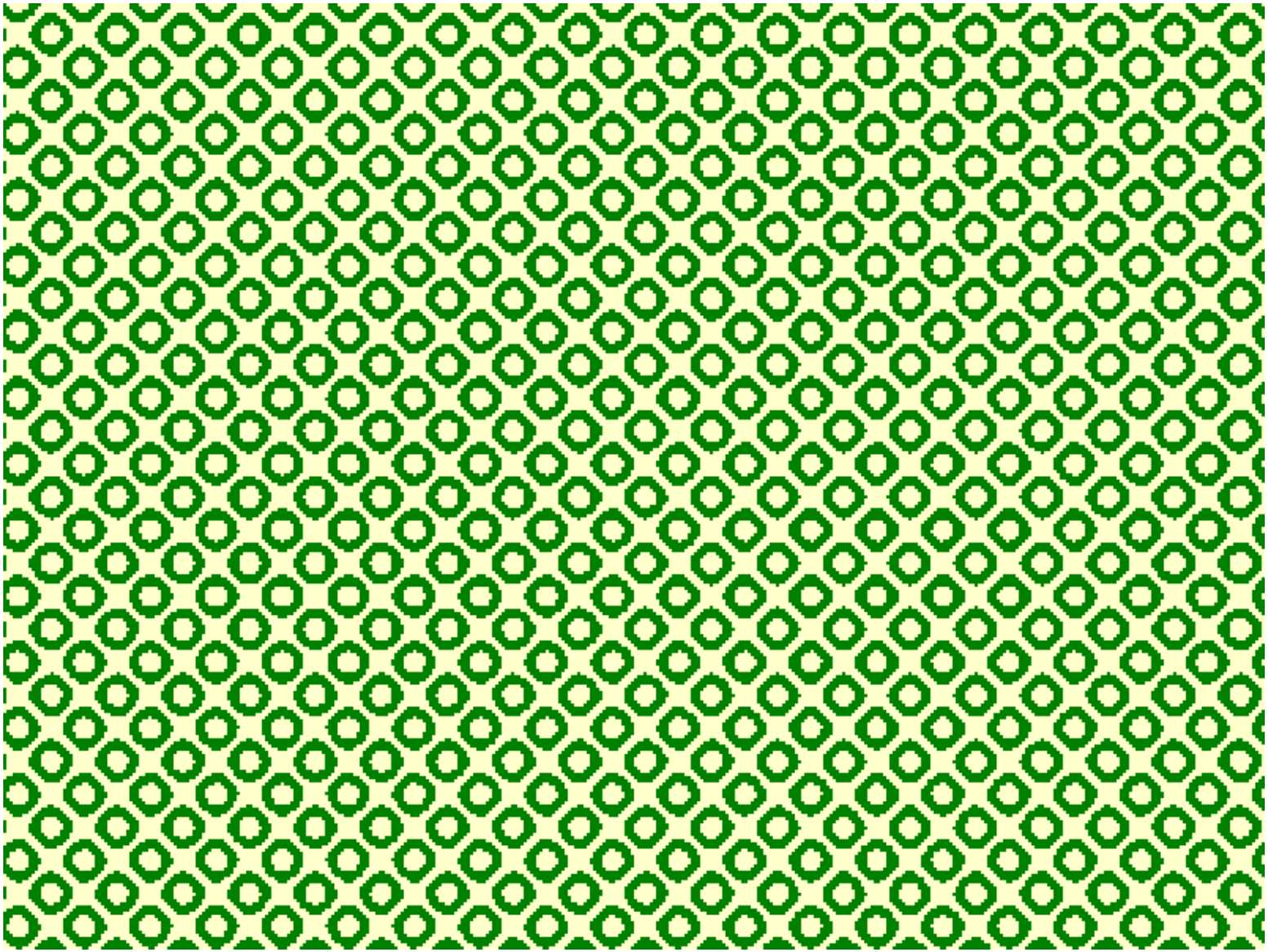


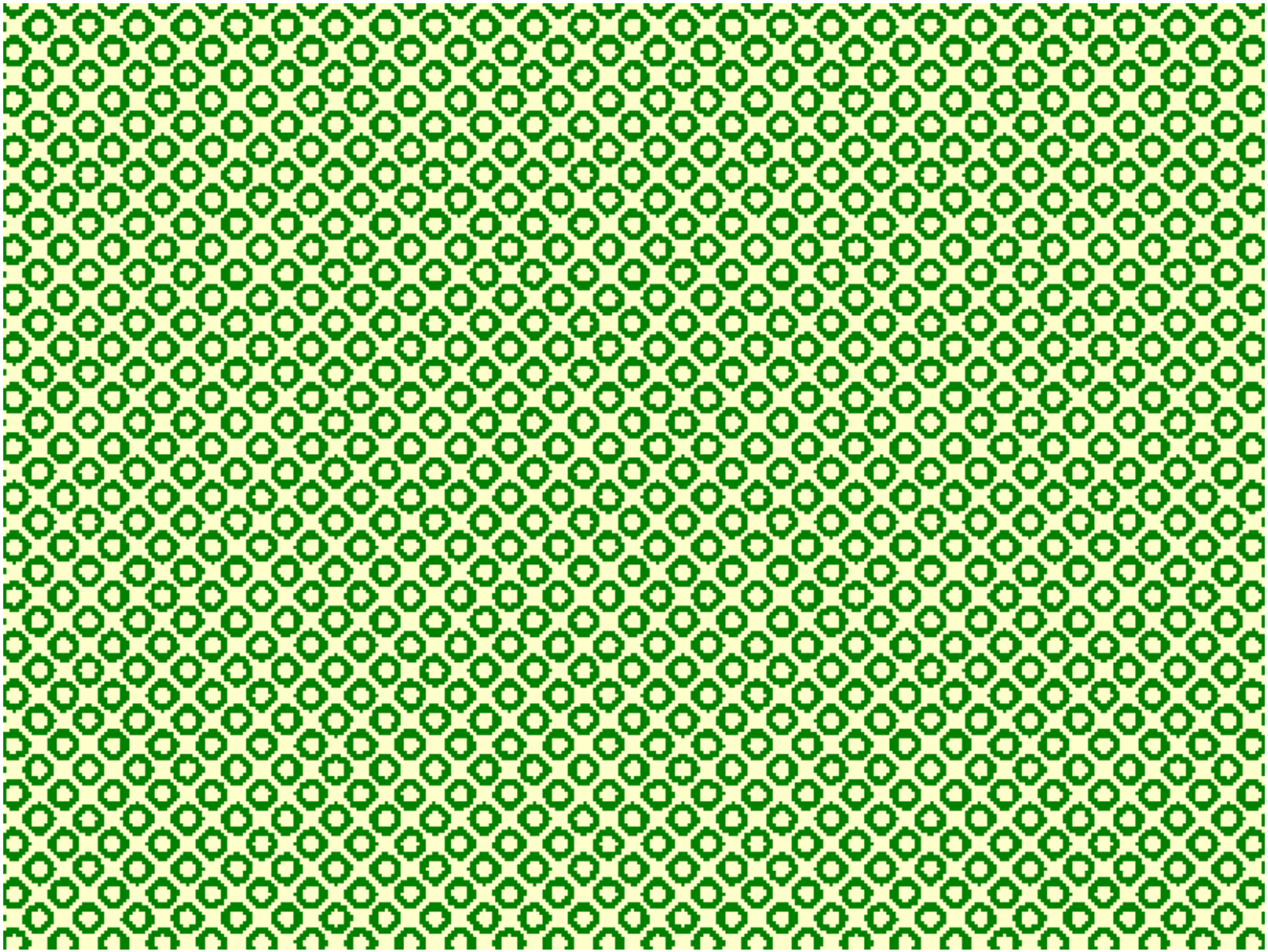


