Dates

Brian Heinold

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Not 2100, 2200, or 2300, but 2400 is.

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Corresponds to a year of 365.25 days

More on the Julian calendar

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This was messing with Easter.

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Eastern Orthodox Christmas – Jan. 7 (still on Julian calendar).

Still not perfect

- Julian: 365.25 days/year
- Gregorian:

Every 4 years:		365.25
Not every 100:	−	.01
Yes every 400:	+	.0025

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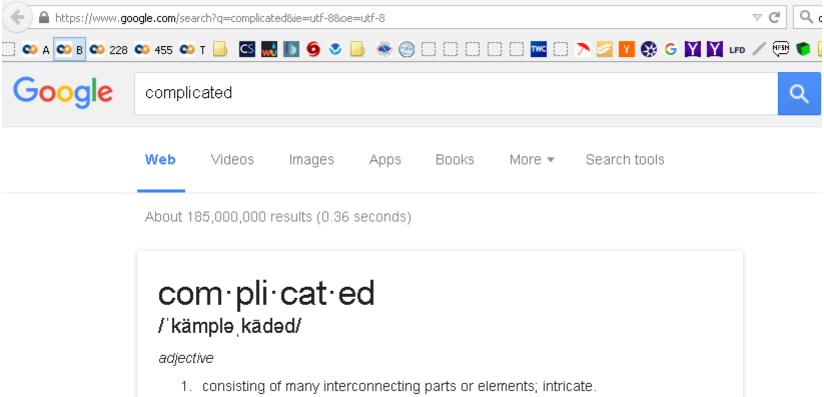
- Off by about 26 seconds each year.
- One fix: years divisible by 4000 aren't leap years → 365.24225.
- Another: If year mod 900 is 200 or 600 then it's a leap year → 364.2422.

This is where things get complicated



Things we think we know but don't

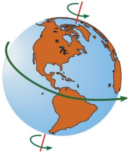
What is a day?

What is a second?

How long is a day?

What is a day?

One full rotation of the earth.



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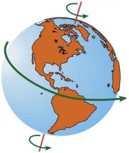

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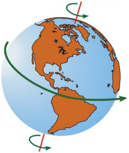

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Solar day: Put a vertical pole in ground. Solar noon is when shadow is perfectly north/south. Time between one solar noon and the next solar noon is one solar day.

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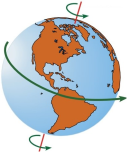

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Problems:

- Depends on where on earth you are
- Depends on the time of year, varying by nearly 50 seconds because earth's orbit is an ellipse, not a circle

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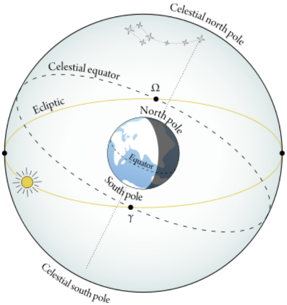

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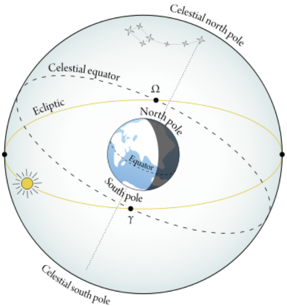


Even these are complicated by something called *precession of equinoxes* (earth wobbles over 1000s of years like a spinning top in its orbit)

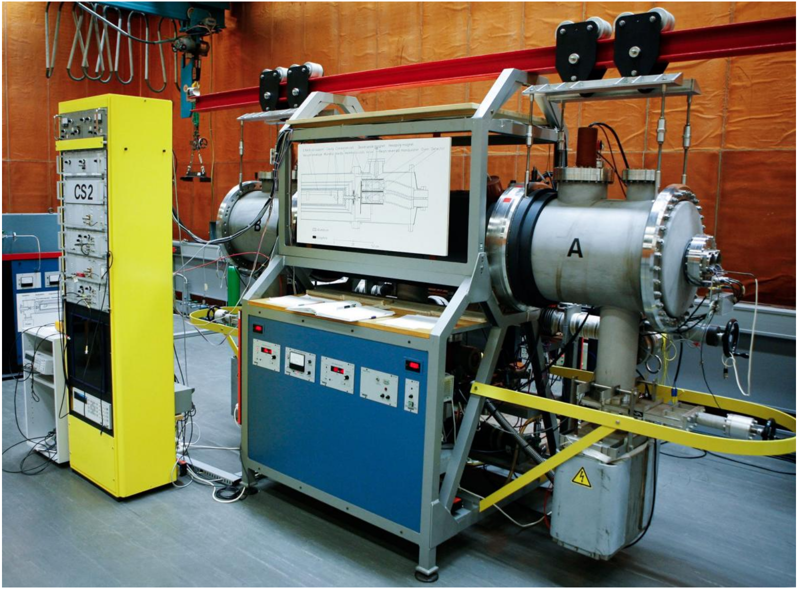
It took several tries to get things right

From Wikipedia:

