Functions

A function takes inputs and transforms them according to some rule into outputs. Many calculus functions are given by formulas like $f(x) = x^2$ or $g(x) = 2x + 3$.

Evaluating functions

Much of the time, evaluating a function means plugging a value into the function. For instance, if $f(x) = x^2$, then $f(4)$ is gotten by plugging 4 into the formula to get $f(4) = 4^2 = 16$.

Sometimes it is useful to plug in things other than numbers. Here are some examples using $f(x) = x^2$

1. $f(a) = a^2$

2. $f(x + h) = (x + h)^2$

Addition, subtraction, etc. of functions

Sometimes when given two functions $f(x)$ and $g(x)$, people will use the notation $(f + g)(x)$ to denote $f(x) + g(x)$. The idea is that $(f + g)(x)$ represents a new function gotten from adding two other functions. For instance, if $f(x) = x^2$ and $g(x) = 2x + 3$, then $(f + g)(x) = x^2 + 2x + 3$.

The notations $(f - g)(x)$, $(fg)(x)$ and $(f/g)(x)$ are defined similarly.

Function composition

Function composition is the important idea of taking the output from one function and plugging it into another. Given two functions $f(x)$ and $g(x)$, the notation for the composition is $(f \circ g)(x)$. Often this is written as $f(g(x))$. Here are a couple of examples.

1. If $f(x) = x^2$ and $g(x) = 2x + 3$, then

$$(f \circ g)(x) = f(g(x)) = f(2x + 3) = (2x + 3)^2.$$

2. If $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x}$, then

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = \frac{1}{\sqrt{x}}.$$

3. If $f(x) = x^2 + 1$ and $g(x) = 4x$, and we want to know $(f \circ g)(3)$, we do the following:

$$(f \circ g)(3) = f(g(3) = f(4 \cdot 3) = f(12) = 145.$$

Exercises

1. For the following, find $f(4)$ and $f(x + h)$.

   (a) $f(x) = 2x + 3$

   (b) $f(x) = \sqrt{x}$

2. For the following, find $(f + g)(x)$, $(fg)(x)$, and $(f \circ g)$.

   (a) $f(x) = x^3$, $g(x) = 1 + x$

   (b) $f(x) = x^2 + 1$, $g(x) = \sqrt{x}$

Answers

1. (a) $f(4) = 2(4) + 3 = 11$ and $f(x = h) = 2(x + h) + 3$

   (b) $f(4) = \sqrt{4} = 2$ and $f(x + h) = \sqrt{x + h}$

2. (a) $(f + g)(x) = x^3 + 1 + x$, $(fg)(x) = x^3(1 + x)$, and $(f \circ g)(x) = f(g(x)) = (1 + x)^3$

   (b) $(f + g)(x) = x^2 + 1 + \sqrt{x}$, $(fg)(x) = (x^2 + 1)\sqrt{x}$, and $(f \circ g)(x) = f(g(x)) = \sqrt{x}^2 + 1 = x + 1$.