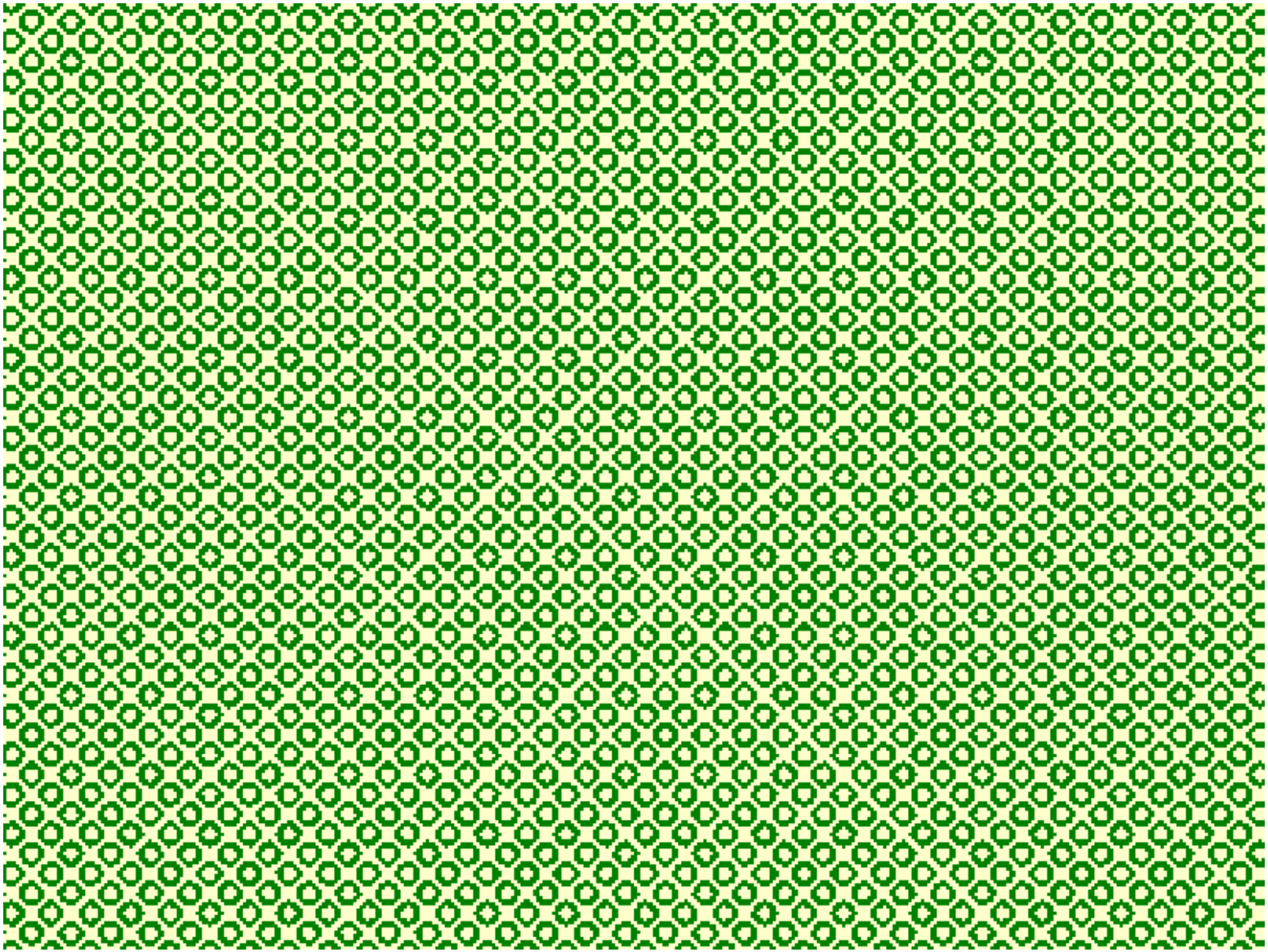


