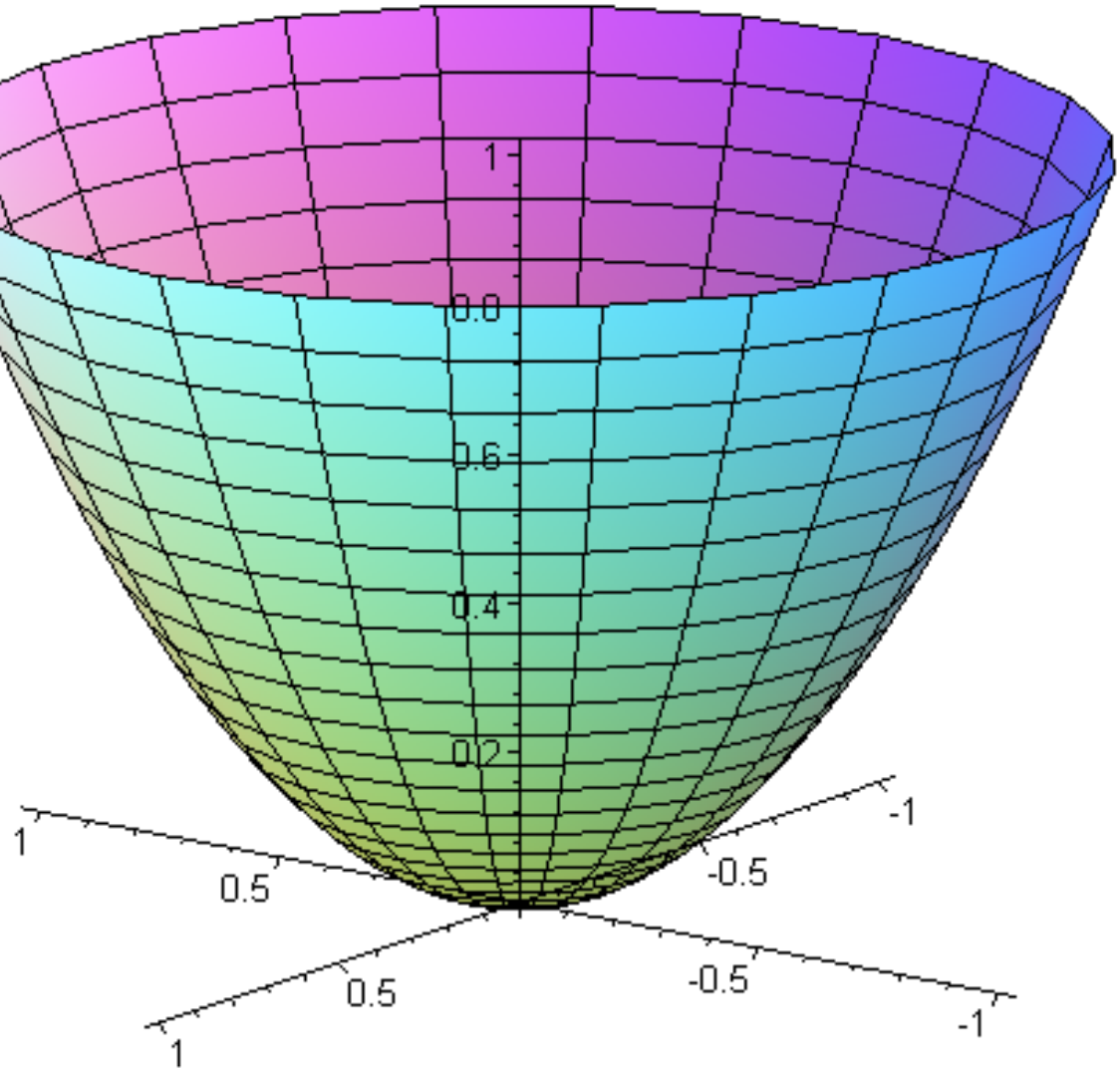
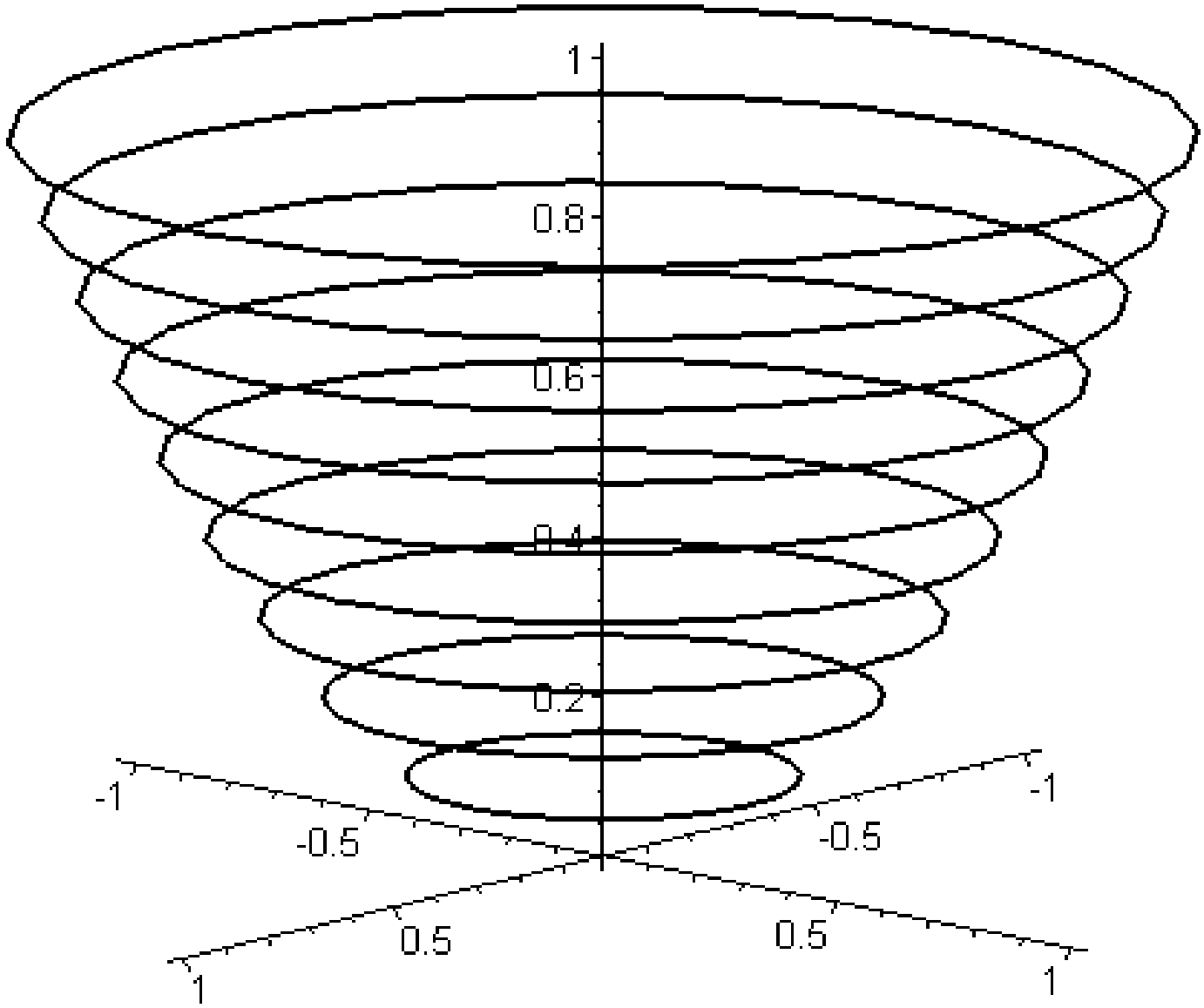
Functions of two variables from an artistic perspective

