Using Python in a Numerical Methods Course

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About the class

- Mix of Math and CS students (counts as an elective for both)
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- We’re a smallish liberal arts school, graduating about 10 total math and CS majors a year
What is Python

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- You already have it if you have a Mac. Easy download on Windows.
Easy to show floating point gotchas:

```python
>>> .2 + .1
0.30000000000000004
```
Using the Python shell

Easy to show floating point gotchas:

```python
>>> .2 + .1
0.30000000000000004

>>> "{:.50f}".format(.1))
0.10000000000000000555111512312578270211815834045410
```
Using the Python shell

Easy to show floating point gotchas:

```python
>>> .2 + .1
0.30000000000000004

>>> "{:0.50f}".format(.1)
0.1000000000000000555111512312578270211815834045410

>>> x = (1.000000000000001 - 1) * 100000000000000
0.11102230246251565
```
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>>> .2 + .1
0.30000000000000004

>>> "{:50f}".format(.1))
0.10000000000000000555111512312578270211815834045410

>>> x = (1.000000000000001 - 1) * 10000000000000000
0.11102230246251565

>>> s = 0
>>> for i in range(1000000):
    s = s + .1
>>> s
999999.9998389754
```
```python
from math import cos
x = 2
for i in range(20):
    x = cos(x)
    print(x)
```

```
-0.4161468365471424
0.9146533258523714
0.6100652997429745
0.8196106080000903
0.6825058578960018
... 
0.7394108086387853
0.7388657151407354
0.7392329180769628
```
Python reads like pseudocode:

```python
def bisection(f, a, b, n):
    for i in range(n):
        m = (a + b) / 2
        if f(a)*f(m) < 0:
            b = m
        else:
            a = m
    return m
```
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```

Can use anonymous functions passed as arguments:

```python
bisection(lambda x:x*x-2, 0, 2, 20)
```
More Examples We Build in Class

```python
def secant(f, a, b, toler=1e-10):
    while f(b) != 0 and abs(b-a) > toler:
        a, b = b, b - f(b) * (b-a)/(f(b)-f(a))
    return b

def trapezoid(f, a, b, n):
    dx = (b-a) / n
    return dx/2 * (f(a) + f(b) +
    2*sum(f(a+i*dx) for i in range(1,n)))

def euler(f, y_start, t_start, t_end, h):
    t, y = t_start, y_start
    ans = [(t, y)]
    while t < t_end:
        y += h * f(t,y)
        t += h
    ans.append((t,y))
    return ans
```

from tkinter import *
from math import *

def plot():
    v, y = 3, 1
    h = .0005
    while True:
        v, y = v + h*f(y,v), y + h*v
        a = 100*sin(y)
        b = 100*cos(y)
        canvas.coords(line, 200, 200, 200+a, 200+b)
        canvas.coords(bob, 200+a-10, 200+b-10, 200+a+10, 200+b+10)
        canvas.update()

f = lambda y, v: -9.8/1*sin(y)-v/10
root = Tk()
canvas = Canvas(width=400, height=400, bg='white')
canvas.grid()
line = canvas.create_line(0, 0, 0, 0, fill='black')
bob = canvas.create_oval(0, 0, 0, 0, fill='black')
plot()
Homework usually consists of

(a) Conceptual questions
(b) Questions asking students to walk through an algorithm
(c) Choice of a few programming or trickier math problems
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For some problems, I give the option to use a programming language or Excel.

For other problems, I give the choice to do a programming problem or a mathematical problem.
Example Exercises

- Write a Python program that implements Simpson’s rule in a manner analogous to the program we wrote in class for the trapezoid rule.
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- Modify the Python code for adaptive quadrature to build up a list of all the points at which the algorithm evaluates the function while doing its thing.
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- Modify the Python code for adaptive quadrature to build up a list of all the points at which the algorithm evaluates the function while doing its thing.

- Modify the Adams-Bashforth two-step program on Moodle to implement the four-step method.
Use the Python `Decimal` class, the Java `BigDecimal` class, or another programming language’s decimal class to estimate the solution of $1 - 2x - x^5 = 0$ correct to 50 decimal places.
Tricky Exercises

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- Use one of the numerical methods we’ve learned to write a method in your favorite programming language called `my_sqrt` that computes $\sqrt{n}$ as accurately as the programming language’s own `sqrt` function (but without using the language’s `sqrt` or power functions).
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- Write a function that takes a sequence (a list), and returns a new sequence gotten by applying Aitken’s $\Delta^2$ method to it.
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Write a function that takes a sequence (a list), and returns a new sequence gotten by applying Aitken’s $\Delta^2$ method to it.

Implement the method for estimating $\ln x$ discussed on page 33 of the notes to accurately approximate the natural log of any positive number.
Write a function in a programming language that is given a list of data points, an x-value, and uses Newton’s divided differences to compute the value of the interpolating polynomial at x. It’s up to you how to specify how the data points are passed to your function, but make sure that it works for any number of data points.
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Write a program that returns the \( n \)th Chebychev polynomial, nicely formatted as a string. For instance, \( \text{cheb}(5) \) should return \( 16x^5-20x^3+5x \).
Write a function in a programming language that is given a list of data points, an $x$-value, and uses Newton’s divided differences to compute the value of the interpolating polynomial at $x$. It’s up to you how to specify how the data points are passed to your function, but make sure that it works for any number of data points.

Write a program that returns the $n$th Chebychev polynomial, nicely formatted as a string. For instance, `cheb(5)` should return $16x^5-20x^3+5x$.

Write a Python function called `mc_integrate` that estimates $\int_a^b \int_c^d f(x,y) \, dy \, dx$. Its arguments should include the function $f$; the bounds $a, b, c, \text{ and } d$; and the bounds of a box enclosing the region of integration; and an integer $n$ specifying how many iterations to do, having a default value of 10000.
Project Ideas

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- Write a program that simulates a physical system, like the pendulum programs we worked on in class. The program should graphically display the motion of the system.
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- Simulations (graphical traffic flow, spread of disease, …)

- Various non-programming ones involve writing a paper, comparing methods, …
Diff Eq Plotter I Wrote for Class
class Dual:
    def __init__(self, a, b):
        self.a = a
        self.b = b

    def __add__(self, y):
        if type(y) == int or type(y) == float:
            return Dual(self.a + y, self.b)
        else:
            return Dual(y.a + self.a, y.b + self.b)

    def __mul__(self, y):
        if type(y) == int or type(y) == float:
            return Dual(self.a * y, self.b * y)
        else:
            return Dual(y.a * self.a, y.b * self.a + y.a * self.b)

    def __pow__(self, e):
        return Dual(self.a ** e, self.b * e * self.a ** (e-1))

# various other operator definitions omitted...
```
def create_func(f, deriv):
    return lambda D: Dual(f(D.a), D.b*deriv(D.a))
        if type(D)==Dual else f(D)

def autoderiv(s, x):
    f = eval('lambda x: ' + s.replace("^", "**"))
    return (f(Dual(x,1)) - f(Dual(x,0))).b

sin = create_func(math.sin, math.cos)
exp = create_func(math.exp, math.exp)
# various other function defs omitted...

print(autoderiv("sin(x^2+exp(x+1))", 2))
```
Magic, continued

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```

This is called automatic differentiation.

Results are always accurate to within machine $\epsilon$!
Thanks!

See www.brianheinold.net these slides.