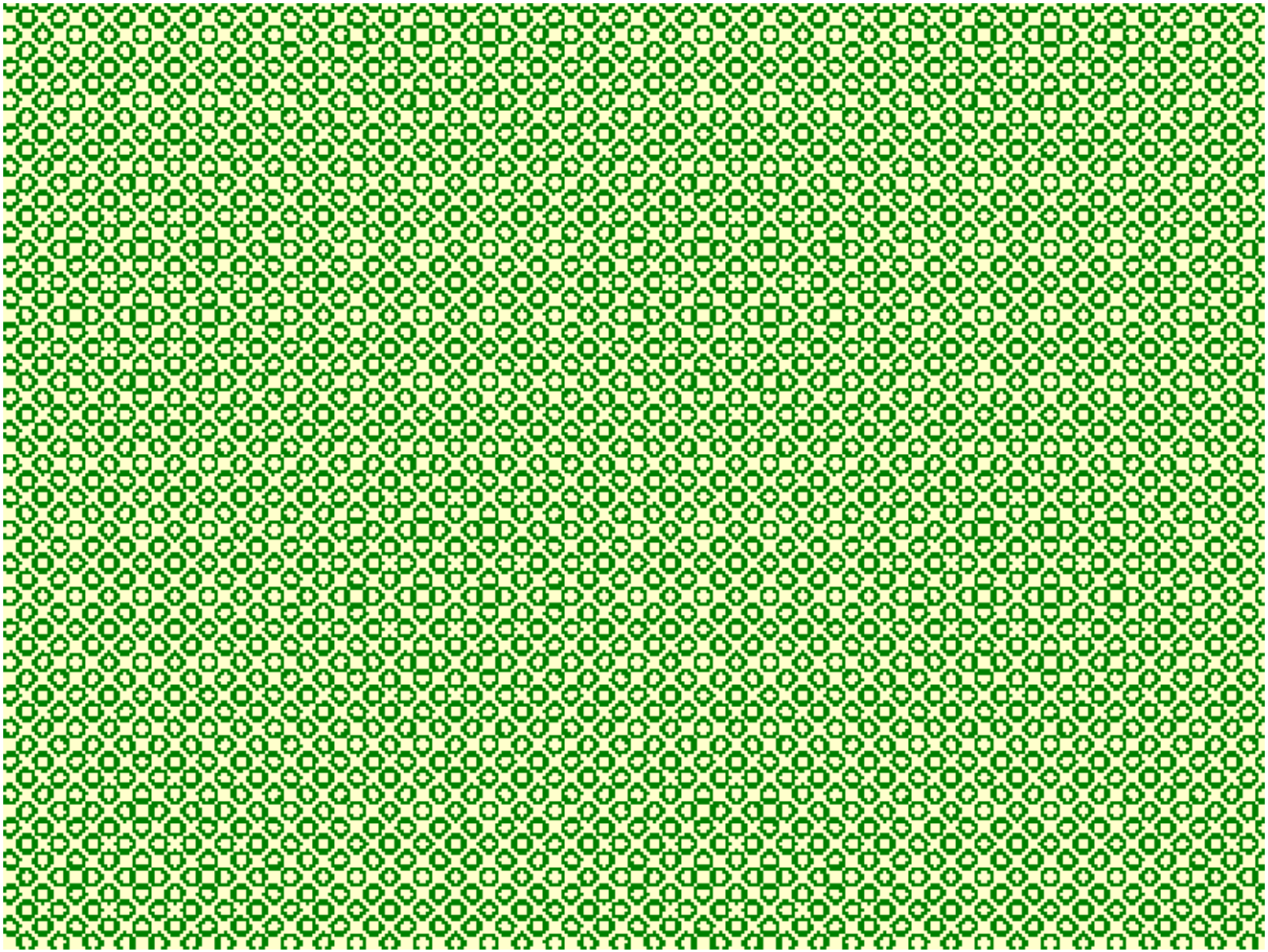