Brian Heinold

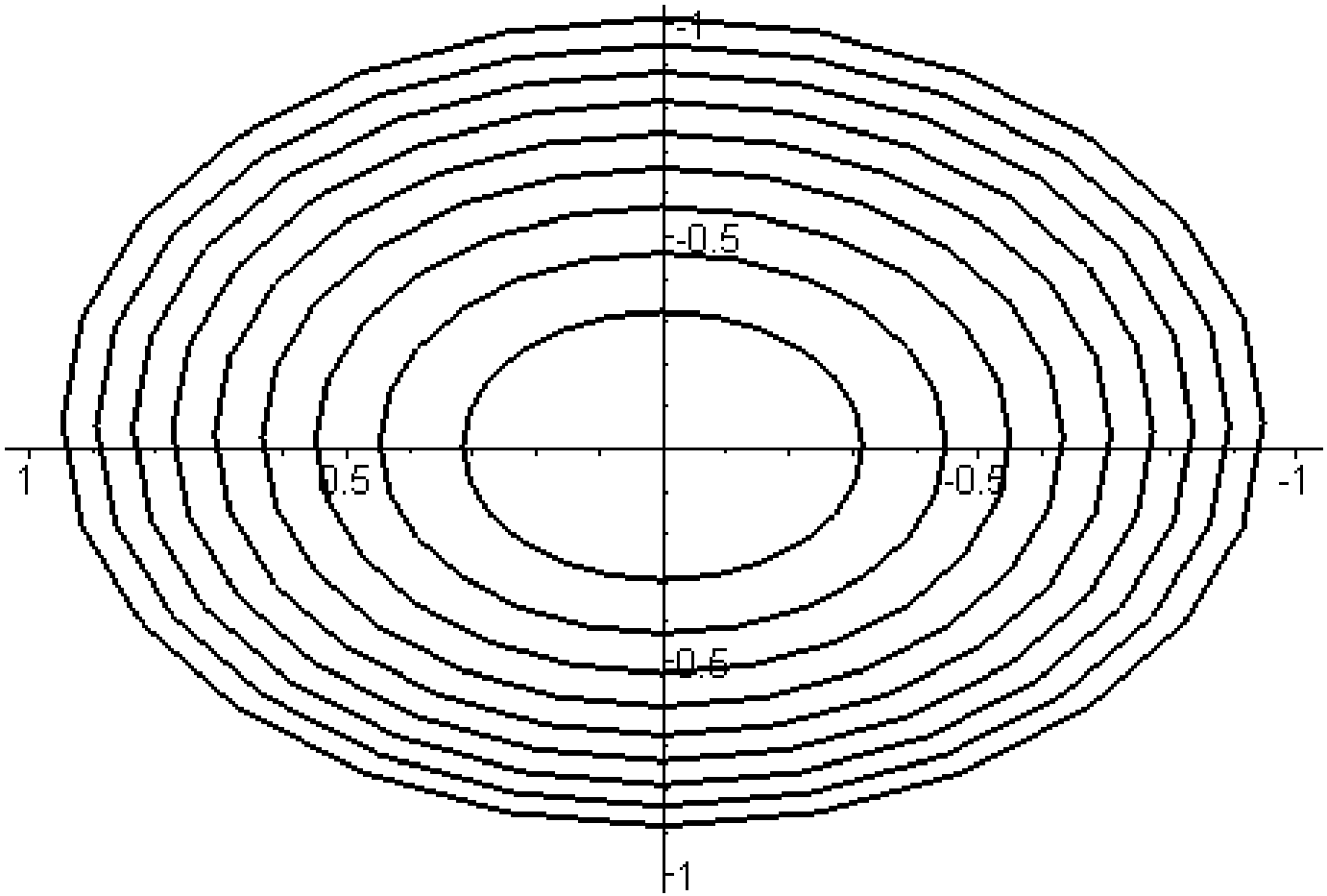
Ordinary plot of $f(x, y) = x^2 + y^2$