About 140 AD, Ptolemy, the Alexandrian astronomer, sexagesimally subdivided both the mean solar day and the true solar day to at least six places after the sexagesimal point, and he used simple fractions of both the equinoctial hour and the seasonal hour, none of which resemble the modern second.[3] Muslim scholars, including al-Biruni in 1000, subdivided the mean solar day into 24 equinoctial hours, each of which was subdivided sexagesimally, that is into the units of minute, second, third, fourth and fifth, creating the modern second as $\frac{1}{60}$ of $\frac{1}{60}$ of $\frac{1}{24}$ = $\frac{1}{86400}$ of the mean solar day in the process.[4] With this definition, the second was proposed in 1874 as the base unit of time in the CGS system of units.[5] Soon afterwards Simon Newcomb and others discovered that Earth's rotation period varied irregularly,[6] so in 1952, the International Astronomical Union (IAU) defined the second as a fraction of the sidereal year. Because the tropical year was considered more fundamental than the sidereal year, in 1955, the IAU redefined the second as the fraction $\frac{1}{31,556,925.9747}$ of the 1900.0 mean tropical year, which was adopted in 1956 by the International Committee for Weights and Measures and in 1960 by the General Conference on Weights and Measures, becoming a part of the International System of Units (SI).[7]

Eventually, this definition too was found to be inadequate for precise time measurements, so in 1967, the SI second was again redefined as 9,192,631,770 periods of the radiation emitted by a caesium-133 atom in the transition between the two hyperfine levels of its ground state.[8] That value agreed to 1 part in 10^{10} with the astronomical (ephemeris) second then in use.[9] It was also close to $\frac{1}{86400}$ of the mean solar day as averaged between years 1750 and 1892.

An atomic clock



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In another billion years, one day will be 45 *days* long, which will also be the length of the month.

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Last one was this past June 30. Before that was June 30, 2012.

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Without leap seconds, after several centuries those seconds will add up to minutes and eventually hours.

Other factors affect the earth's rotation

Changes in pressure and winds

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All of these mean that leap seconds are needed irregularly.

Date of Easter

$$C = \text{century } (1900\text{'s} \to C = 19)$$
$$Y = \text{year (all four digits)}$$
$$m = (15 + C - \lfloor \tfrac{C}{4} \rfloor - \lfloor \tfrac{8C+13}{25} \rfloor) \bmod 30$$
$$n = (4 + C - \lfloor \tfrac{C}{4} \rfloor) \bmod 7$$
$$a = Y \bmod 4$$
$$b = Y \bmod 7$$
$$c = Y \bmod 19$$
$$d = (19c + m) \bmod 30$$
$$e = (2a + 4b + 6d + n) \bmod 7$$

Easter is either March $(22 + d + e)$ or April $(d + e - 9)$. There is an exception if $d = 29$ and $e = 6$. In this case, Easter falls one week earlier on April 19. There is another exception if $d = 28$, $e = 6$, and $m = 2, 5, 10, 13, 16, 21, 24$, or 39. In this case, Easter falls one week earlier on April 18.

Day of the week calculations

First, memorize this table of month codes:

Jan	6	Jul	5
Feb	2	Aug	1
Mar	2	Sep	4
Apr	5	Oct	6
May	0	Nov	2
Jun	3	Dec	4

Note: Jan is 5 and Feb is 1 in leap years

Year codes: 2015 is 4, 2016 is 6

Then compute

$$(\text{day} + \text{month code} + \text{year code}) \bmod 7$$

Result of 0 means Sunday, 1 means Monday, etc.

Examples

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Year code: 2015 is 4, 2016 is 6

(day+month code+year code) mod 7

0=Sunday, 1=Monday, etc.

October 2, 2015
$(6 + 2 + 4) \bmod 7 = 5$ (Friday)

May 15, 2016
$(0 + 15 + 6) \bmod 7 = 0$ (Sunday)

Year code for any year

Year code $= (\lfloor A/4 \rfloor + A + C) \bmod 7$

$A =$ Last two digits of year
Julian calendar: $C = -(\text{century number} + 2)$
Gregorian calendar: $C = 0, 5, 3,$ or 1 depending on if century number mod 4 is 0, 1, 2, or 3.

1700s	$C = 5$
1800s	$C = 3$
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Example: 1997

Start with $\lfloor 97/4 \rfloor = 24$
97 mod 7 is 6
$24 + 6 + 1$ is 30
30 mod 7 is <u>2</u>

Thank you.

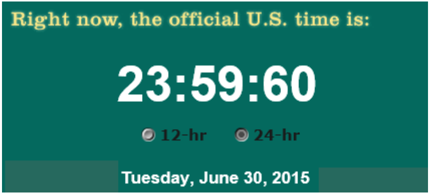
Right now, the official U.S. time is:
23:59:60
12-hr
24-hr
Tuesday, June 30, 2015

Image credits

People on a date: http://trelford.com/Medjool.jpg

Date, the fruit: http://www.goingmobo.com/wp-content/uploads/2011/10/DatingShake.jpg

Calendar: http://mysouthlakenews.com/wp-content/uploads/2011/10/calendarart2.jpg

Calvin and Hobbes calendar:
https://m2.behance.net/rendition/pm/7624743/disp/afcba32b3750adec2711af114ee9f350.jpg

Earth's rotation: http://www.physicalgeography.net/fundamentals/images/earth_rotation.jpg

Celestial equator: https://upload.wikimedia.org/wikipedia/commons/thumb/4/47/Celestial_equator_and_ecliptic.svg/353px-Celestial_equator_and_ecliptic.svg.png

Screenshot from Wikipedia: https://en.wikipedia.org/wiki/Leap_second

Atomic clock: http://news.nationalgeographic.com/content/dam/news/2015/06/25/leapsecond/02leapsecond.ngsversion.1435674600230.adapt.1190.1.jpg

Moon:
https://upload.wikimedia.org/wikipedia/commons/f/f0/Full_Moon_as_Seen_From_Denmark.jpg

Leap second:
https://en.wikipedia.org/wiki/Leap_second#/media/File:Leap_Second_-_30_June_2015.png