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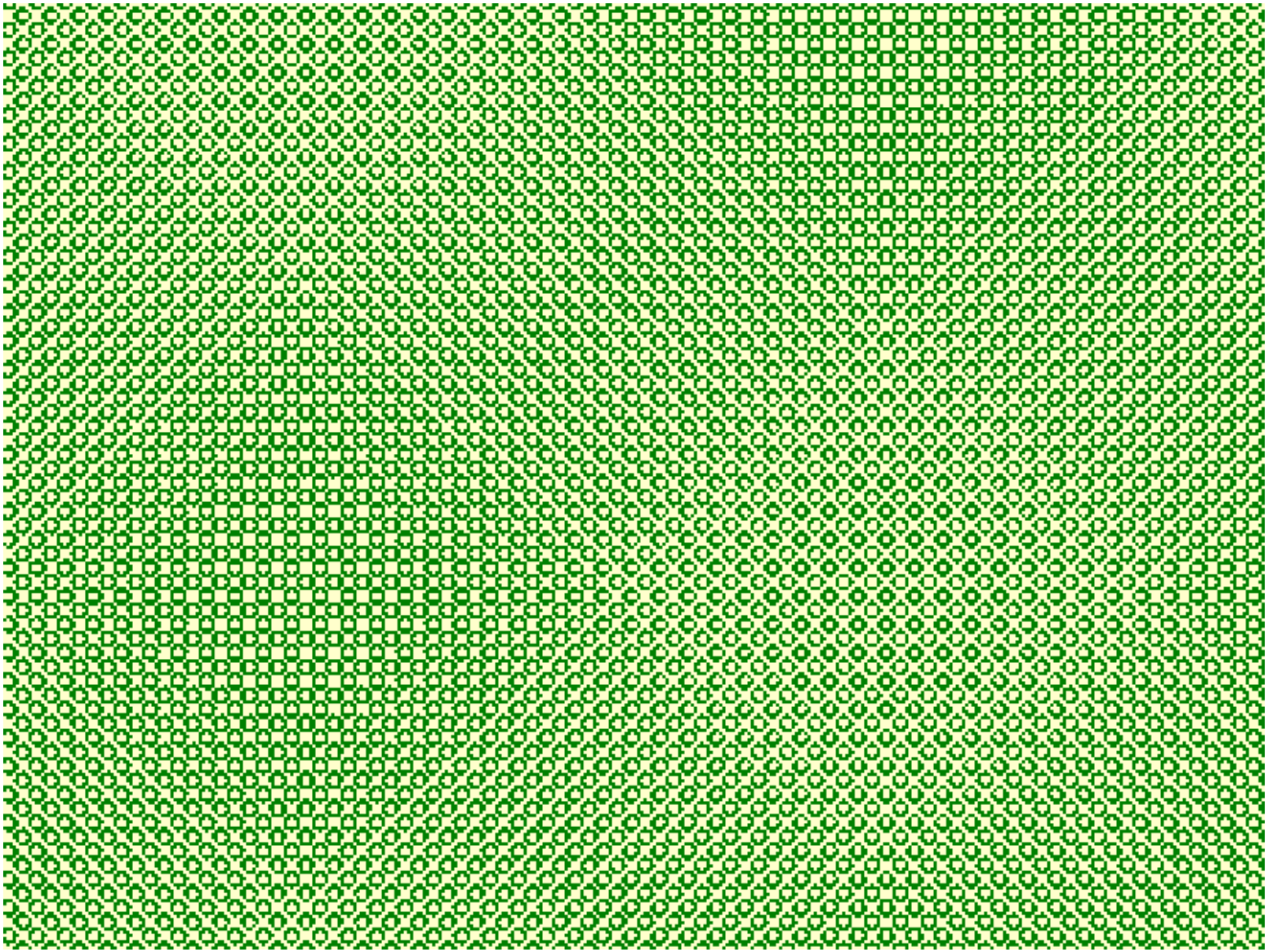

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