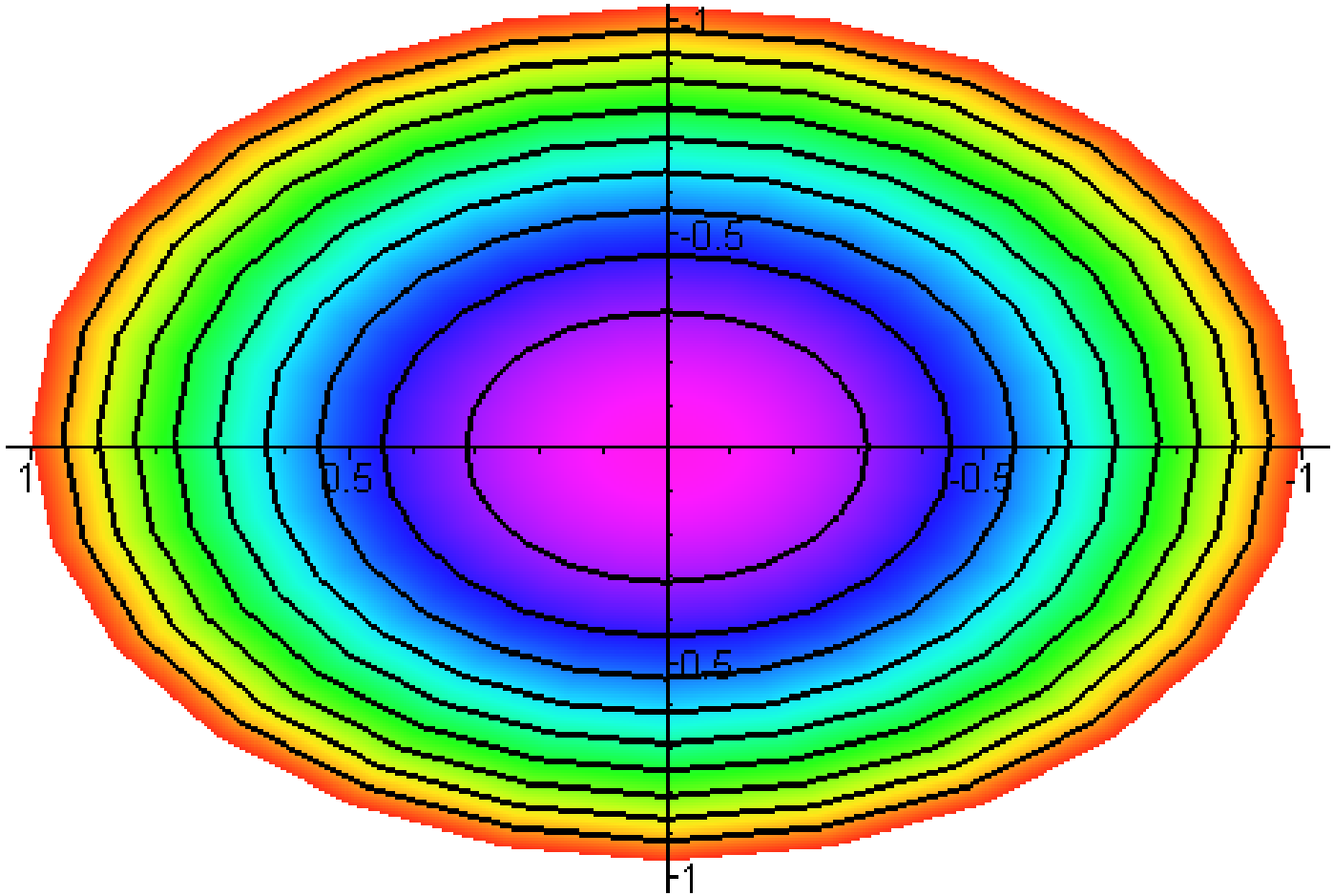
Contour plot of $f(x, y) = x^2 + y^2$



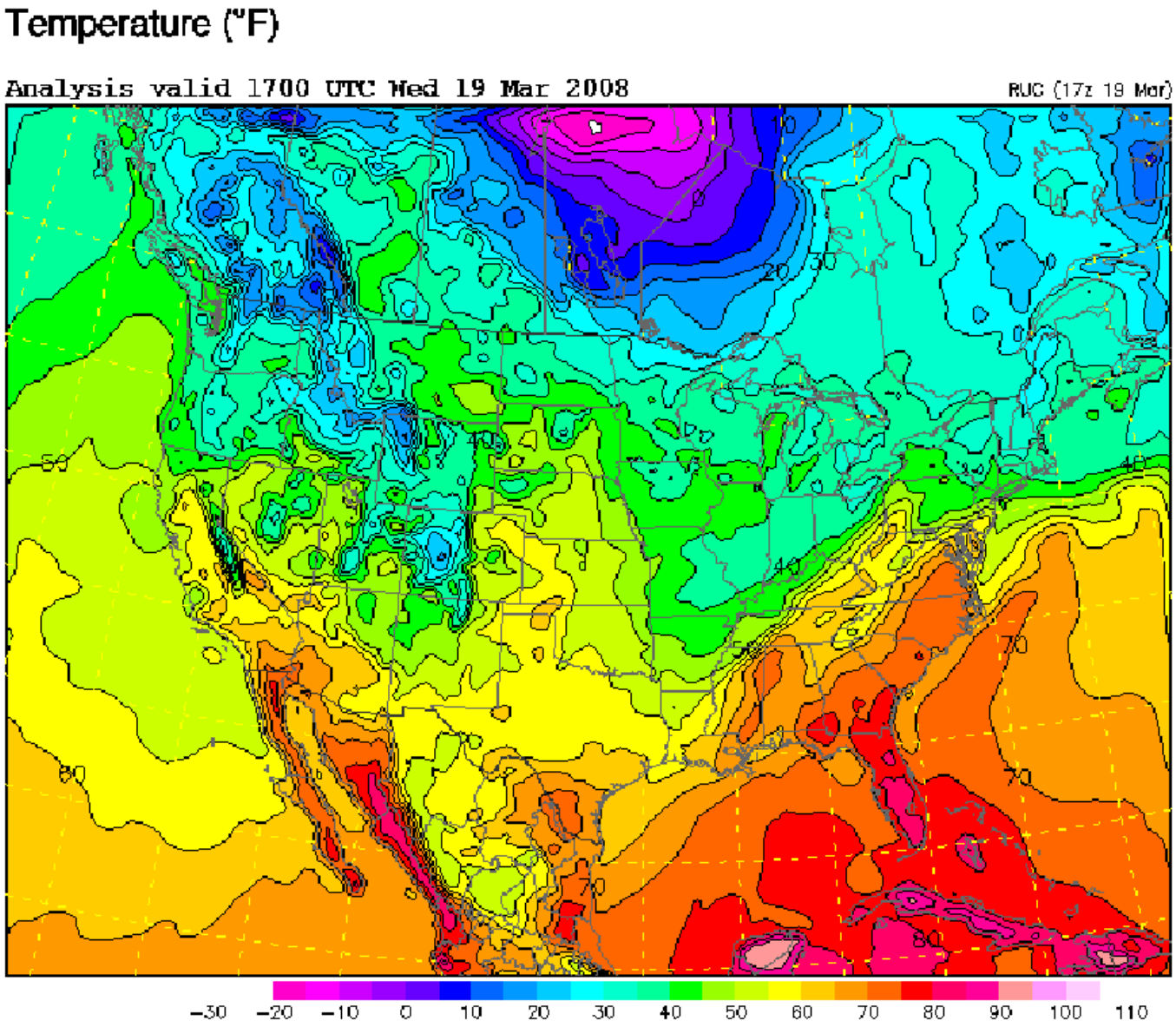
