Solving Quadratic Equations

Quadratic equations can be solved in a number of ways, including basic algebra and factoring. When in doubt, though, the quadratic formula always works, even if it is a little tedious to use. The quadratic formula says that the solutions to $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here are several examples of solving quadratics.

Example 1 Solve $x^2 - 9 = 0$ for x.

Solution: If a quadratic has an x^2 term but no x term, like this one, then get the x term alone and take a square root. Here we get $x^2 = 9$, and then $x = \pm\sqrt{9}$ or $x = \pm 3$.

The $\pm$ is important, since $\sqrt{9}$ only stands for the positive root 3. But both $x = -3$ and $x = 3$ work in the equation.

Example 2 Solve $16 - 2x^2 = 0$ for x.

Solution: Get the x^2 term alone on one side by adding $2x^2$ to both sides and then dividing by 2 to get $x^2 = 8$. Then take the square root of both sides to get $x = \pm\sqrt{8}$.

Example 3 Solve $x^2 + 5x - 6 = 0$ for x.

Solution: If a quadratic is easy to factor, then that gives a quick solution. In this case, we can factor $x^2 + 5x + 6 = 0$ into $(x - 1)(x + 6) = 0$. Therefore, the solutions are $x = 1$ and $x = -6$.

Example 4 Solve $x^2 - 7x = 0$ for x.

Solution: Factor this into $x(x - 7) = 0$. Then the solutions are $x = 0$ and $x = 7$.

Note that if the equation were $x^2 - 7x = 1$, this would not work. It only works if the right side is 0.

Example 5 Solve $3x^2 + 9x - 5 = 0$ for x.

Solution: If a quadratic is not easily factorable (or if you hate factoring), then use quadratic formula. In this case $a = 3$, $b = 9$, and $c = -5$. Plugging into the formula gives

$$x = \frac{-9 \pm \sqrt{9^2 - (4)(3)(-5)}}{(2)(3)} = \frac{-9 \pm \sqrt{141}}{6}.$$

Example 6 Solve $x^2 + 8 = 0$ for x.

Solution: Sometimes equations have no solution. This is one of those times. Subtracting 8 from both sides gives $x^2 = -8$. However, you can't square a number and get a negative, so there is no solution. The equation does have solutions involving imaginary numbers, but in calculus we don't use those.

Exercises

Solve the following equations for x.

1. $x^2 - 16 = 0$
2. $7 - x^2 = 0$
3. $(x - 1)^2 = 16$
4. $x^2 - 6x - 7 = 0$
5. $x^2 + 9x - 1 = 0$

Answers

1. Add 16 to both sides to get $x^2 = 16$. Take the square root of both sides to get $x = \pm 4$. Alternately, factor the left side into $(x-4)(x+4)$ and set those to equal to 0 separately to get the $x = \pm 4$.

2. Add x^2 to both sides to get $x^2 = 7$. Then take the square root of both sides to get $x = \sqrt{7}$.

3. Take the square root of both sides to get $x - 1 = \pm 4$. That is, $x - 1 = 4$ and $x - 1 = -4$. Solve these to get $x = 5$ and $x = -3$.

4. Factor it into $(x-7)(x+1)$. So we have $x - 7 = 0$ and $x + 1 = 0$, giving $x = 7$ and $x = -1$.

5. This can't be factored. Use the quadratic formula to get $x = \dfrac{-9 \pm \sqrt{9^2 - 4(1)(-1)}}{2(1)} = \dfrac{-9 \pm \sqrt{85}}{2}$.