Looking down from above



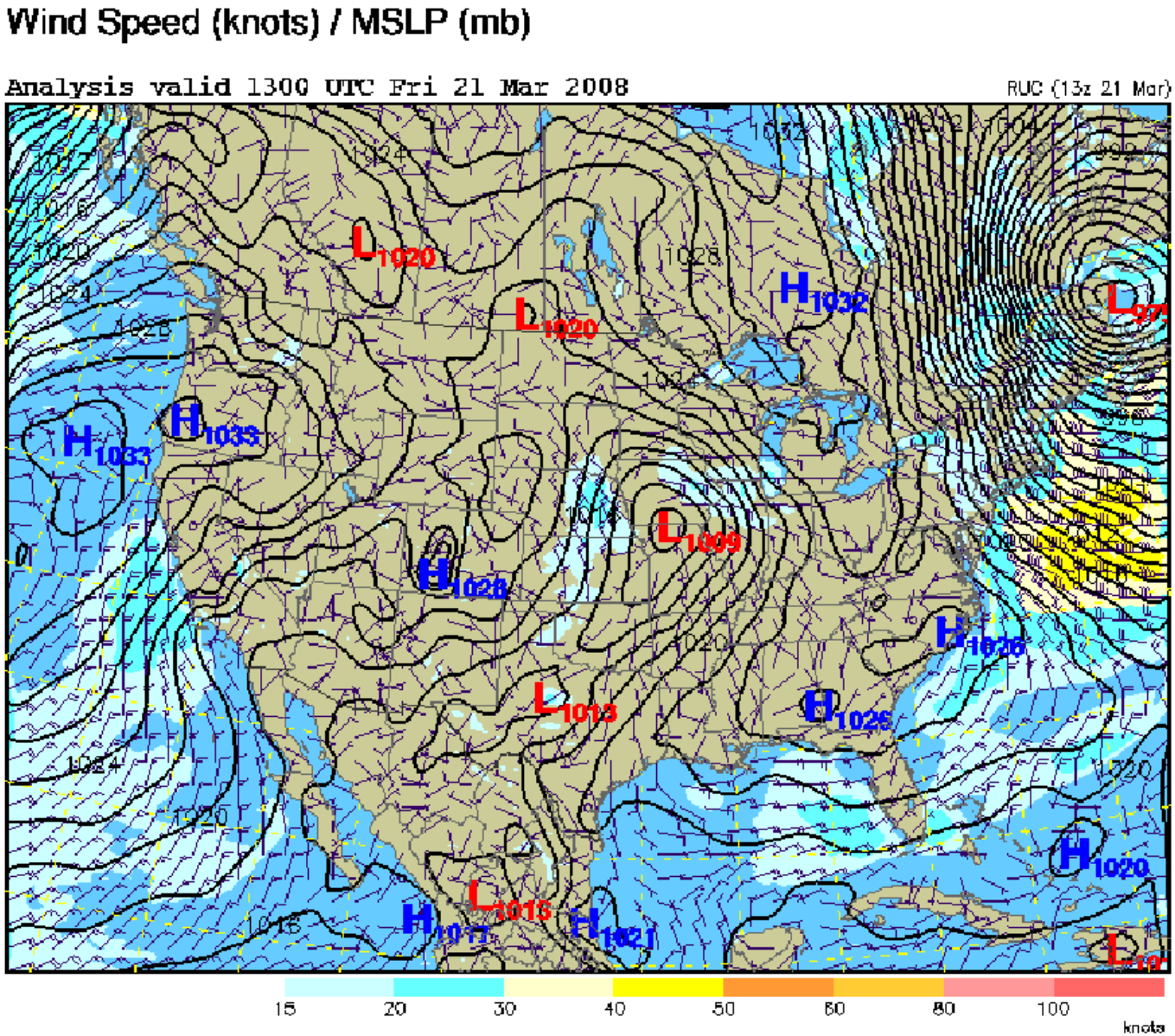
Colored contour map



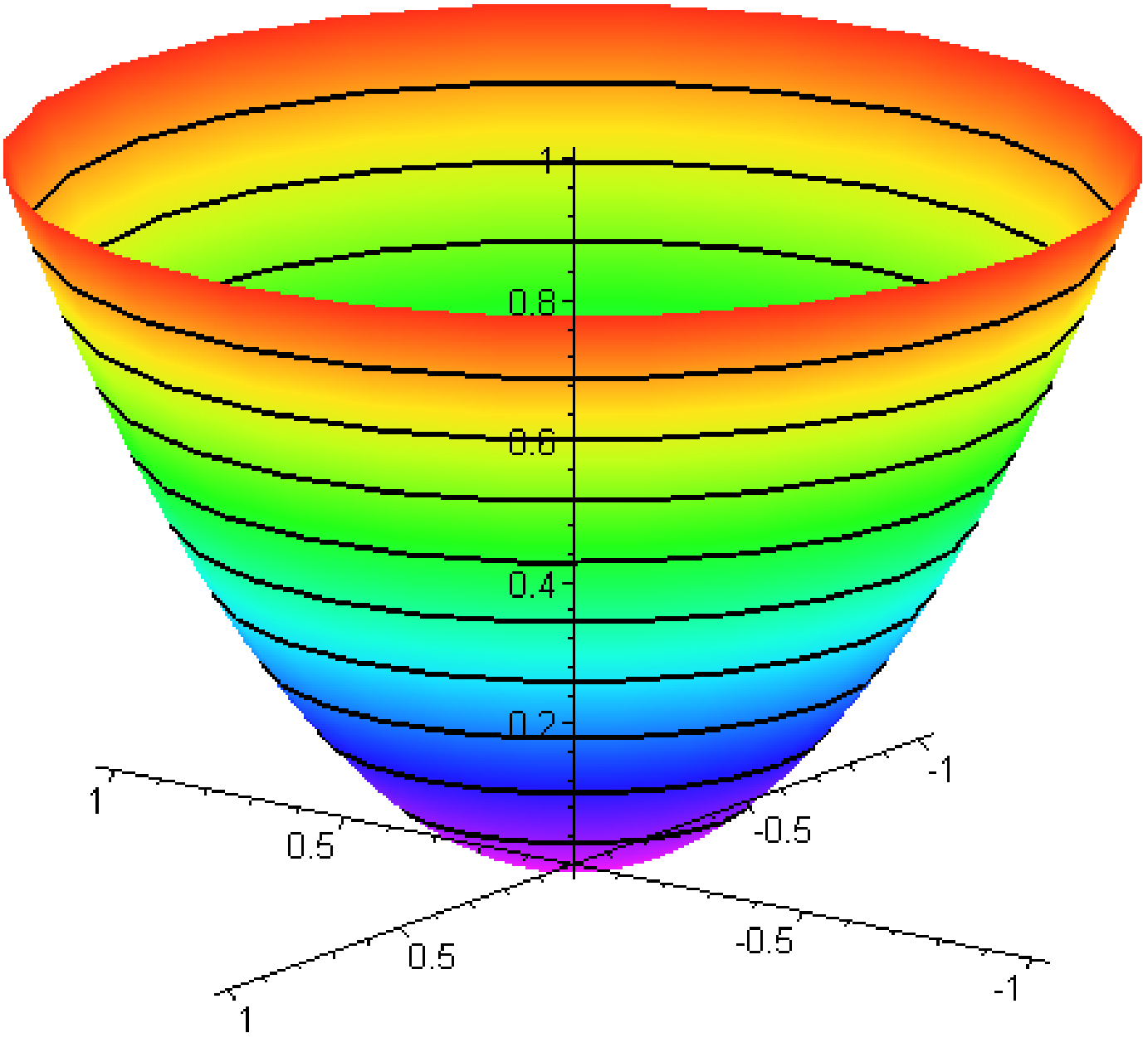
Temperatures maps are contour maps.



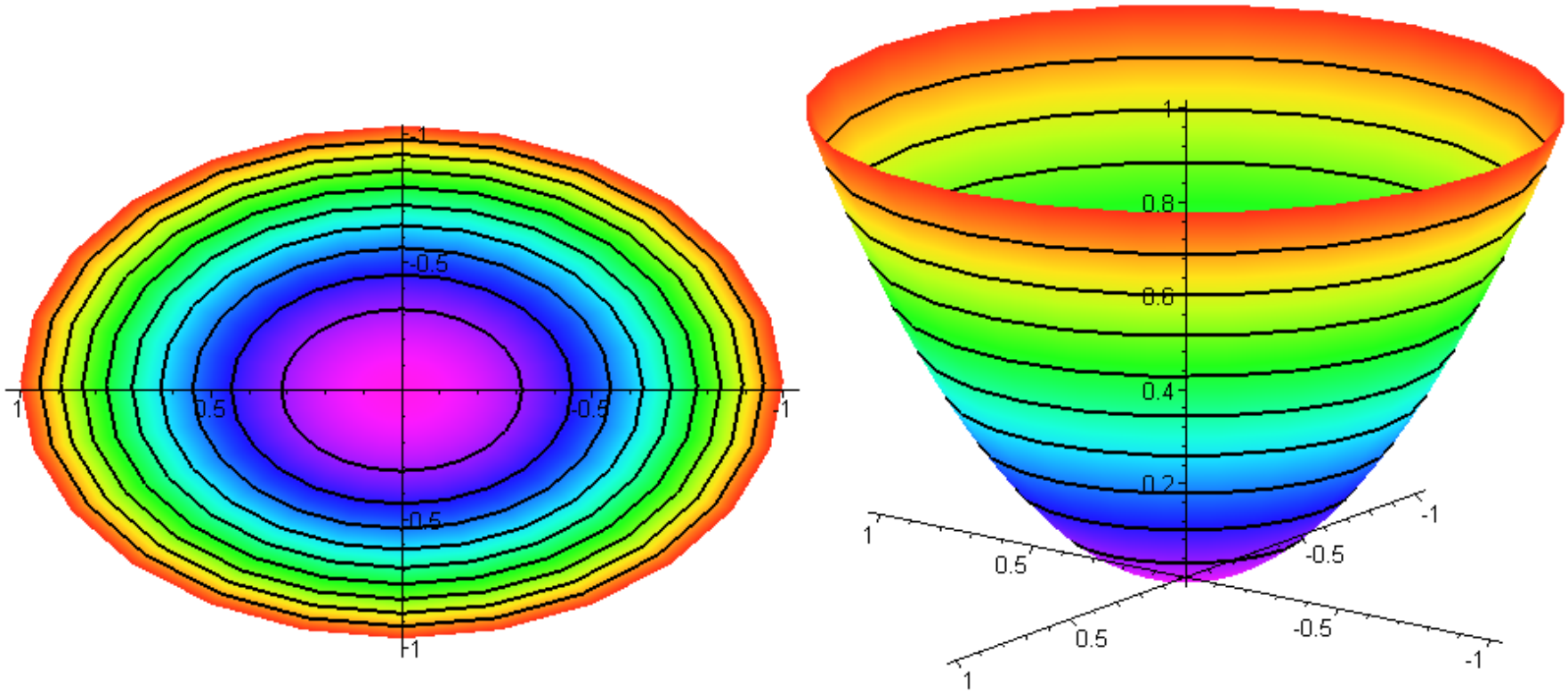
Pressure maps are, too.



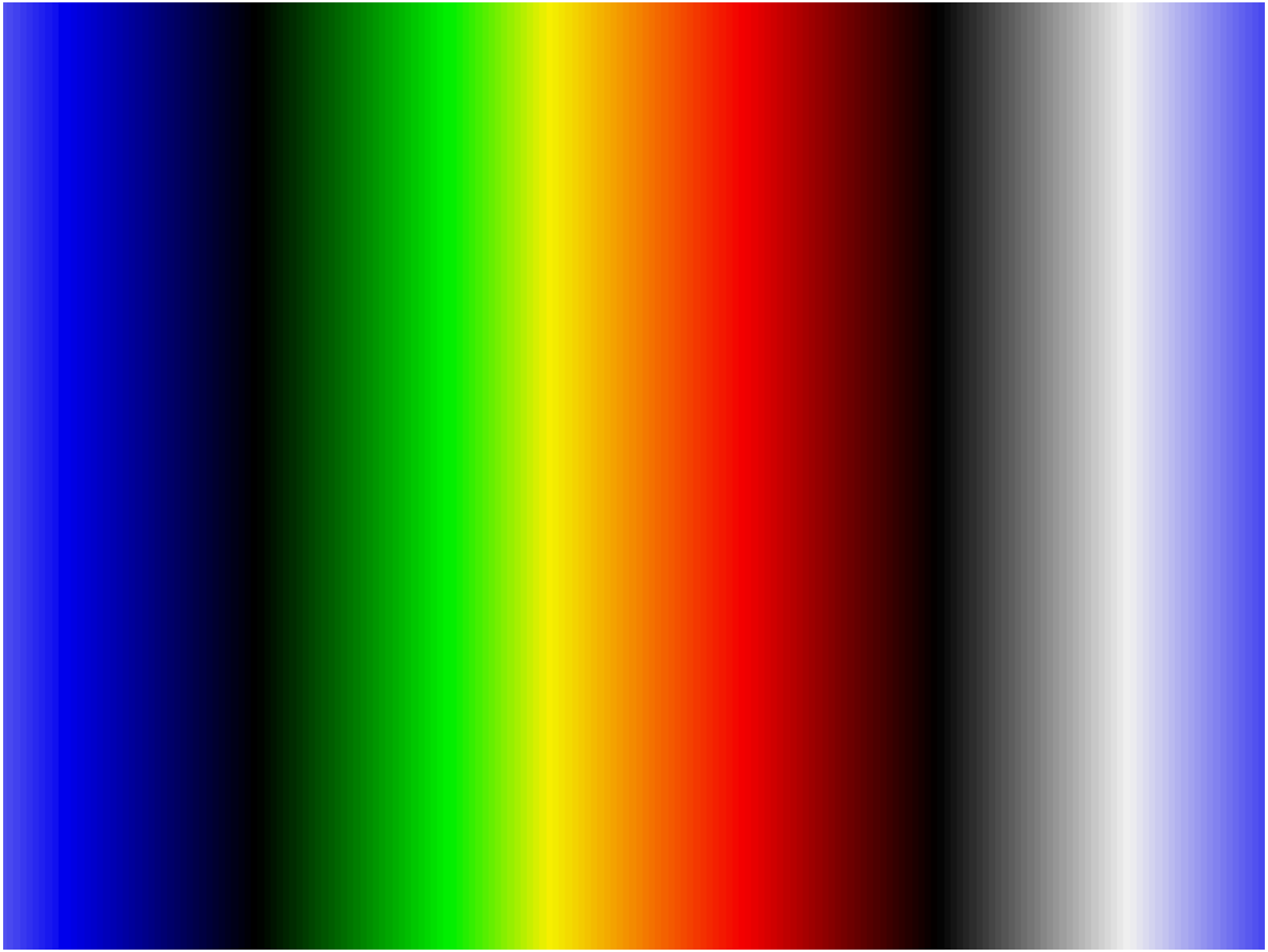
Back to $f(x, y) = x^2 + y^2$



Graph vs. contour plot

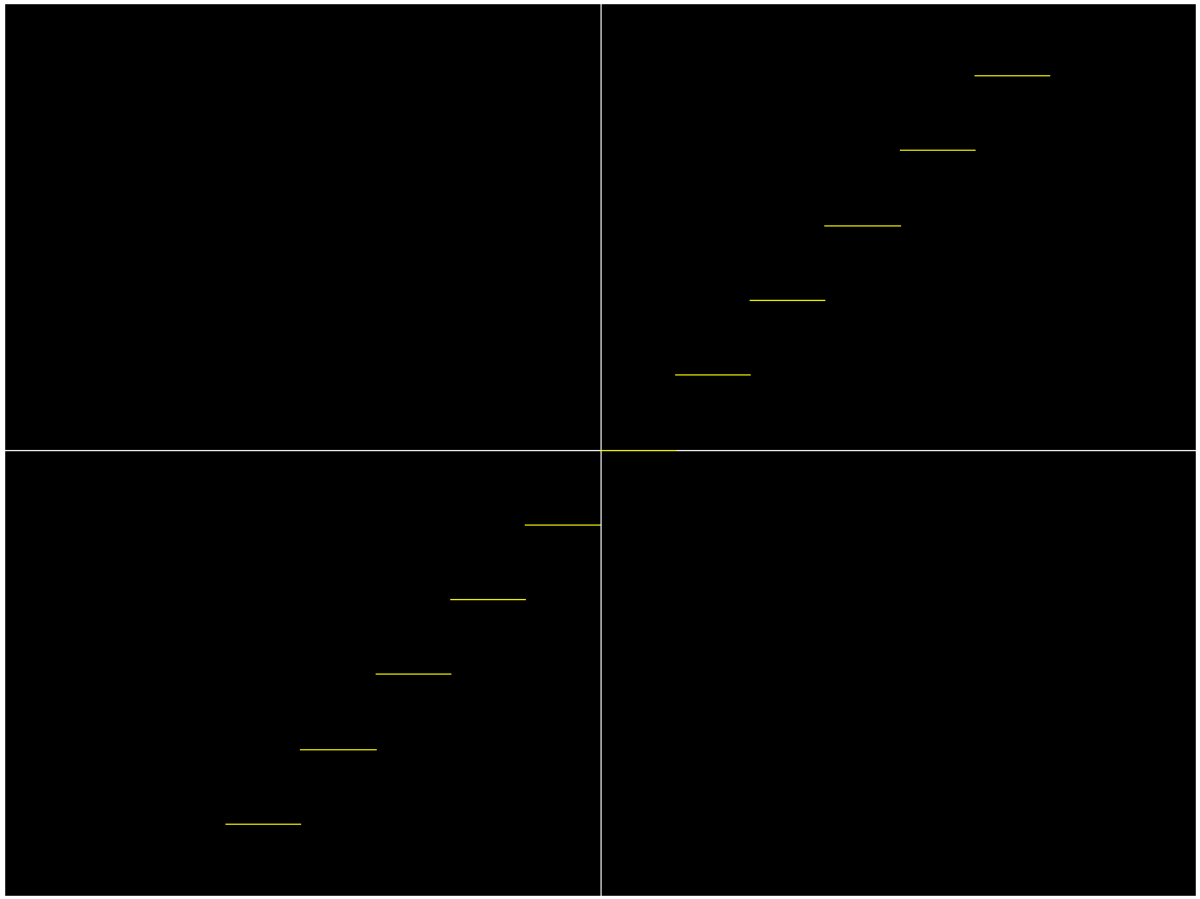


Color scheme



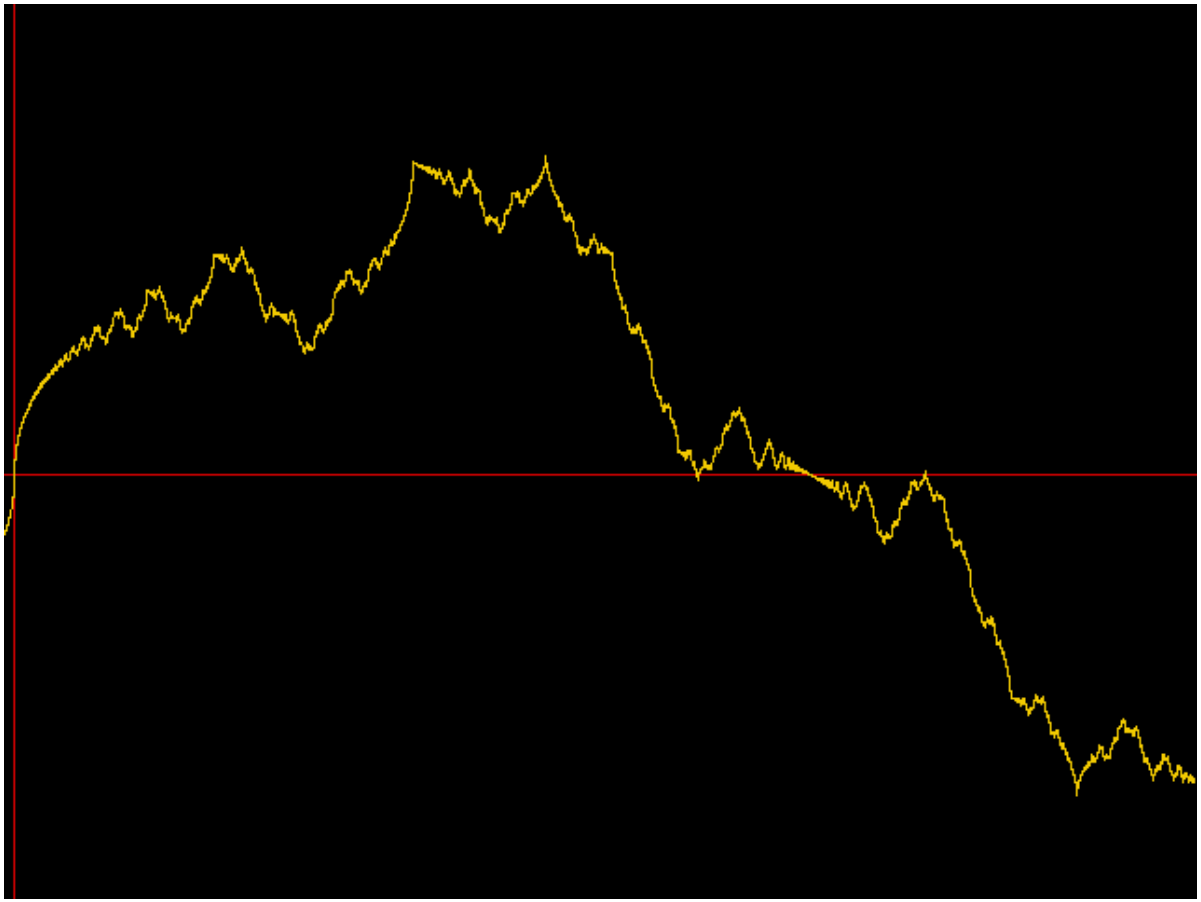
Floor function

The floor function returns the greatest integer less than or equal to the given number.
$\text{floor}(2.56) = 2$, $\text{floor}(3.98) = 3$, and $\text{floor}(-3.98) = -4$



Riemann function

$$\mathrm{riem}(x) = \sum_{n=1}^{\infty} \frac{\sin\left(n^2 x\right)}{n^2}$$



Riemann function

It is continuous everywhere, but differentiable almost nowhere.

The best we can do is approximate it:

$$\mathrm{riem}(x,k) = \sum_{n=1}^{k} \frac{\sin\left(n^2 x\right)}{n^2}$$

$$\mathrm{riemc}(x,k) = \sum_{n=1}^{k} \frac{\cos\left(n^2 x\right)}{n^2}$$

Mod function

It returns the remainder when a number is divided by another.

$20 \bmod 7 = 6$ because the remainder when 20 is divided by 7 is 6.

It is represented by % in the formulas.

Bitwise AND function

We represent it by the symbol &

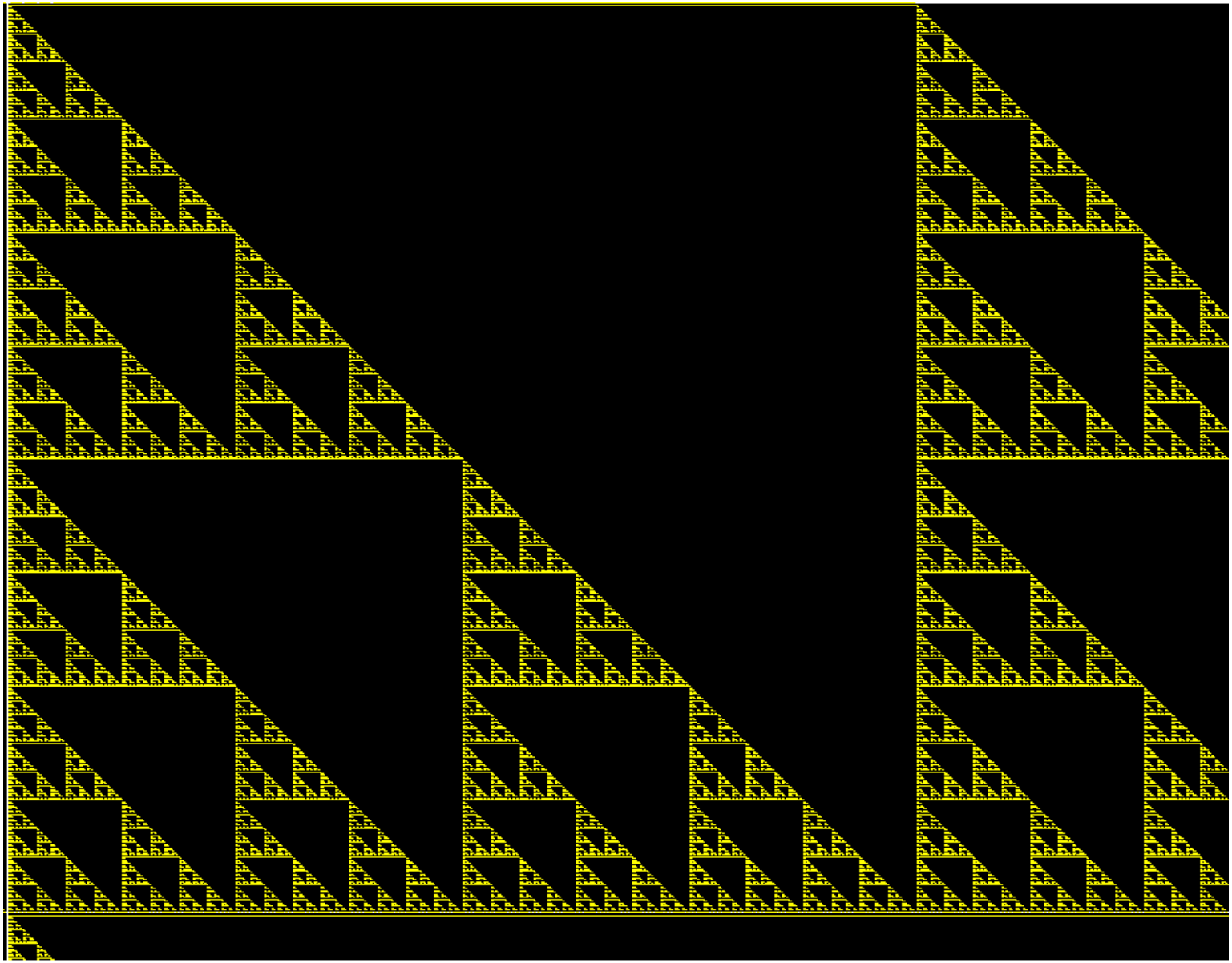
1=True, 0 = False

$1 \,\&\, 1 = 1, \qquad 1 \,\&\, 0 = 0, \qquad 0 \,\&\, 1 = 0, \qquad 0 \,\&\, 0 = 0$

To compute 11&14:

1. Convert each to binary $\rightarrow 1011 \,\&\, 1110$
2. AND the corresponding digits $\rightarrow 1010$
3. Convert back to decimal $\rightarrow 10$

Plot of $x \,\&\, y = 0$



Bitwise NOT function

It is the logical not function, represented by !.

$!1 = 0$ and $!0 = 1$

Extend this to $\mathbb{R}$ by defining $!x$ to equal 1 if $-1 < x < 1$ and 0 otherwise.